Winter School on Secure Computation and Efficiency
Bar-Ilan University, Israel 30/1/2011-1/2/2011

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Efficient Secure Computation with an Honest Majority

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Technion

MPC with an Honest Majority

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- **Several potential advantages**
  - Unconditional security
  - Guaranteed output and fairness
  - Universally composable security
  - This talk: efficiency
- **Main feasibility results**
  - Perfect security with t<n/3 [BGW88,CCD88]
  - Statistical security with t<n/2 (assuming broadcast) [RB89]
- **Goal: minimize complexity**
  - Communication
  - Computation

What can we hope for?

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- **Communication**
  - Match insecure communication complexity?
    - Possible (in theory, up to poly(k) overhead) using FHE
    - Big open question in information-theoretic setting
  - A more realistic goal
    - Allow communication for each gate
    - Minimize amortized cost as a function of n
      - Ignore additive terms that do not depend on circuit size
    - Ideally, O(1) bits per gate
- **Computation**
  - O(1) computation per gate?

What can we get?

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- **Essentially what we could hope for**
  - At most polylog(n) overhead
  - Work per party decreases with number of parties!
  - Small price in resilience
  - O(depth) rounds
    - or O(1) rounds with poly(k) overhead and comp. security
- **This talk: several simplifying assumptions**
  - Inputs originate from a constant number of "clients"
  - Security with abort
  - Statistical security against static malicious adversary
  - Small fractional resilience
  - Broadcast
- **Assumptions can be removed**

The model

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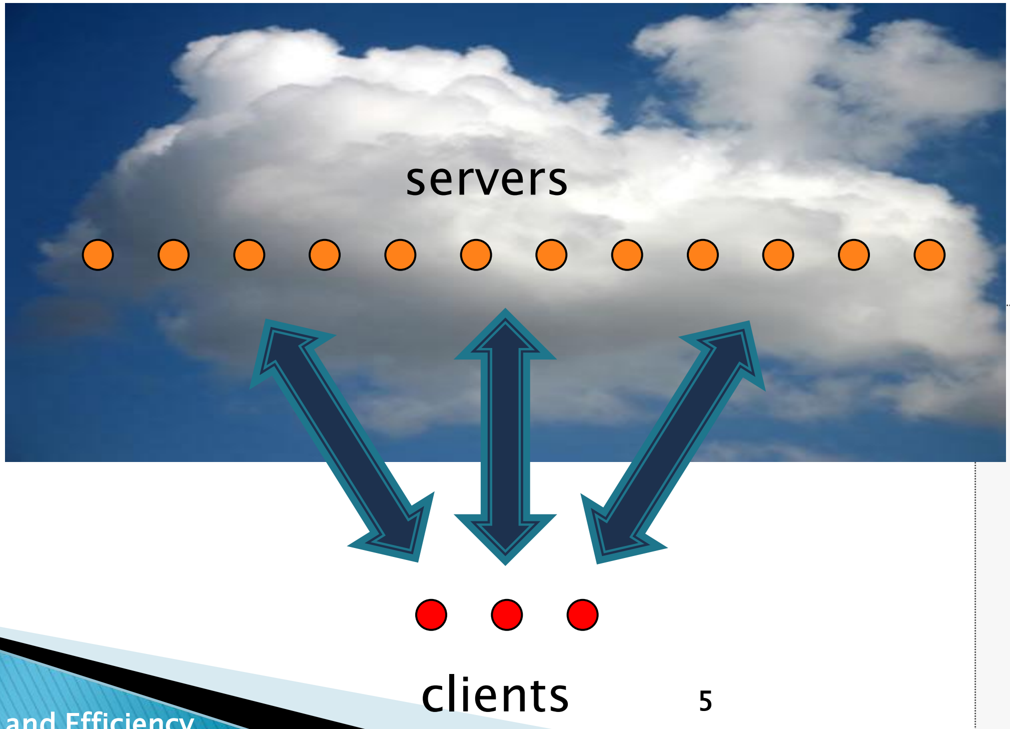
- **m≥2 clients, n servers**
  - Only clients have inputs and outputs
  - Assume m=O(1) in most of this talk
  - Motivated by next talk



The model

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- Synchronous secure point-to-point channels + broadcast
  - Servers only talk to clients
- Malicious adversary corrupting:
  - at most cn servers for some constant $0<c<1/2$
  - any subset of the m clients
- Statistical security with abort

Efficiency in more detail

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- **Functionality represented by a circuit C**
  - Arithmetic circuit over F (with + and x gates)
  - Assume $n \ll |C|$, $depth(C) \ll |C|$
  - Ignore low-order additive terms
- **Goal 1: Minimize communication**
  - Initial protocols [BGW88,CCD88]: $|C| \cdot poly(n)$
  - Best unconditional protocols (this talk): $|C| \cdot O(1)$
  - Using FHE: $|input| + poly(k) \cdot |output|$
- **Goal 2: Minimize computation**
  - Best one can hope for: |C| field ops.
  - Best known (this talk): $|C| \cdot O(\log n)$
    - Assumes large F ($|F| > 2^k$)
    - Polylog(n) overhead possible for any F

Some historical credits

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- Franklin-Yung 92
  - Run several parallel instances of BGW roughly for price of one
  - Small penalty in security threshold
  - Reduces complexity of BGW for some tasks
- Hirt-Maurer 01, Cramer-Damgård-Nielsen 01, Damgård-Nielsen 06
  - Improved overhead of MPC with optimal resilience
- Damgård-I 06, I-Prabhakaran-Sahai 09
  - Extend scope of Franklin-Yung technique to general tasks
  - Optimize computational complexity using technique from Groth 09

Some historical credits

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- Damgård–I–Kroigaard–Nielsen–Smith 08
  efficiency with many clients, boosting resilience using technique of Bracha 87
- Beerliova–Hirt 08, Damgård–I–Kroigaard 10
  perfect security
- Beaver–Micali–Rogaway 90, Damgård–I 05
  constant-round protocols
- Chen–Cramer 06
  using constant-size fields

Starting point: BGW

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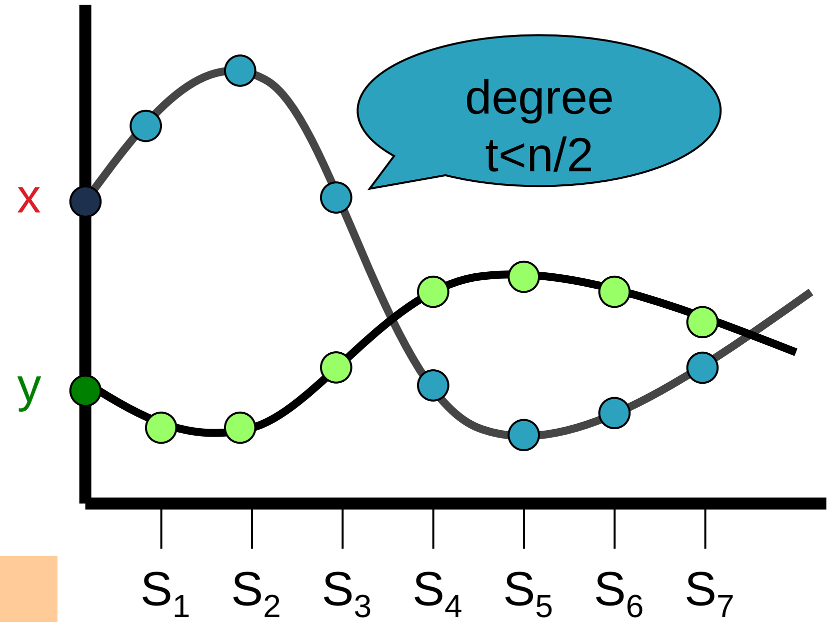
- Secret-share inputs
- Evaluate C on shares
  - Non-interactive addition
  - Interactive multiplication
- Recover outputs



- Secure with $t<n/2$ (semi-honest)
  or $t<n/3$ (malicious)
- Complexity: $|C|\cdot O(n^2)$ (semi-honest)
  $|C|\cdot \text{poly}(n)$ (malicious)

Sources of overhead

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- Each wire value is split into n shares
  - Use "packed secret sharing" to amortize cost
- Multiplication involves communication between each pair of servers
  - Reveal blinded product to a single client
- Expensive consistency checks
  - Efficient batch verification

Share packing

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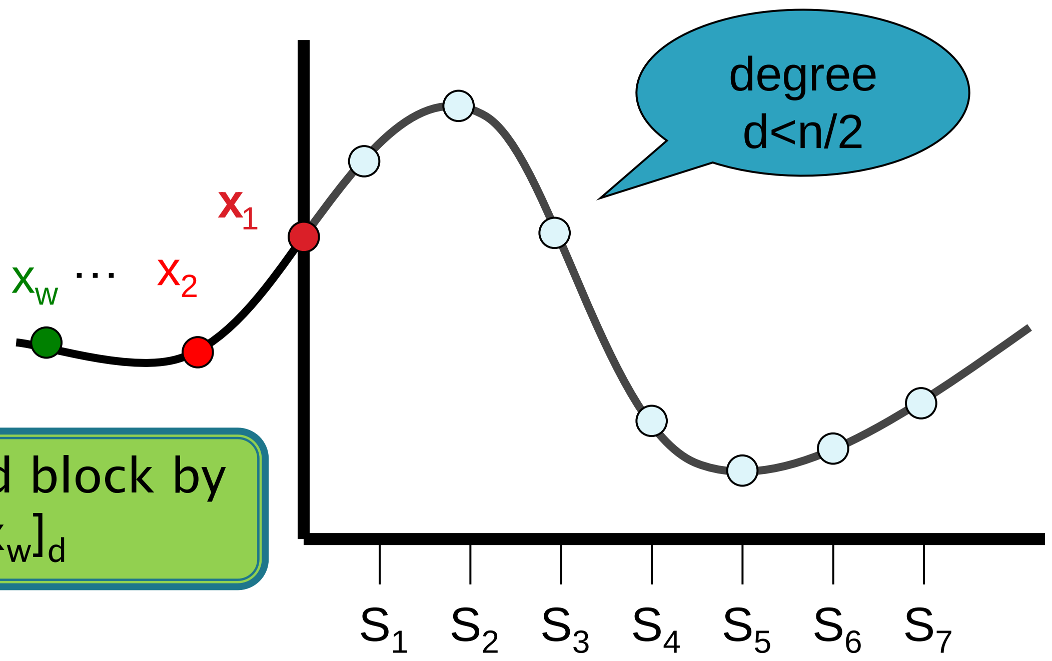

Denote shared block by $[x_1,\ldots,x_w]_d$

- Handle block of w secrets for price of one.
- Security threshold degrades from d to d-w+1
- w=n/10 ➔ Ω(n) savings for small security loss
- Compare with error correcting codes

BGW with share packing?

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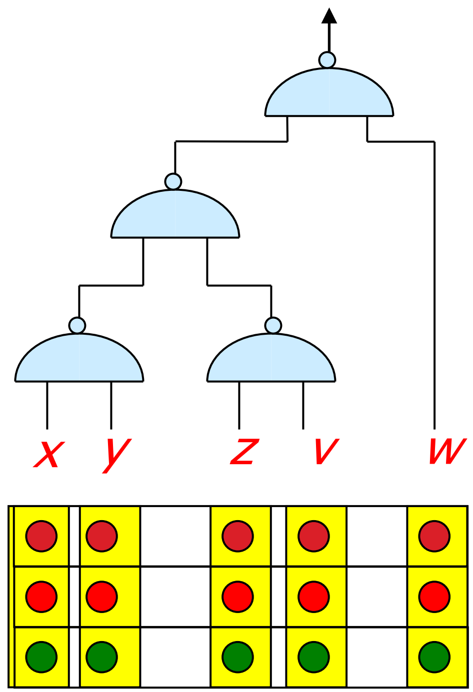
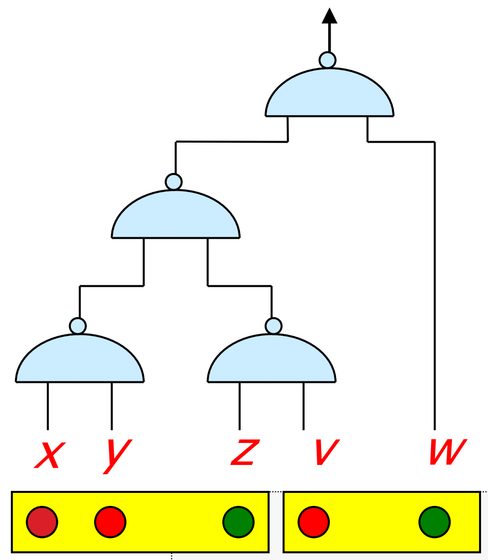
YES: evaluate a circuit on multiple inputs in parallel

NO: evaluate a circuit on a single input



3 inputs

5 blocks



Warmup: Semi-honest, depth 1

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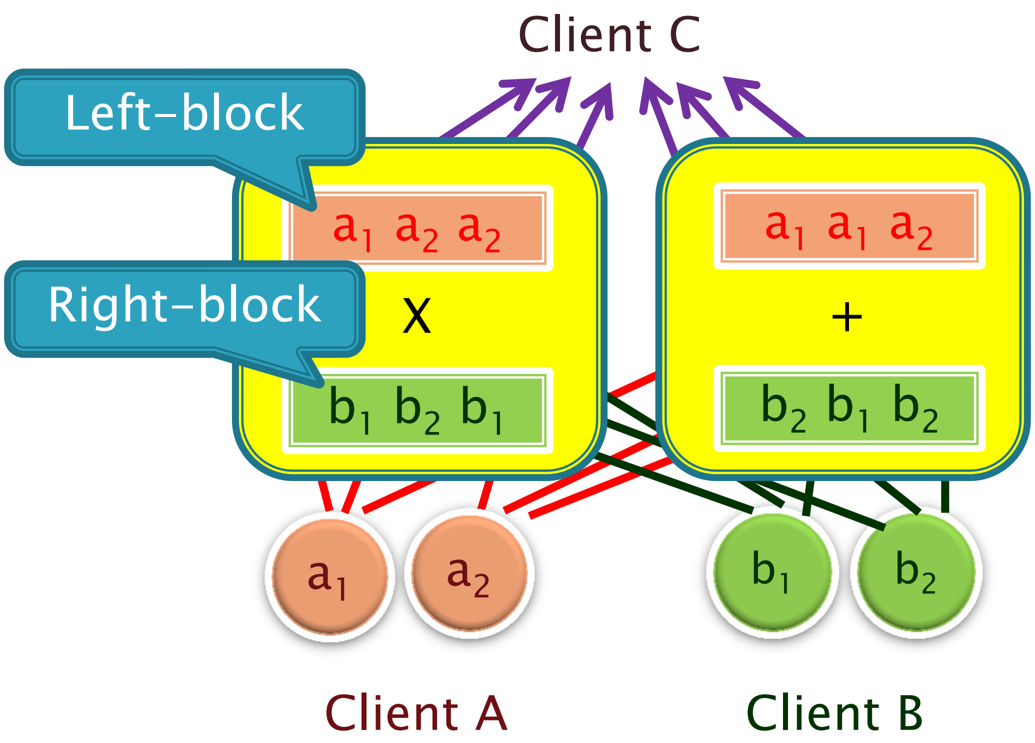

A→S: $p_A=[a_1,a_2,a_2]_d$
$q_A=[a_1,a_1,a_2]_d$
$z_A=[0,0,0]_{2d}$

B→S: $p_B=[b_1,b_2,b_1]_d$
$q_B=[b_2,b_1,b_2]_d$
$z_B=[0,0,0]_{2d}$

S→C: $p_Ap_B+z_A+z_B$
q_A+q_B

- Extends to constant-depth circuits
- Still 2 rounds, $t=\Omega(n)$

Semi-honest, any depth

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- Assume circuit is composed of layers 1,…,H.
- Clients share inputs into $[left^1]_d$ and $[right^1]_d$
- For h=1 to H-1:
  - Clients generate random blocks $[r]_{2d}$, $[left\_r]_d$ and $[right\_r]_d$ replicated according to structure of layer h+1
  - Servers send masked output shares of layer h to Client A: $[y]_{2d}=[left^h]_d*[right^h]_d+[r]_{2d}$ ($* \in \{x,+,-\}$)
  - A decodes, rearranges and reshares y into $[left\_y]_d$, $[right\_y]_d$
  - Servers let
    - $[left^{h+1}]_d=[left\_y]_d-[left\_r]_d$
    - $[right^{h+1}]_d=[right\_y]_d-[right\_r]_d$
- Servers reveal output shares $[left^H]_d*[right^H]_d+[0]_{2d}$

Example

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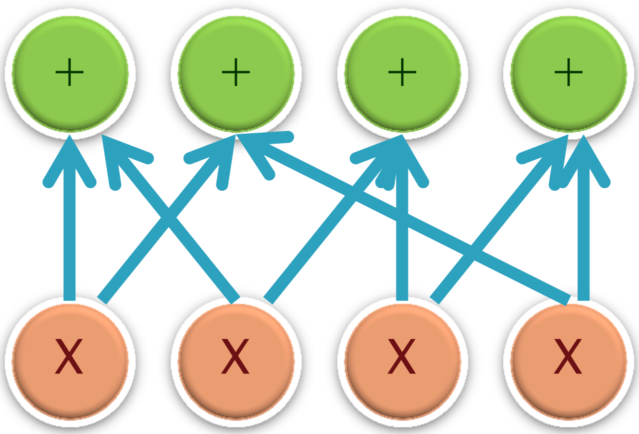

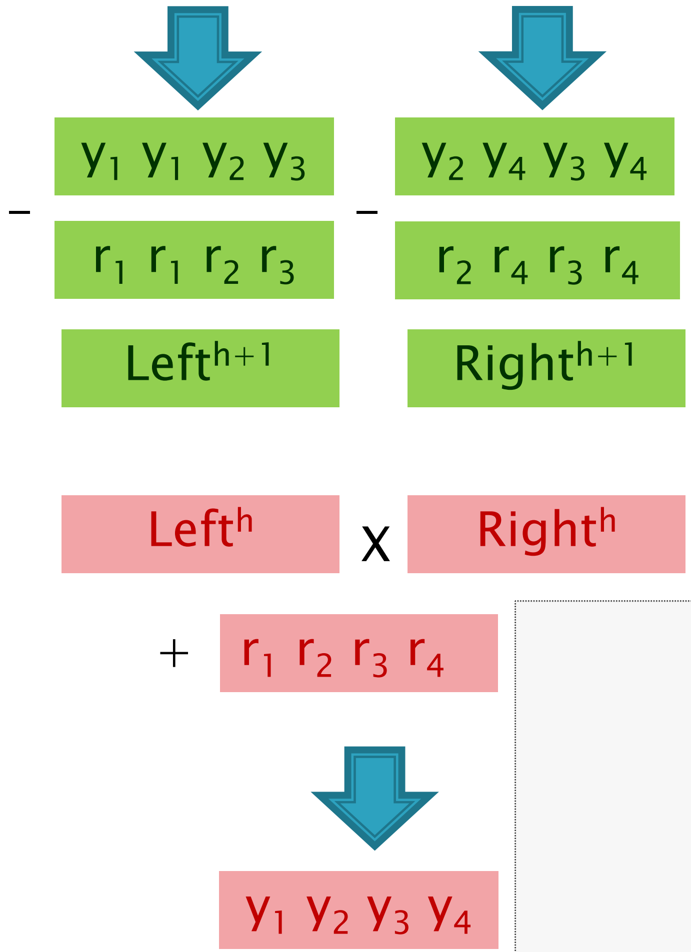

Malicious model

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- Need to protect against $t=\Omega(n)$ malicious servers and $t'<m$ malicious clients.
- Malicious servers handled via error correction
  - Valid shares form a good error-correcting code
  - Error detection sufficient for security with abort
- Malicious clients handled via efficient VSS procedures (coming up)

Efficient statistical VSS

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- Recall: only shoot for security with abort
- Two types of verification procedures
  - Verify that shares lie in a linear space
    - E.g., degree-d polynomials
  - Verify that shared blocks satisfy a given replication pattern
    - E.g., $[r_1,r_1,r_2,r_1]$ $[r_2,r_3,r_1,r_2]$
- Cost is amortized over multiple instances

Verifying membership in a linear space

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- **Suppose Client A distributed a vector v between servers.**
  - S_i holds the i-th entry of v
  - Can be generalized to an arbitrary partition of entries
- **Goal: Prove in zero-knowledge to Client B that v is in some (publicly known) linear space L.**
- **Protocol:**
  - A distributes a random $u \in_r L$
  - B picks and broadcasts $c \in_r F$
  - Servers jointly send $w=cv+u$ to B
  - B checks that $w \in L$
- **ZK: w is a random vector in L**
- **Soundness** (static corruption):
  - consider messages from honest servers
  - $cv+u, c'v+u \in L \rightarrow (c-c')v \in L \rightarrow v \in L$
  - soundness error $\leq 1/|F|$

Amortizing cost

- Can be jointly generated by clients
- Can be pseudorandom (ε-biased)

$c_1 \times v_1$

$c_2 \times v_2$

$c_3 \times v_3$

$c_4 \times v_4$

$c_5 \times v_5$

$+ \; u$

$w \in L$?

Verifying replication pattern

secret	a b c d	e f g h
	inner product	
public	r_1 r_2 s_1 s_2	r_3 s_3 r_4 r_5
	r_2 r_3 s_2 s_3	r_4 s_1 r_5 r_1

a b c d		e f g h		a b c d		e f g h
X	+	X	−	X	−	X
r_1 r_2 s_1 s_2		r_3 s_3 r_4 r_5		r_2 r_3 s_2 s_3		r_4 s_1 r_5 r_1

\+ z_1 z_2 z_3 z_4

Asymptotic efficiency

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- **Communication**
  - O(|C|) field elements (|F|>n) + "low order terms"
  - Low order terms include:
    - Additive term of O(depth·n) for layered circuits
    - depth ➔ # "communicating layer pairs" for general circuits
    - Multiply by k/log|F| for small fields
      (k = statistical security parameter)
- **Computation**
  - Communication x O(log n)
    - Uses FFT for polynomial operations
  - Multiply by k/log|F| for small fields

Boosting security threshold

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- **Goal: small fractional resilience ➔ nearly optimal resilience**
  - without increasing asymptotic complexity!
- **Solution: server virtualization**
  - Example: 0.01n-secure Π ➔ 0.33n-secure Π'
  - Pick n committees of servers such that
    - Each committee is of size s=O(1)
    - If 0.33n servers are corrupted, then > 99% of the committees have < s/3 corrupted members
    - Choose committees at random, or use explicit constructions
- **Π' uses s-party BGW to simulate each server in Π by a committee**
  - Overhead poly(s)=O(1)

Using constant-size fields

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- Consider a boolean circuit C with |C|» depth
- Previous protocol requires |F|>n
  - O(|C| logn) bits of communication
- Can we get rid of the logn term?
- Yes, using algebraic-geometric codes
  - Field size independent of n
  - Small fractional loss of resilience
  - Asymptotically optimal protocols for natural classes of circuits

Other extensions

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- **Many clients**
  - Previous protocol required generating secret blocks
  - Easy to implement by summing blocks generated by all clients
  - Overhead can be amortized if only a constant fraction of clients are corrupted
    - Requires converting circuit into a "repetitive" form
  - Gives protocols with polylog(n) overhead in standard n-party setting with $t=\Omega(n)$.
- **Perfect security**
  - Use efficient variant of BGW VSS with share packing

Constant-round protocols

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- **BMR90: Constant-round version of BGW**
  - Uses garbled circuit technique
  - Black-box use of PRG in semi-honest model (Benny's talk)
  - Non-black-box use of PRG in malicious model
    - Required for zero-knowledge proofs involving "cryptographic relations"
    - In BMR paper: distributed ZK proofs of consistency of seed with PRG output
- **DI05: Black-box use of PRG in malicious model**
  - Uses threshold symmetric encryption

Conclusions

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- **An honest majority can be useful**
  - Unconditional, composable security
  - Fairness
  - Efficiency
- **Open efficiency questions**
  - Break circuit size communication barrier for unconditional security
  - Constant computational overhead
  - Improve additive terms
  - Better constant-round protocols
    - O(1) PRG invocations per gate?
  - Practical efficiency