Lookup Arguments

The 13th BIU Winter School on Cryptography

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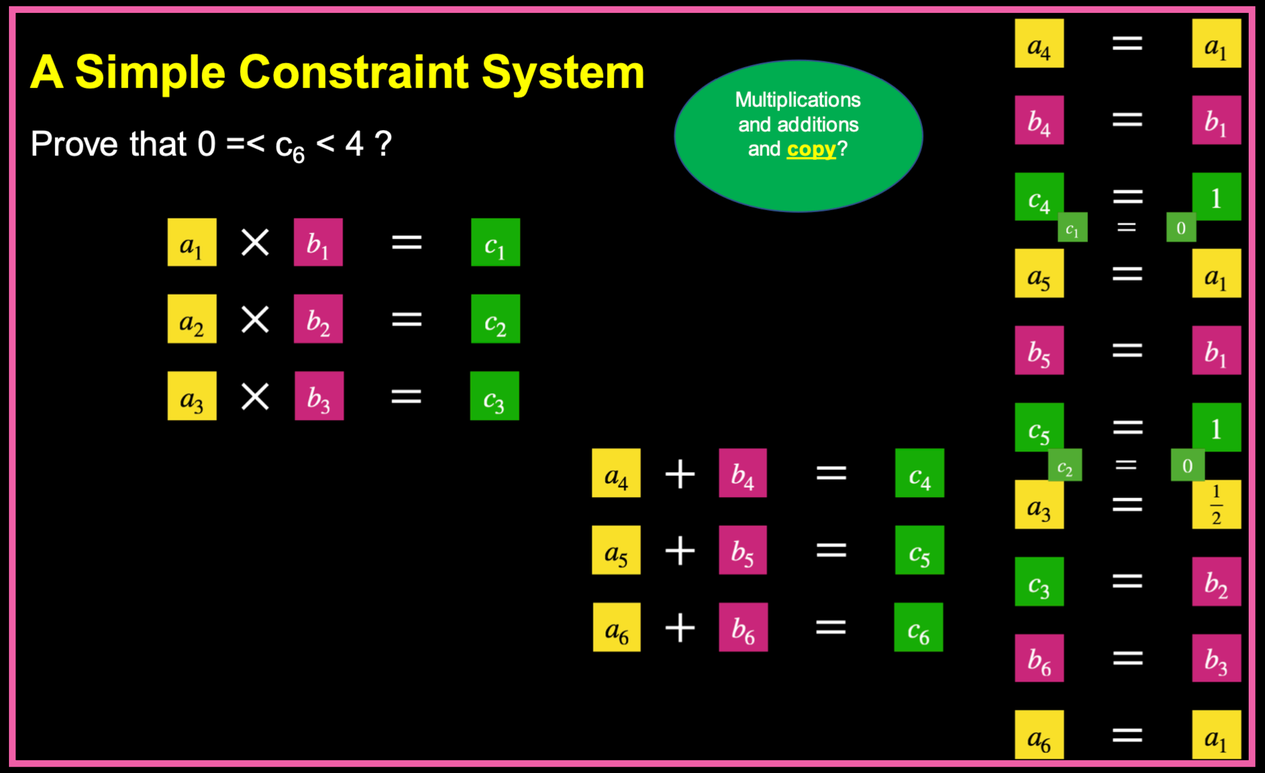
Previously

- Described Plonkish Arithmetisation System in detail:
  - Addition and multiplication constraints
  - Copy constraints
  - Selector polynomials
  - Custom Constraints
- Now we will discuss lookup constraints

Revisiting Simple Constraint System

Prove that $0 \leq x < 4$

Quite a lot of constraints



Revisiting Simple Constraint System

Prove that $0 \leq x \leq 4$

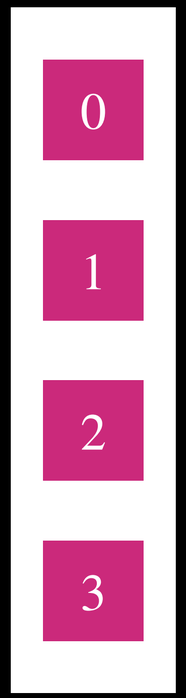
Alternatively

- Prove that x is inside table T

Witness value

x

Table T of allowed range



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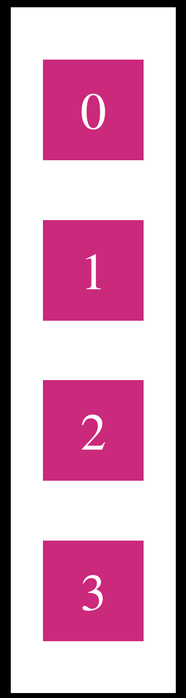

- Prove that x is inside table T
- Polynomial $t(X)$ such that:
  - $t(1) = 0$
  - $t(2) = 1$
  - $t(3) = 2$
  - $t(4) = 3$

Revisiting Simple Constraint System

Prove that $0 \leq x \leq 4$

From 6 constraints + many copy constraints to 1 custom constraint

Alternatively

Witness value

x

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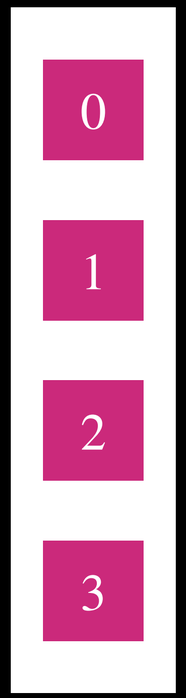

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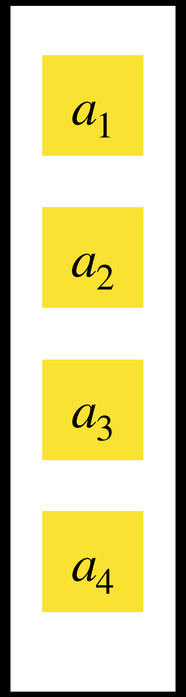
- Prove that $x \in \{t(1), t(2), t(3), t(4)\}$

Revisiting Simple Constraint System

Almost free range constraints

Witness values



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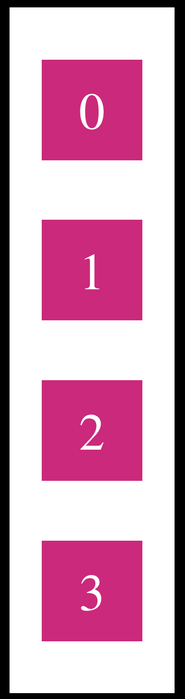

- Prove that a_1, a_2, a_3, a_4 is inside table T
- Polynomial $t(X)$ such that:
  - $t(1) = 0$
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- Witness polynomial $a(X)$ such that:
  - $a(1) = a_1$
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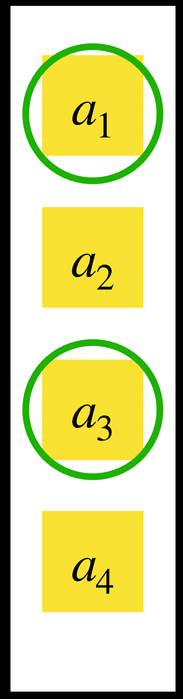
- Prove that $a(i) \in \{t(1), t(2), t(3), t(4)\}$ for $i \in \{0,1,2,3\}$

Revisiting Simple Constraint System

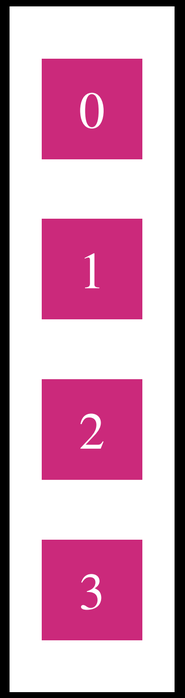
Selector Polynomials

Witness values



We will ignore selector polynomials in this talk.

Table T of allowed range



- Prove that a_1, a_3 is inside table T
- Polynomial $t(X)$ such that:
  - $t(1) = 0$
  - $t(2) = 1$
  - $t(3) = 2$
  - $t(4) = 3$
- Witness polynomial $a(X)$ such that:
  - $a(1) = a_1$
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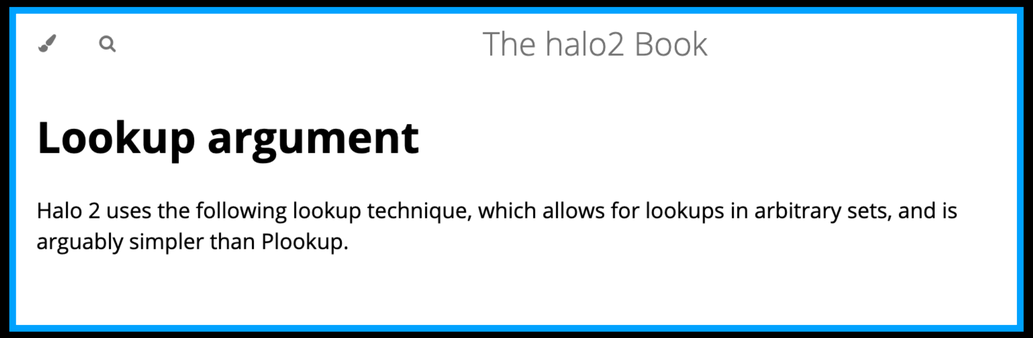
  - $a(4) = a_4$
- Prove that $a(i) \in \{t(1), t(2), t(3), t(4)\}$ for $i \in \{0,2\}$

The Halo2 Strategy

Want to Prove a Lookup Argument.



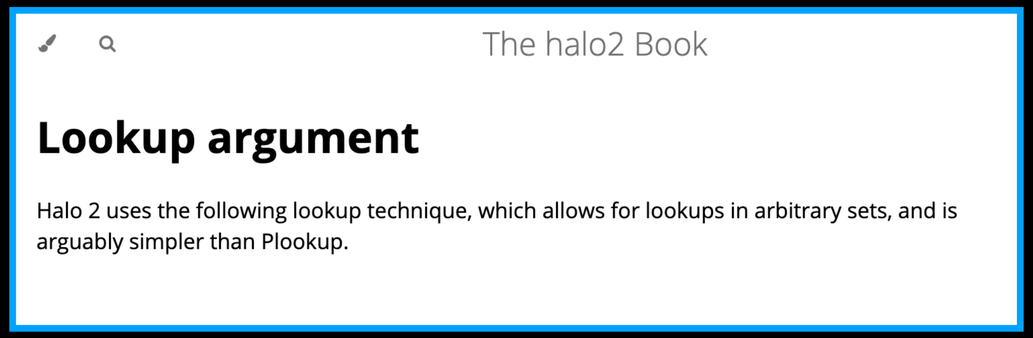
https://zcash.github.io/halo2/design/proving-system/lookup.html

The Halo2 Strategy

Want to Prove a Lookup Argument.

Prove that

$a(i) \in \{t(1), t(2), t(3), t(4)\}$ for $i \in \{0,1,2,3\}$



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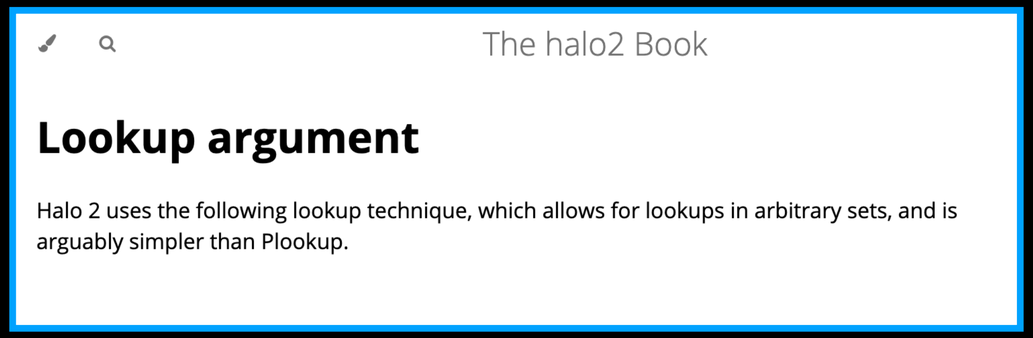

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Prove that

$a(i) \in \{t(1), t(2), t(3), t(4)\}$ for $i \in \{0,1,2,3\}$

- We will try and fail to use a copy argument to prove our lookup constraint.
- We will try and fail to use a permutation argument to prove our lookup constraint.
- We will then discuss how Halo2 actually does it.

The Halo2 Strategy

Can we use a copy argument?

- Polynomial $t(X)$ such that:
  - $t(1) = 0$
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- Witness polynomial $a(X)$ such that:
  - $a(1) = a_1$
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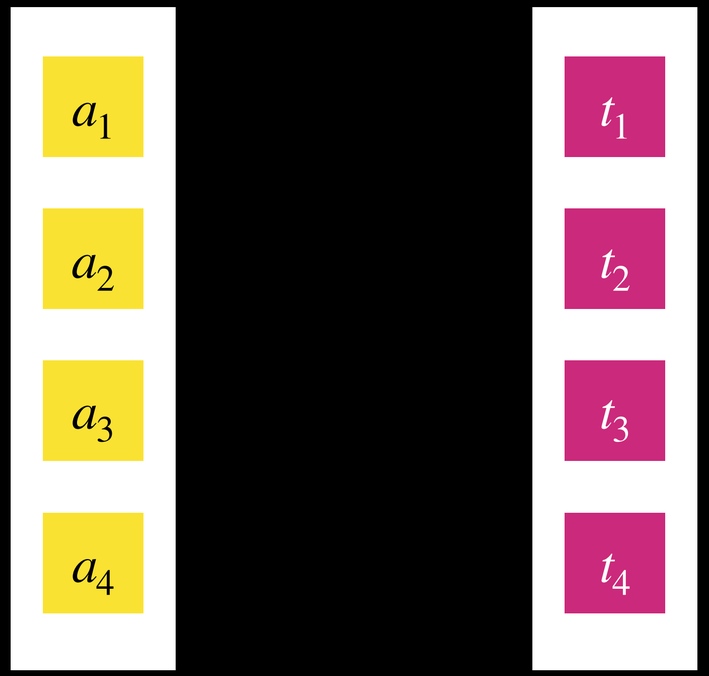

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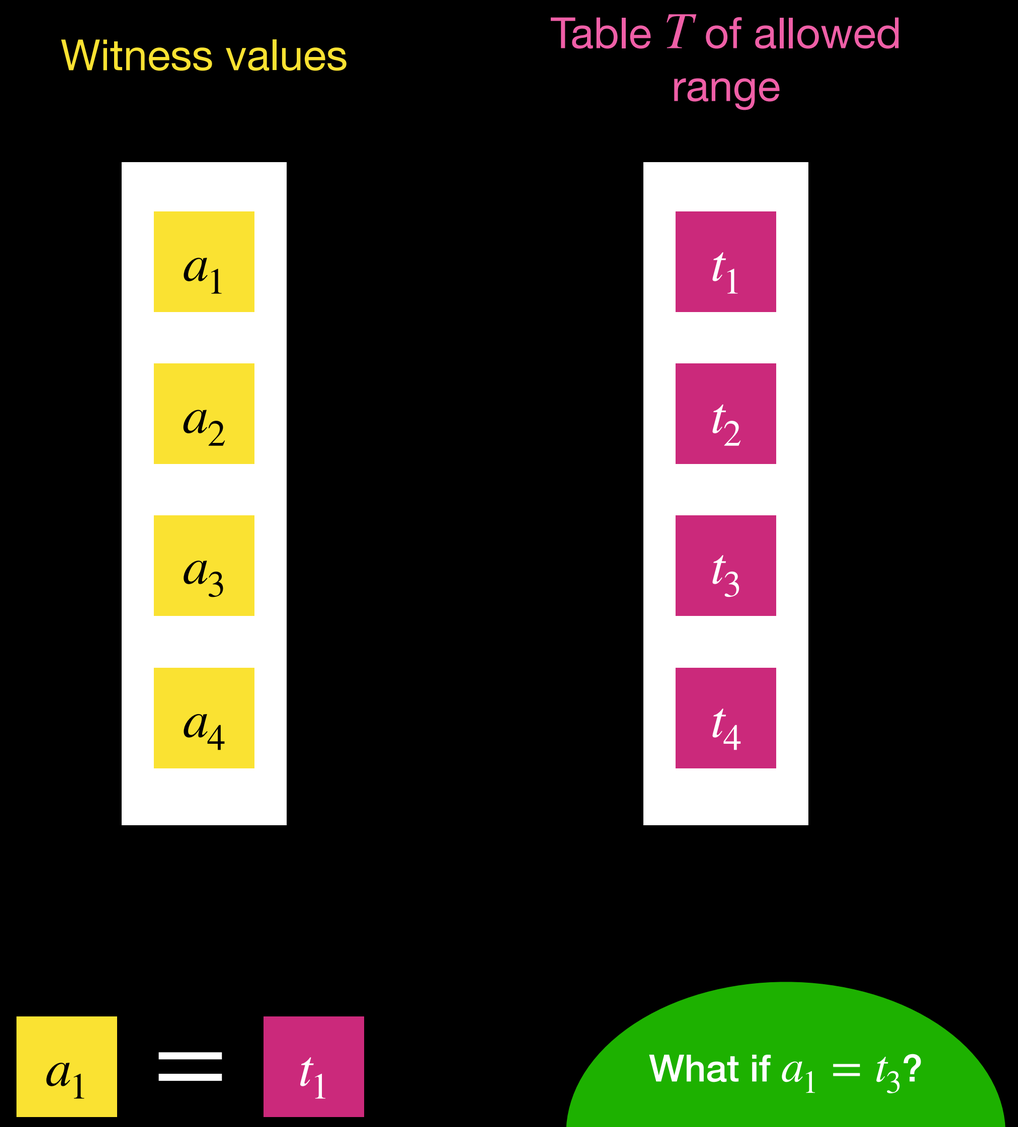

The Halo2 Strategy

Can we use a permutation argument?

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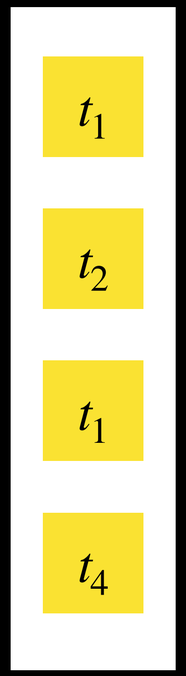
Witness values



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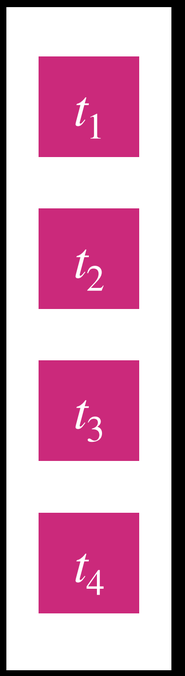

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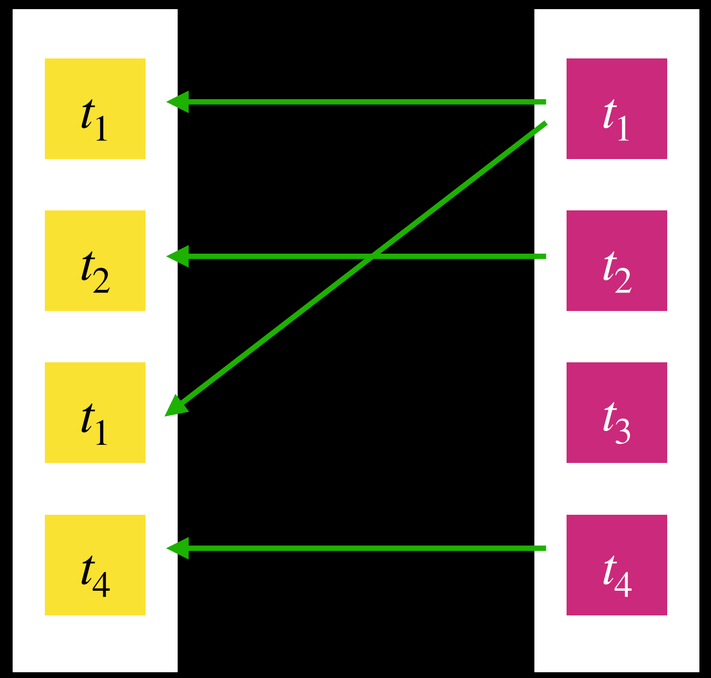

What if there are repeated witness values?

Then permutation is not satisfied.

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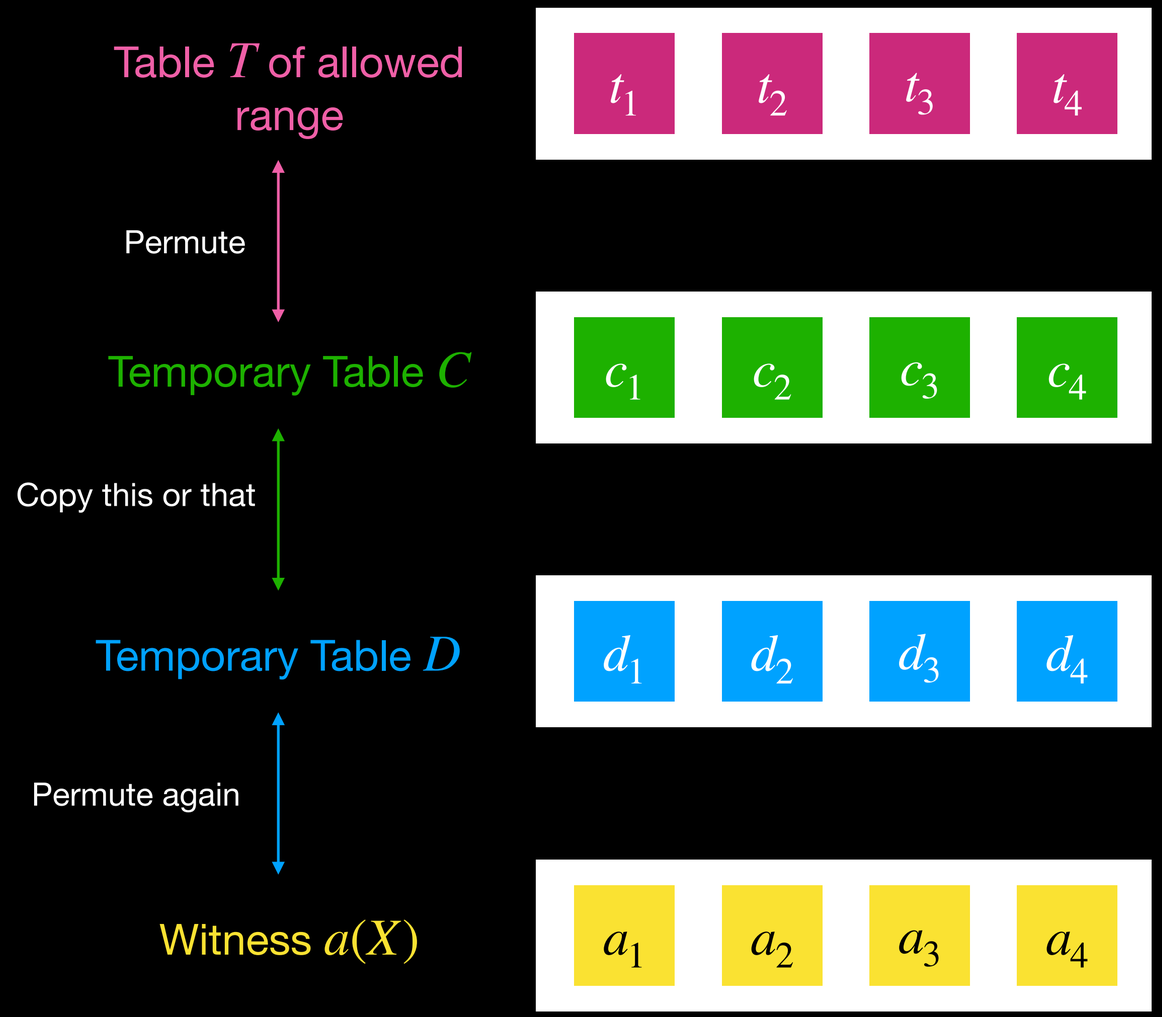

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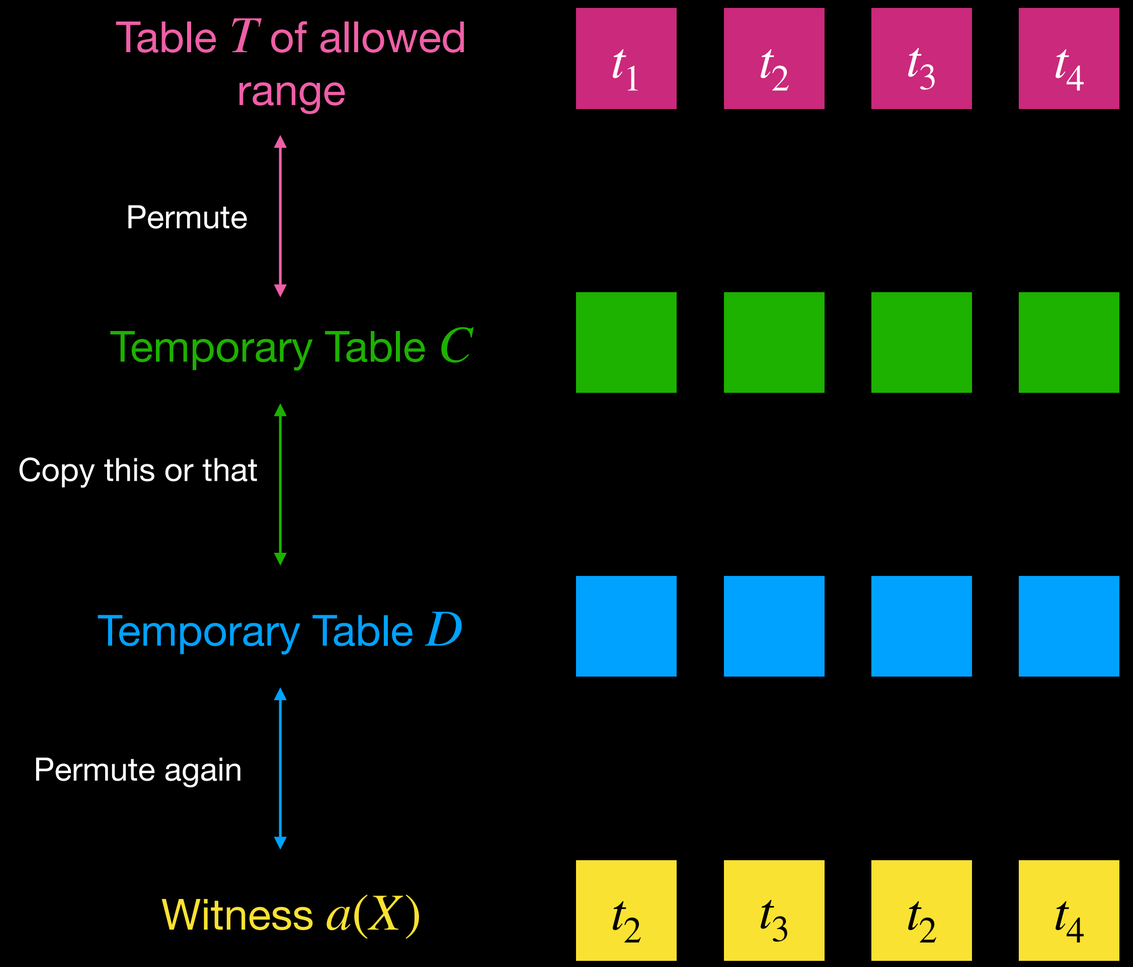

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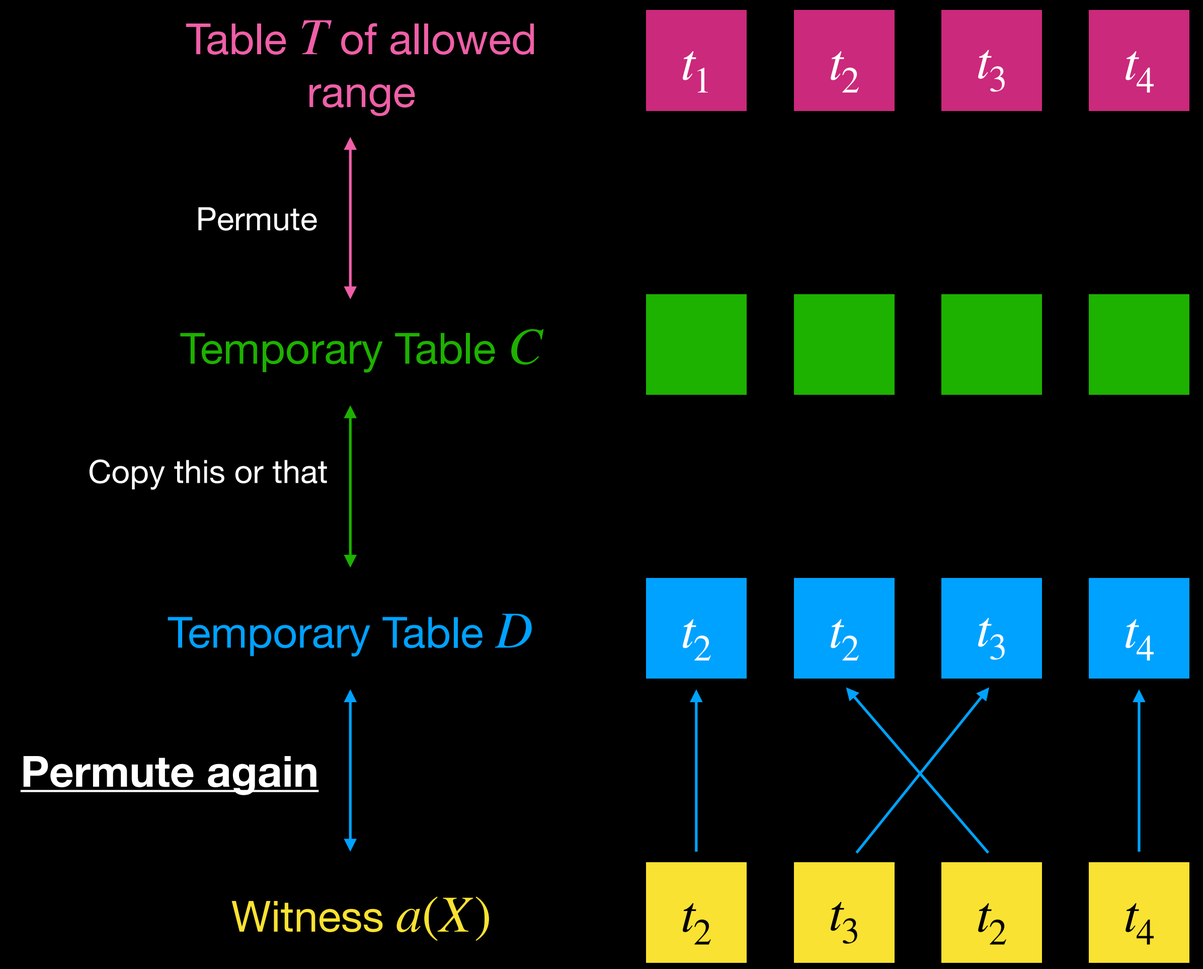

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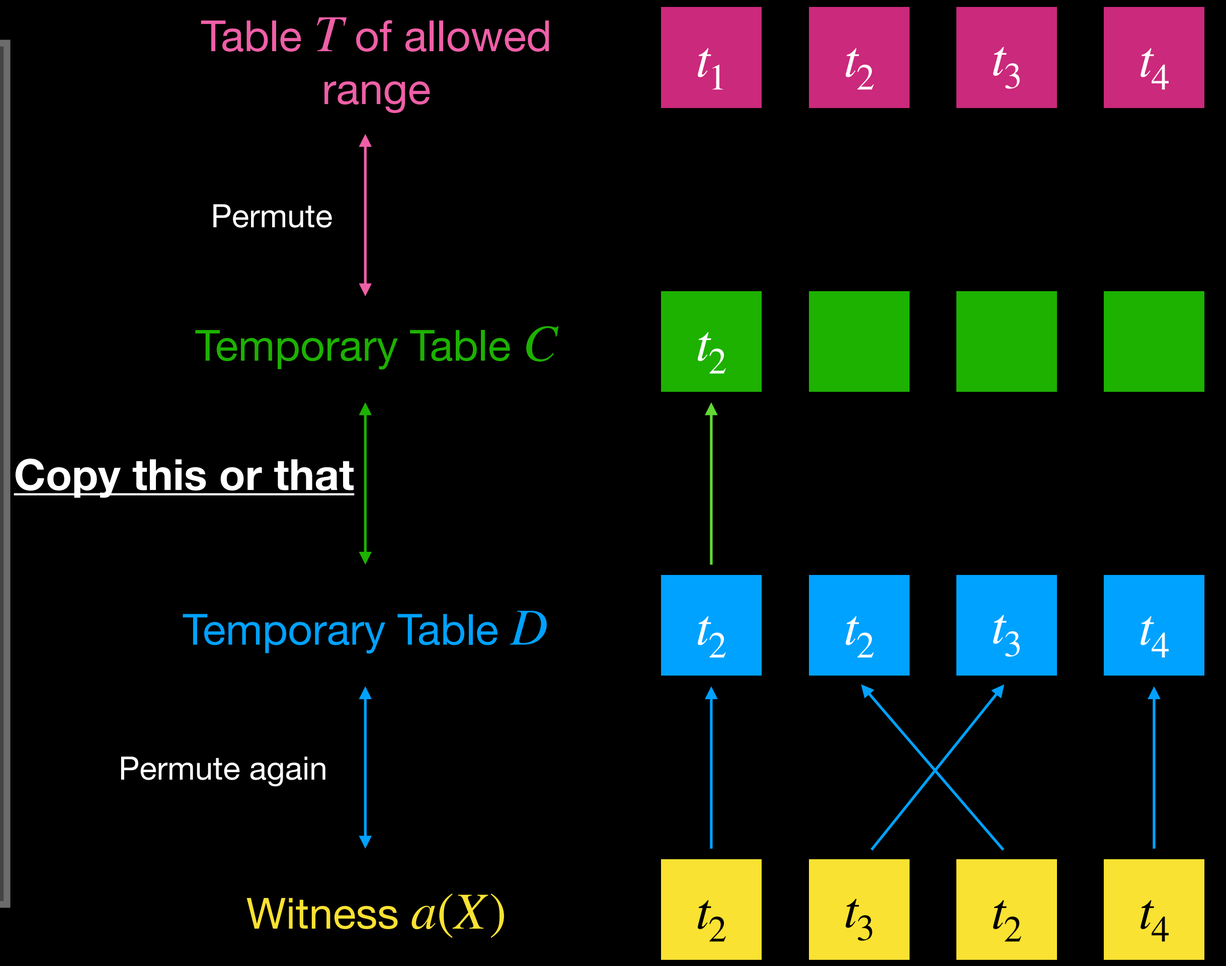

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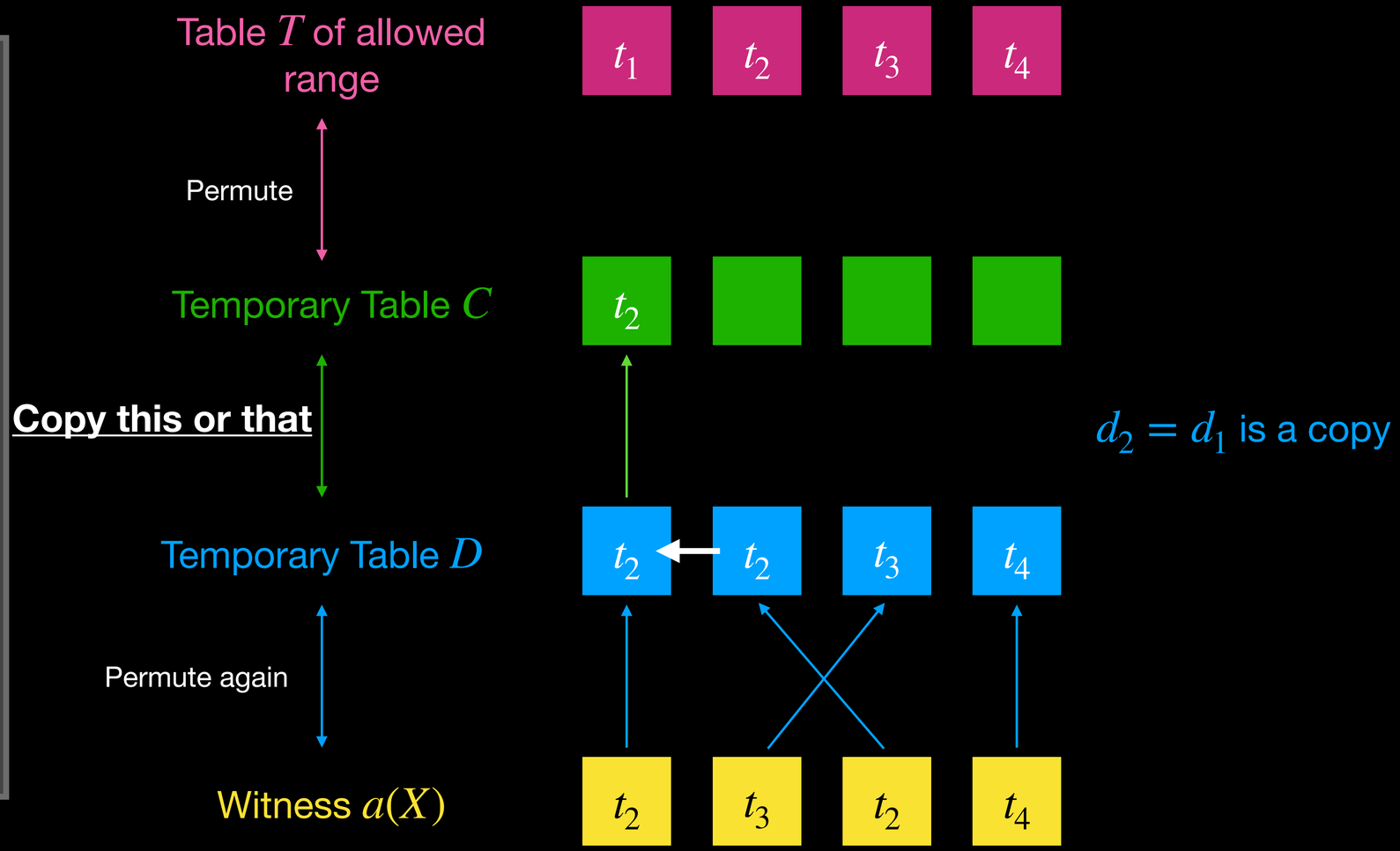

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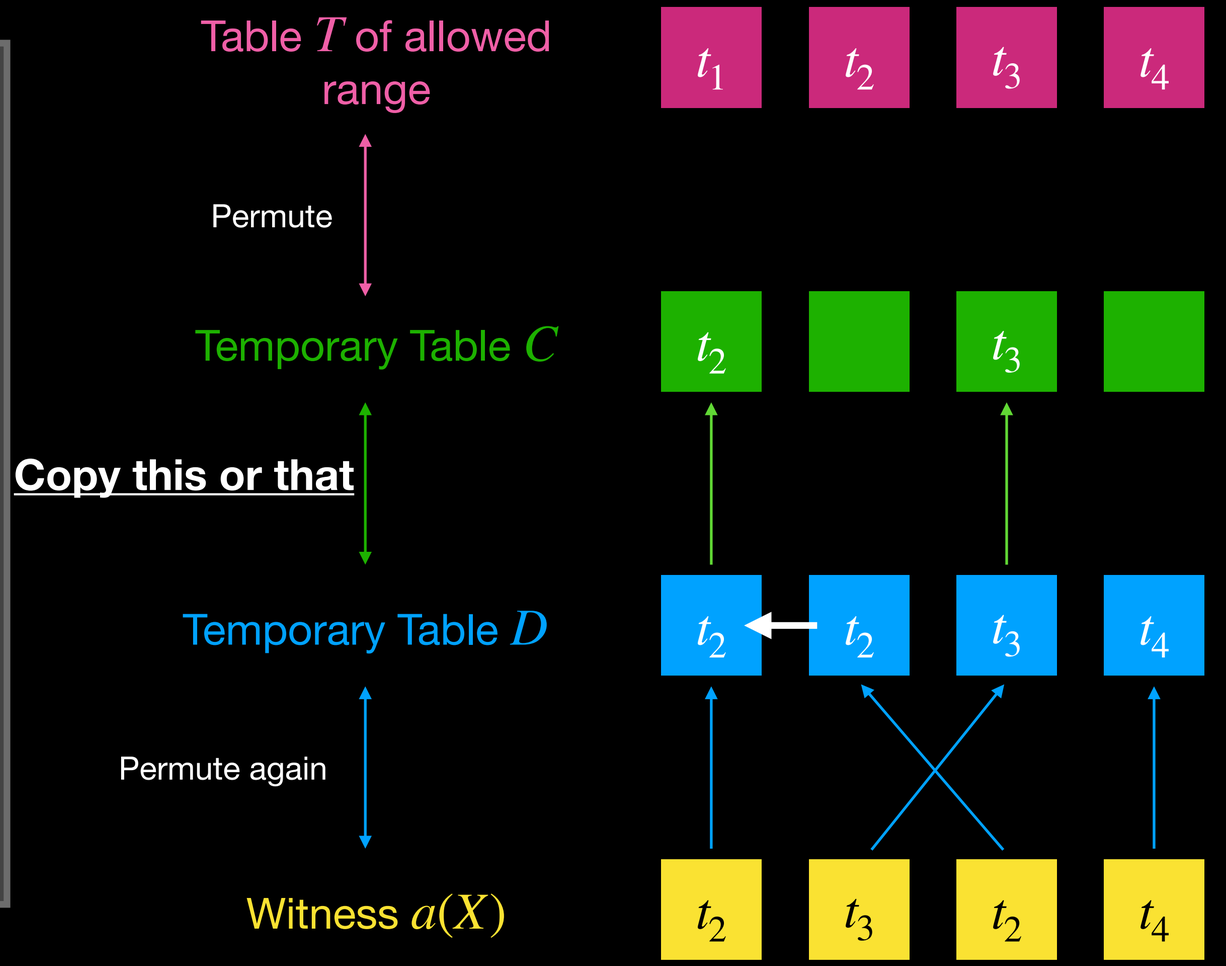

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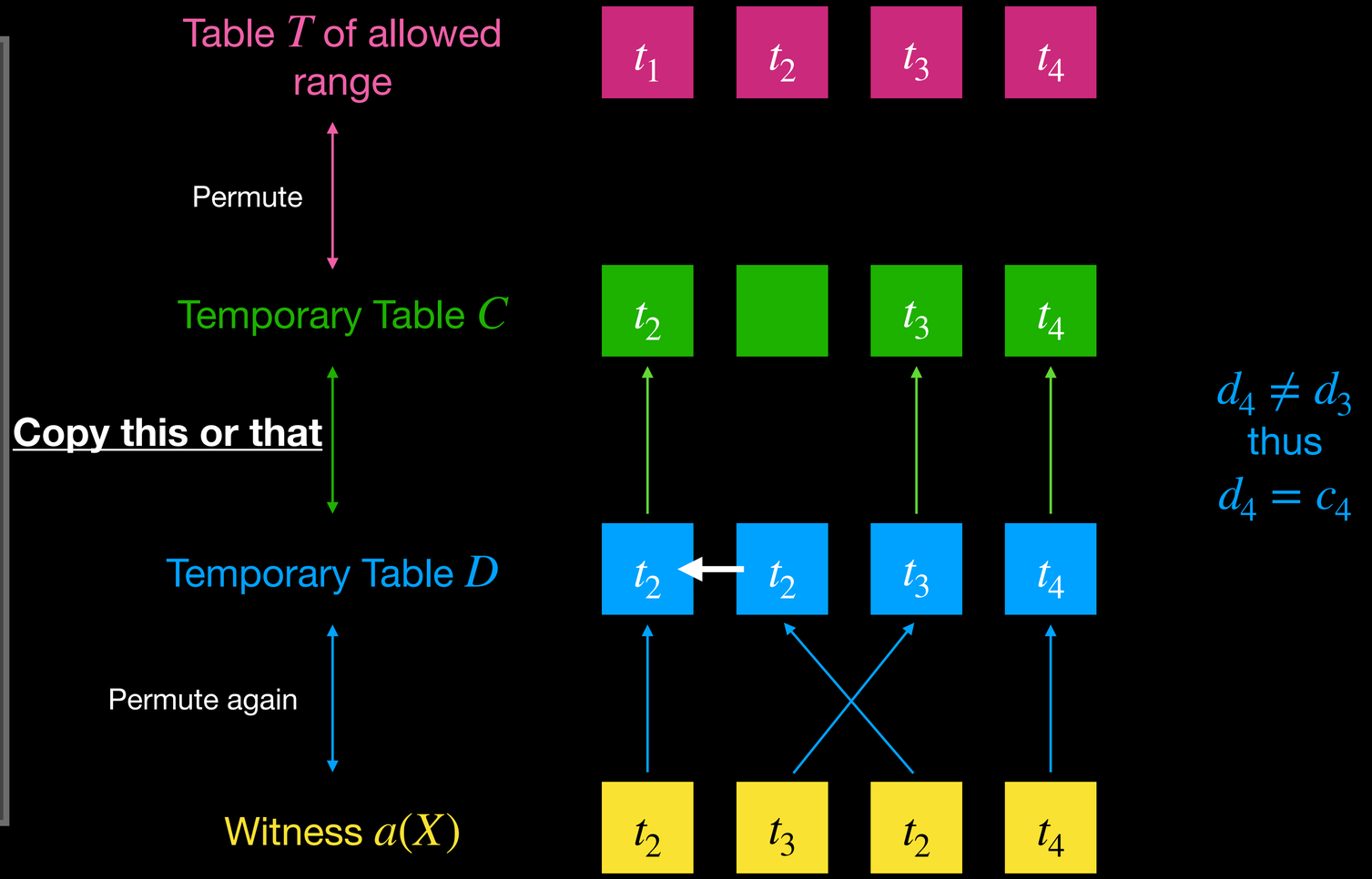

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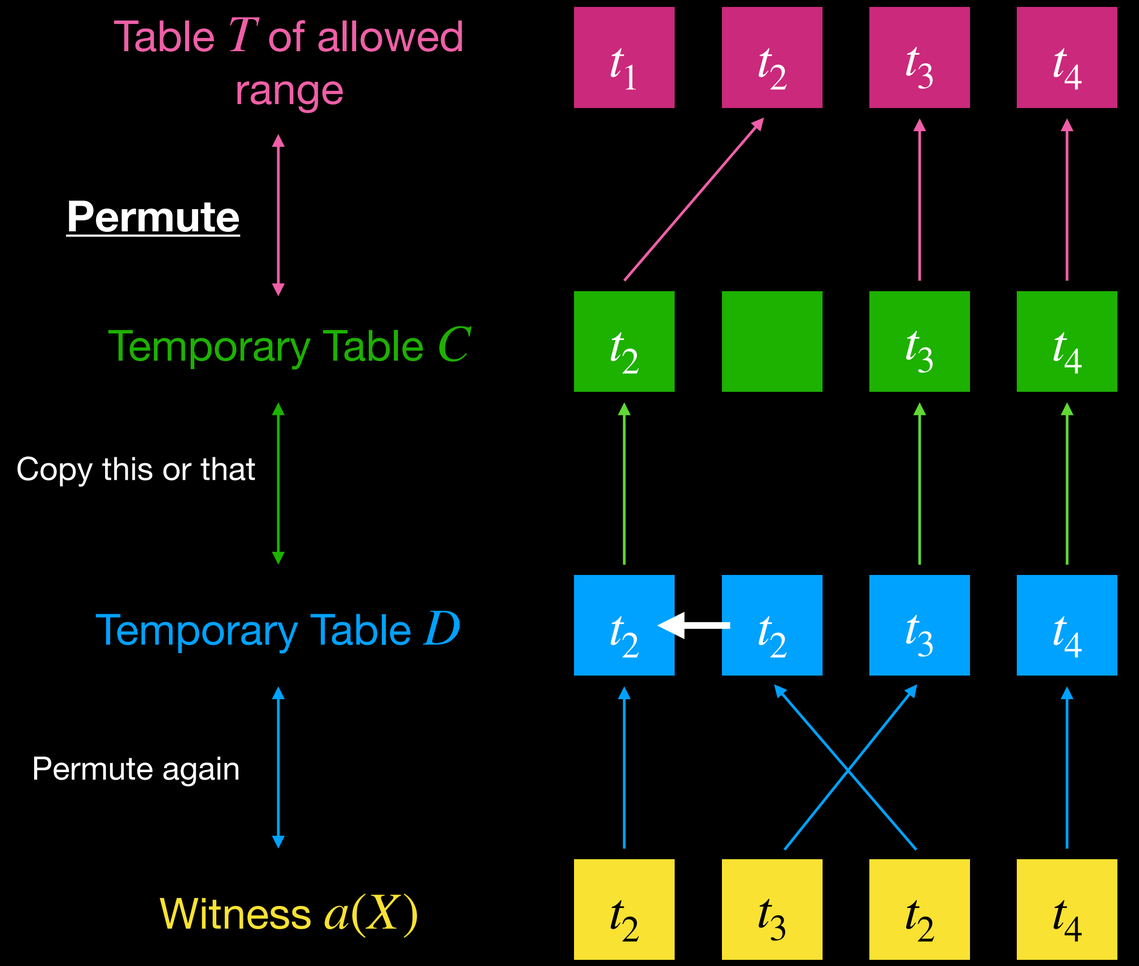

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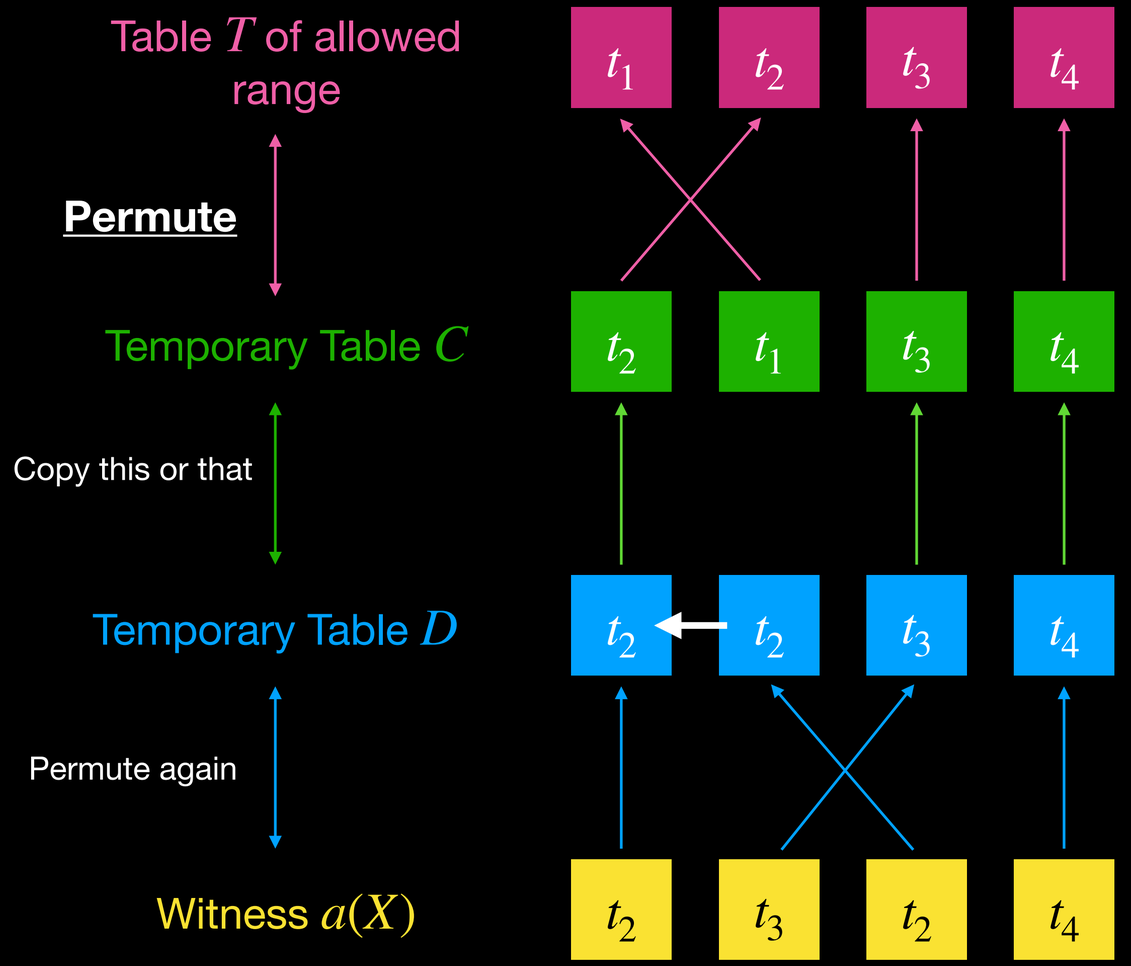

The Halo2 Strategy

Can we use a permutation argument?

Permutation arguments discussed in prior talks.

- Polynomial $t(X)$ such that:
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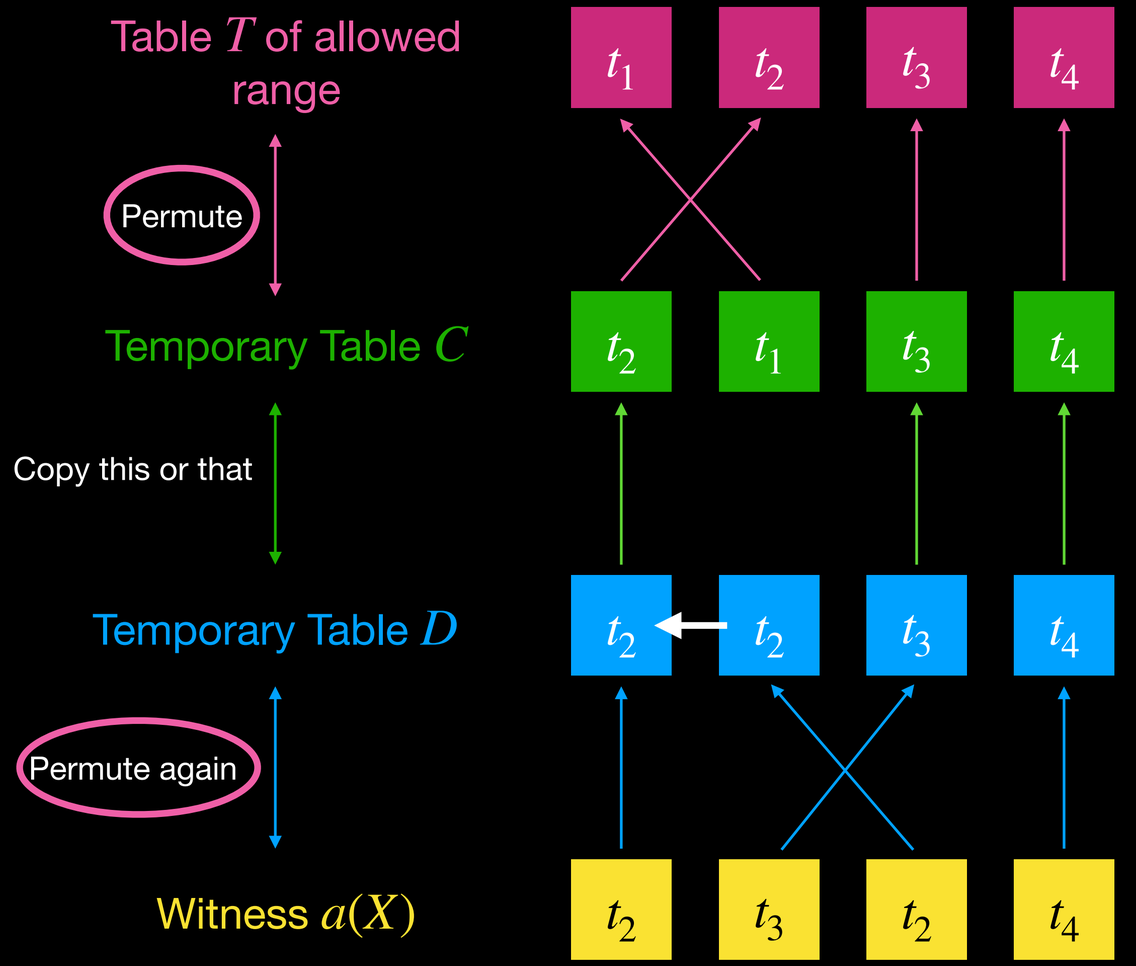
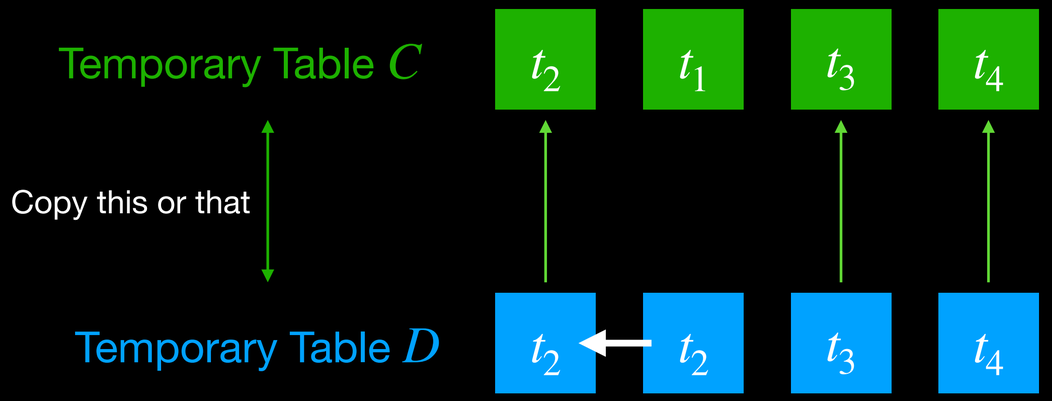


The Halo2 Strategy

Copy this or that constraints?



- Aim to show that $c(X)$, $d(X)$ is such that:
  - $c(i) = d(i)$
  - OR $d(i+1) = d(i)$
- Prove that
  - $b(X)[c(X) - d(X)] + (\mathrm{id}(X) - b(X))[d(X) - d(X-1)] = q'(X)z(X)$
- Prove that
  - $b(X)(\mathrm{id}(X) - b(X)) = q(X)z(X)$

The Halo2 Strategy

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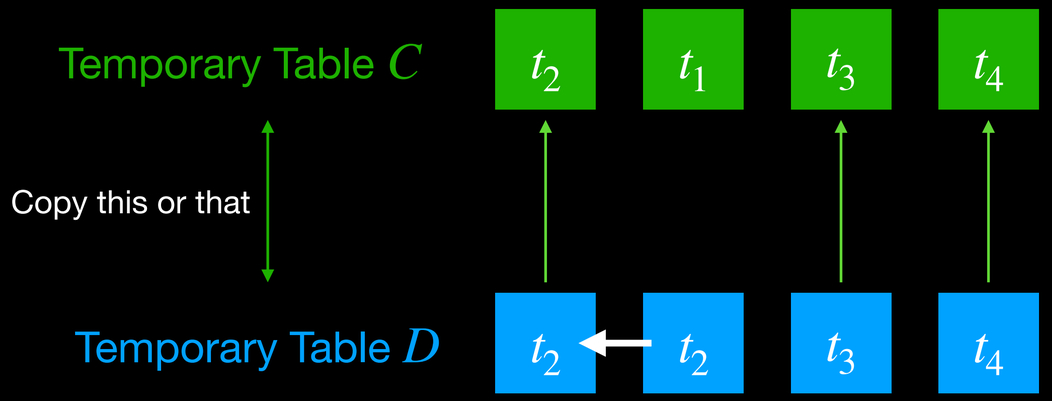

- Aim to show that $c(x),\ d(X)$ is such that:
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  - OR $d(i+1) = d(i)$
- Prove that
  - $[c(X) - d(X)] = q'(X)z(X)$
- Prove that
  - $b(X)(\mathrm{id}(X) - b(X)) = q(X)z(X)$

When $c(i) = d(i)$ have $c(i) - d(i) = 0$

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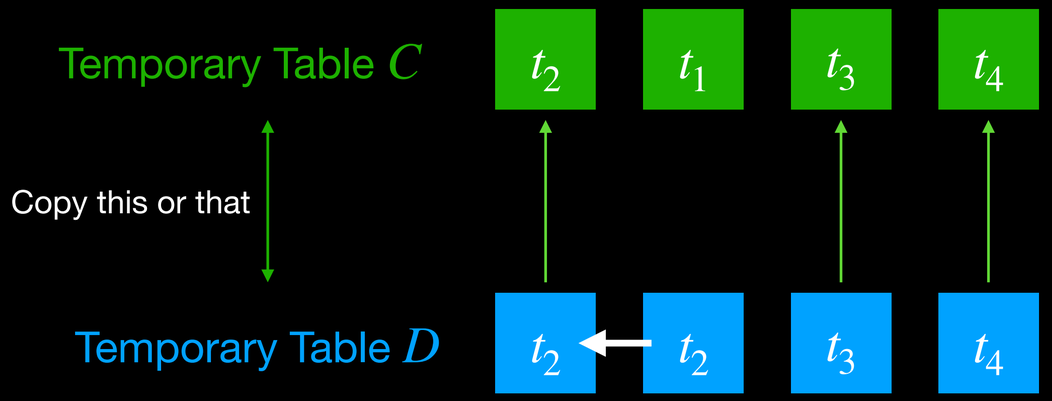

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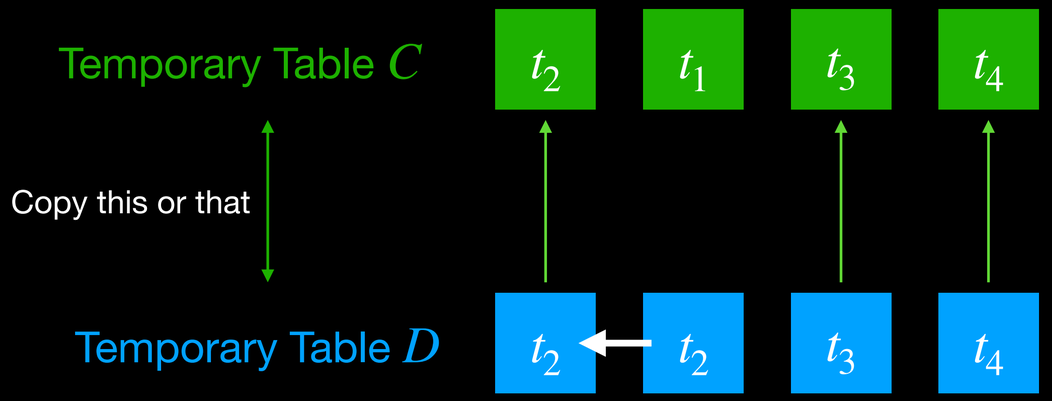
Recall that $z(X) = (X-1)(X-2)(X-3)(X-4)$

Vanishing polynomial

When $c(i) = d(i)$ have $c(i) - d(i) = 0$

The Halo2 Strategy

Copy this or that constraints?



- Aim to show that $c(x),\ d(X)$ is such that:
  - $c(i) = d(i)$
  - OR $d(i+1) = d(i)$
- Prove that
  - $[c(X) - d(X)] = q'(X)z(X)$
- Prove that
  - $b(X)(\mathrm{id}(X) - b(X)) = q(X)z(X)$

Recall that $z(X) = (X-1)(X-2)(X-3)(X-4)$

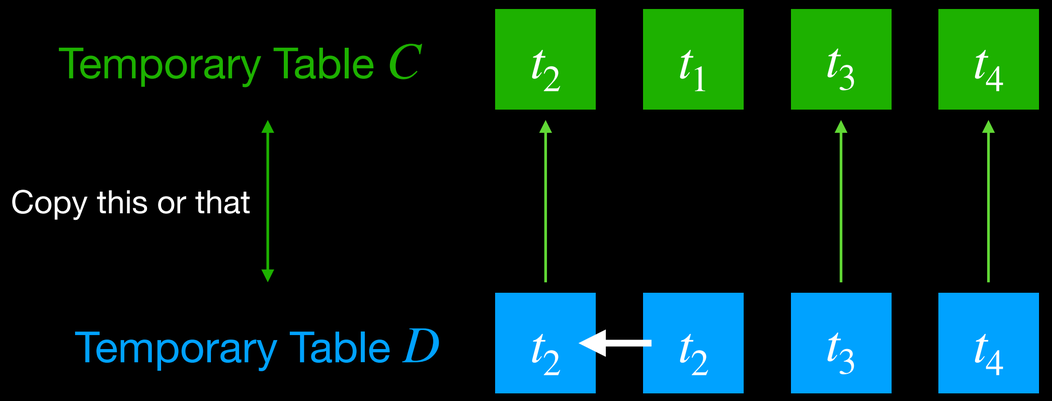
Vanishing polynomial

When $c(i) = d(i)$ have $c(i) - d(i) = 0$

Then copy this or that is not satisfied when $d(i+1) = d(i)$

The Halo2 Strategy

Copy this or that constraints?



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When $c(i) = d(i)$ have $b(i) = 1$

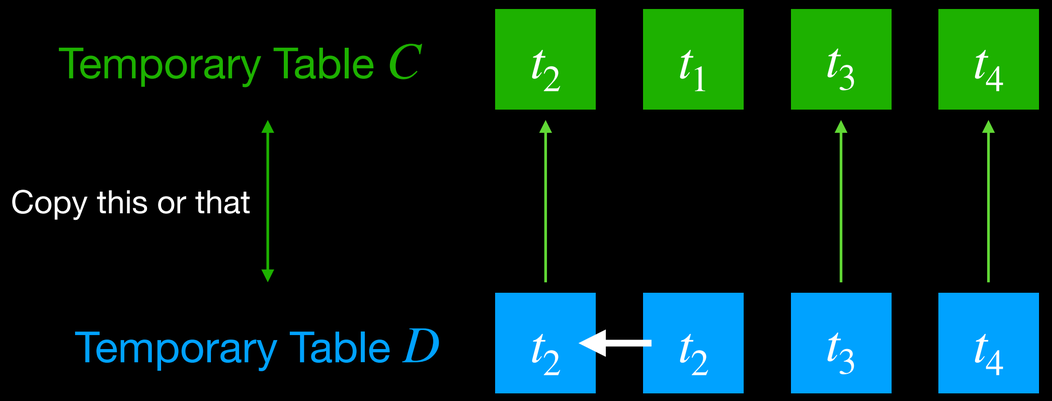
When $c(i) \neq d(i)$ have $b(i) = 0$

The identity polynomial $\mathrm{id}(i) = 1$

The vanishing polynomial $z(i) = 0$

The Halo2 Strategy

Copy this or that constraints?



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Then copy this or that is always satisfied when $b(i) = 0$

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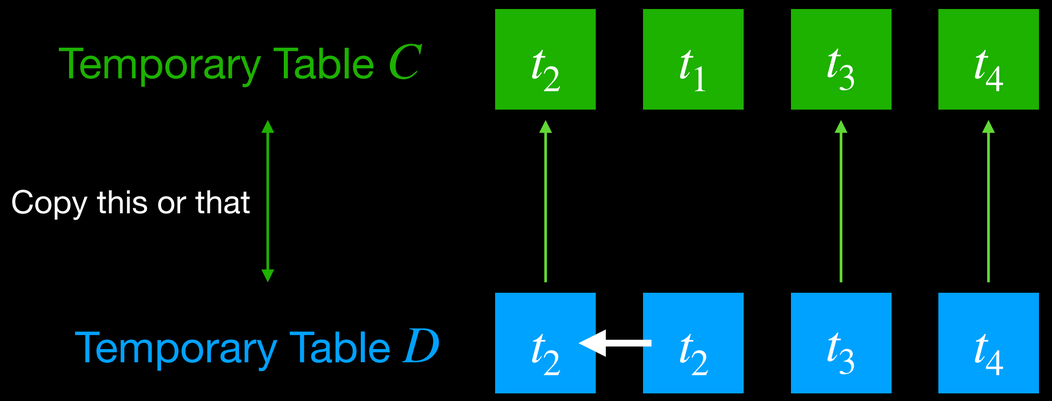
The identity polynomial $\mathrm{id}(i) = 1$

The vanishing polynomial $z(i) = 0$

Polynomial $b(X)$ *is such that* $b(i)(1 - b(i)) = 0 \Rightarrow b(i) \in \{0,1\}$

The Halo2 Strategy

Copy this or that constraints?



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$b(i) \in \{0,1\}$

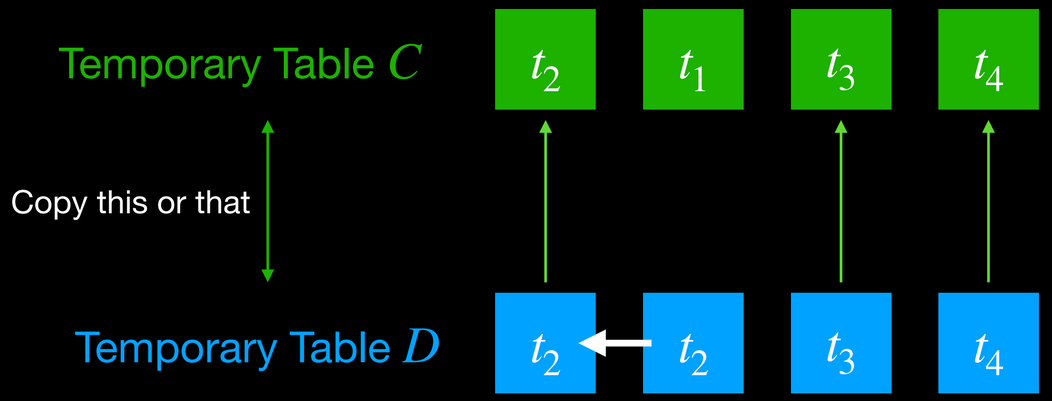
When $c(i) = d(i)$ have $b(i) = 1$

When $c(i) \neq d(i)$ have $b(i) = 0$

When $c(i) = d(i)$ have $1 \times [c(i) - d(i)] = 0$

The Halo2 Strategy

Copy this or that constraints?



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$b(X)$ *is such that* $b(i) \in \{0,1\}$

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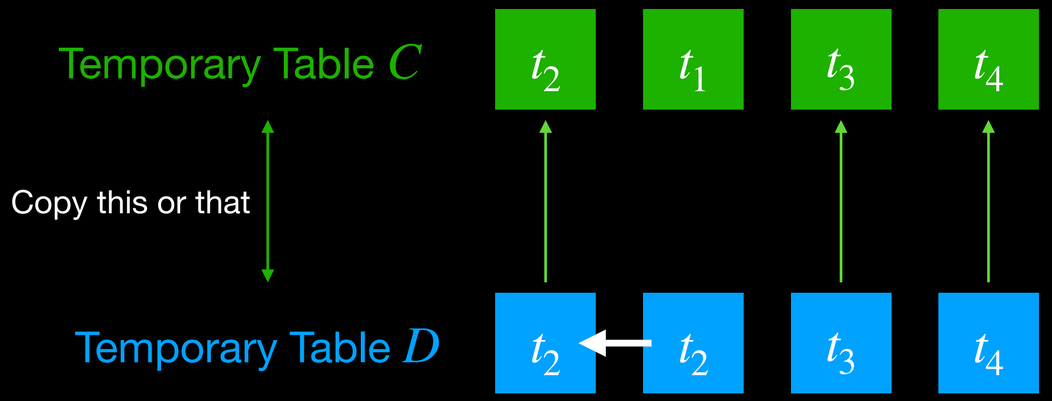
When $c(i) \neq d(i)$ have $b(i) = 0$

When $c(i) = d(i)$ have $1 \times [c(i) - d(i)] + 0 = 0$

When $d(i+1) = d(i)$ have $0 + (1-0) \times [d(i+1) - d(i)] = 0$

The Halo2 Strategy

Copy this or that constraints?



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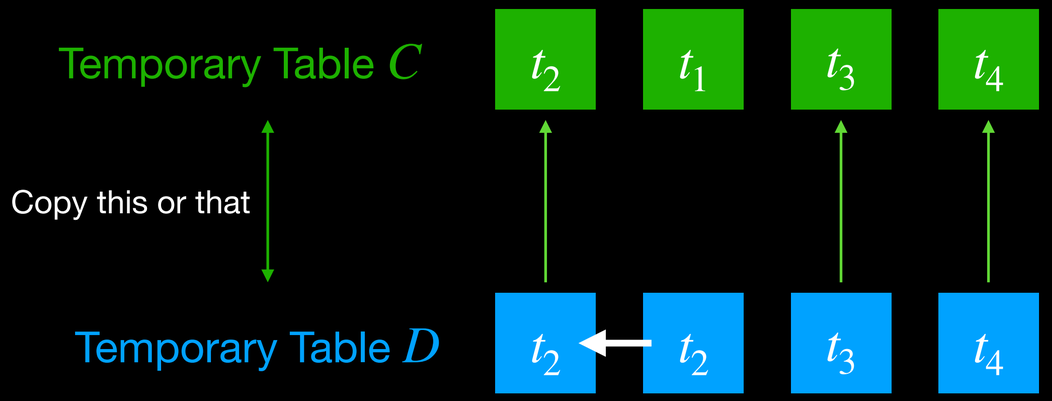
The Halo2 Strategy

Copy this or that constraints?



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- Prove that
  - $b(X)(\mathrm{id}(X) - b(X)) = q(X)z(X)$

Works

$b(X)$ *is such that* $b(i) \in \{0,1\}$

When $c(i) = d(i)$ have $b(i) = 1$

When $c(i) \neq d(i)$ have $b(i) = 0$

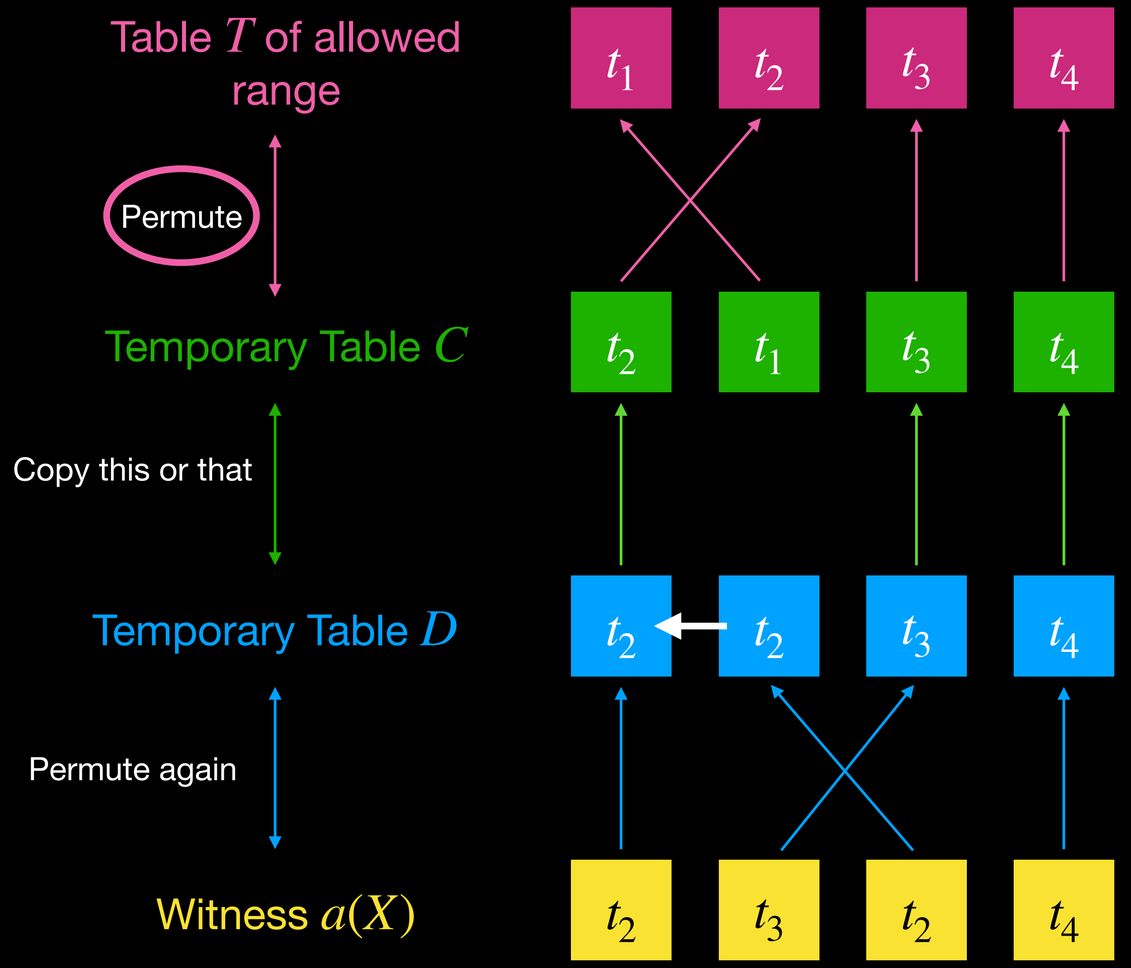
When $c(i) = d(i)$ have $1 \times [c(i) - d(i)] + 0 = 0$

When $d(i+1) = d(i)$ have $0 + (1-0) \times [d(i+1) - d(i)] = 0$

The Halo2 Strategy

Slow for big tables?

- Halo2 runs a permutation argument over $t(X)$.
- Prover time depends linearly on $t(X)$.
- Want large tables e.g. $\{t_1, \ldots, t_{2^{27}}\}$



The Caulk+ Strategy

Fast for big tables.

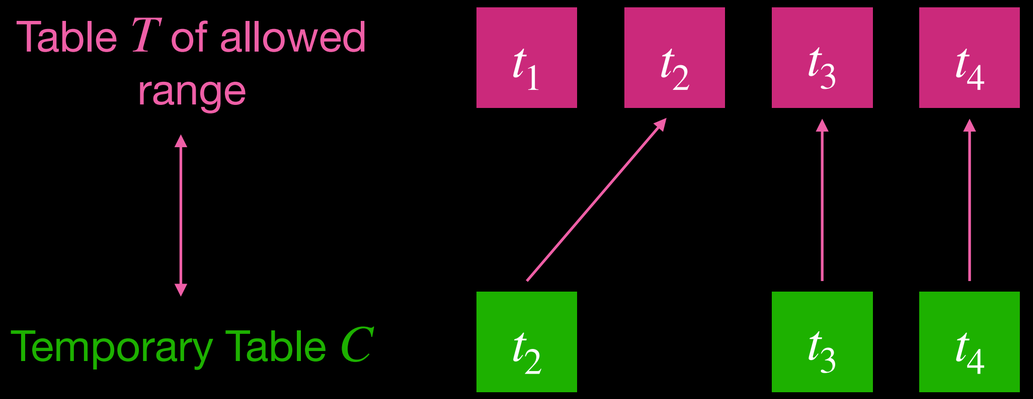
Caulk+: Table-independent lookup arguments

Jim Posen[1] and Assimakis A. Kattis[2]

[1] Ulvetanna jimpo AT ulvetanna.io
[2] New York University kattis AT cs.nyu.edu

- Halo2 runs a permutation argument over $t(X)$.
- Prover time depends linearly on $t(X)$.
- Want large tables e.g. $\{t_1, \ldots, t_{2^{27}}\}$
- We would like a temporary table $c(X)$ that is independent of the size of $t(X)$.

Possible with preprocessing.



The Caulk+ Strategy

Preprocessing.

Fast amortized KZG proofs

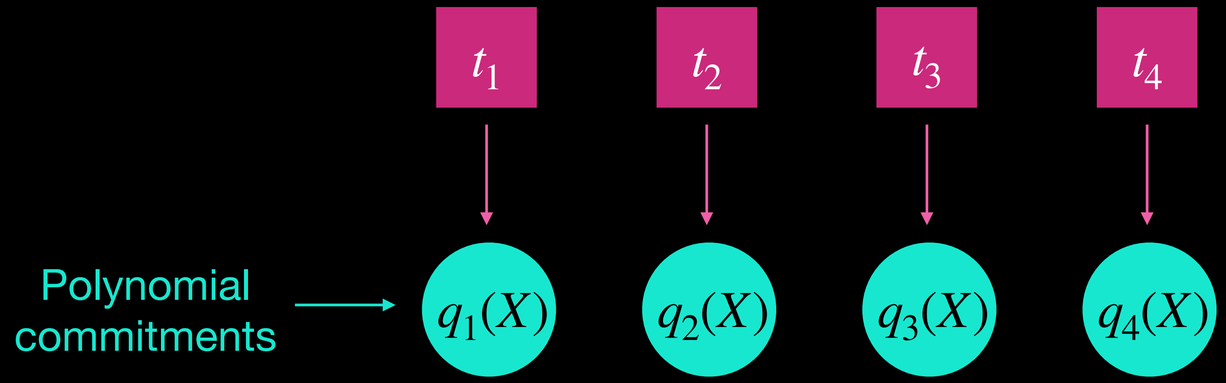
Dankrad Feist[*1] and Dmitry Khovratovich[†2]

[1,2]Ethereum Foundation

January 11, 2023

Abstract

In this note we explain how to compute n KZG proofs for a polynomial of degree d in time superlinear of $(t + d)$. Our technique is used in lookup arguments and vector commitment schemes.



$$t(X) - t(i) = q_i(X)(X - i)$$

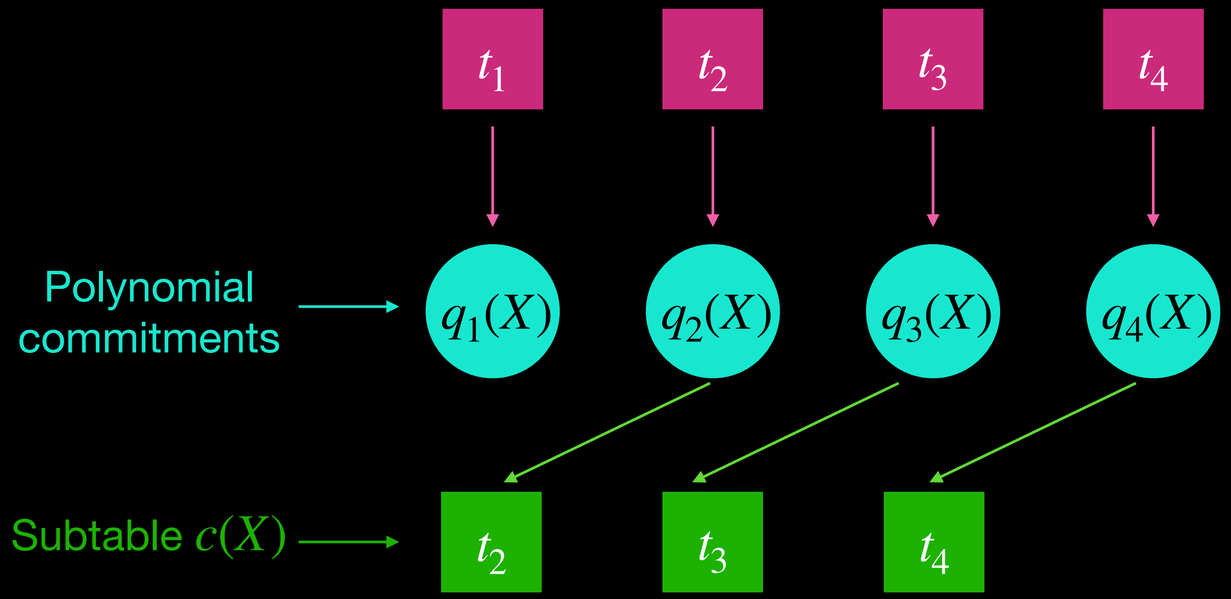
- Polynomial commitments $q_i(X)$ can be precomputed quickly.

The Caulk+ Strategy

Obtaining a subtable

If $c(X)$ defined over I is a subtable then

$$t(X) - c(X) = q(X)z_I(X)$$



- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$

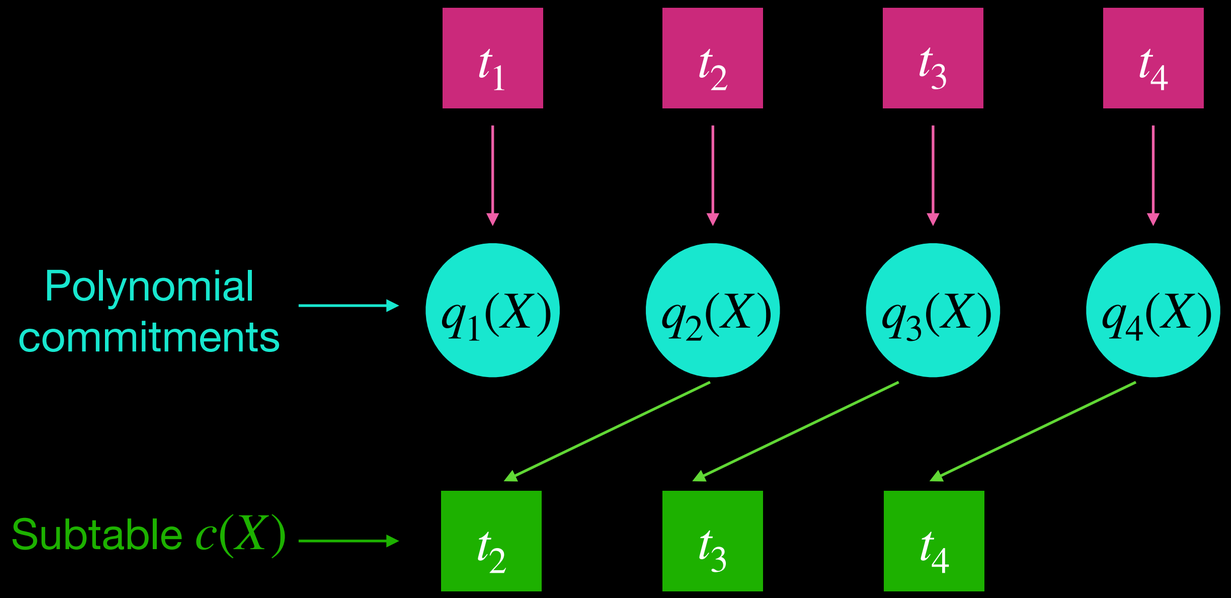
The Caulk+ Strategy

Obtaining a subtable

$$t(X) - c(X) = q(X)z_I(X)$$

Vanishing polynomial over $I = \{2,3,4\}$

$$z_I(X) = (X-2)(X-3)(X-4)$$



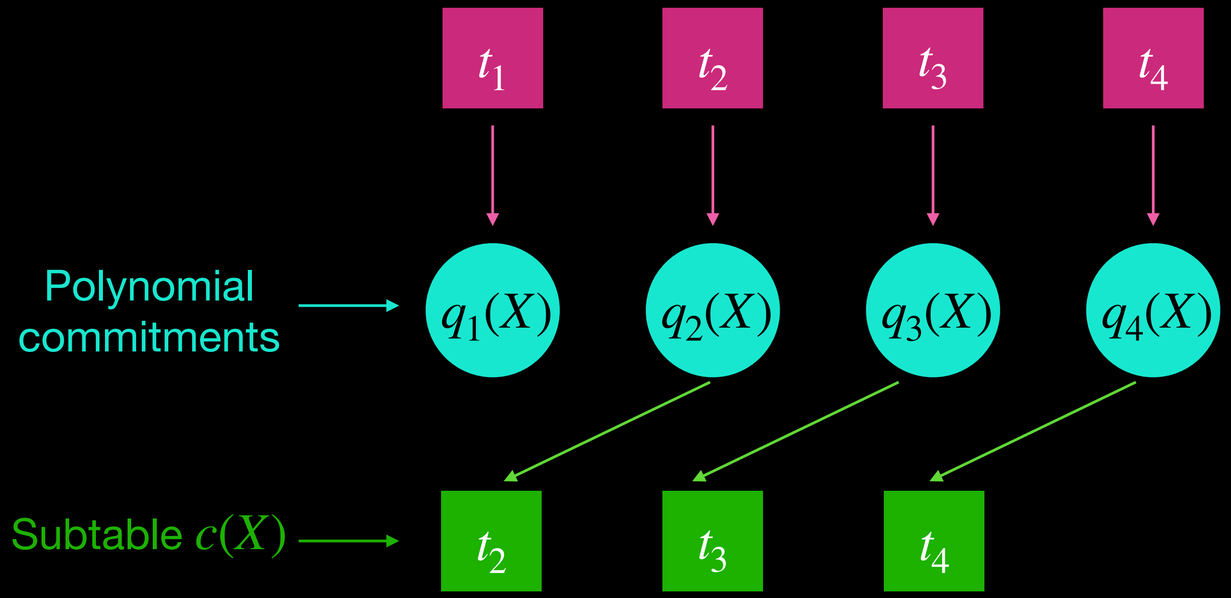
- At all points in $i \in I$ require $c(i) = t(i)$.
- Can prove $z_I(X)$ divides $z(X)$ and thus is valid.

- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$

The Caulk+ Strategy

Obtaining a subtable

$$t(X) - c(X) = q(X)z_I(X)$$



Precomputed values such that

- $t(X) - c(2) = q_2(X)(X - 2)$
- $t(X) - c(3) = q_3(X)(X - 3)$
- $t(X) - c(4) = q_4(X)(X - 4)$

- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$

The Caulk+ Strategy

Obtaining a subtable

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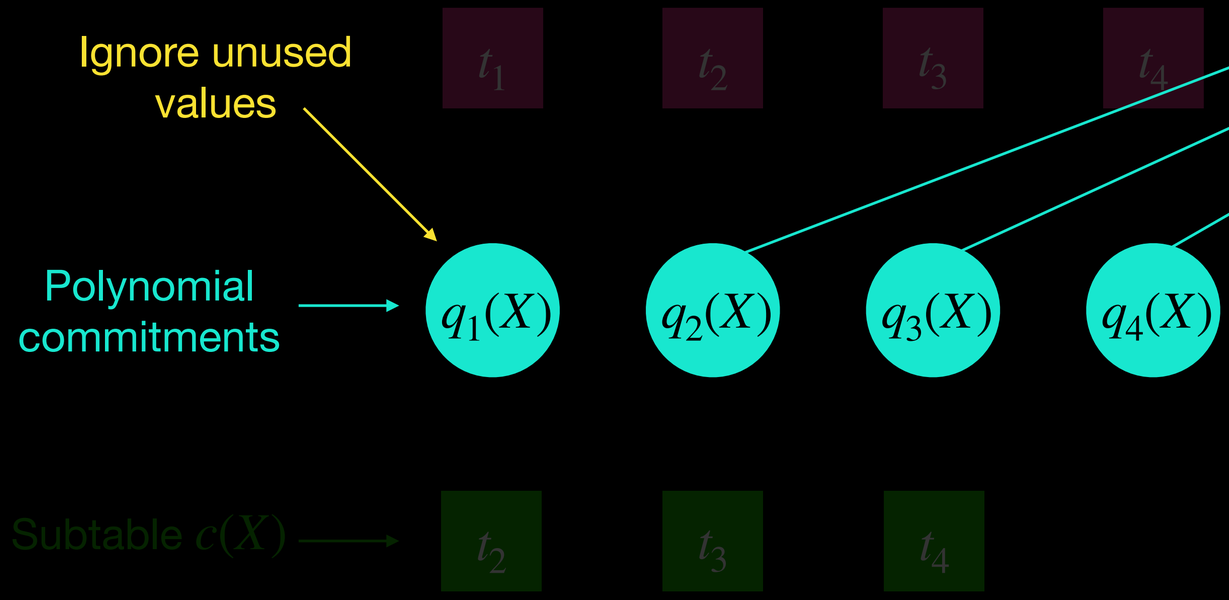

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Can quickly compute

$$q(X) = f(\ q_2(X),\ q_3(X),\ q_4(X)\ )$$

- Polynomial commitments $q_i(X)$ can be precomputed quickly.
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The Caulk+ Strategy

Obtaining a subtable

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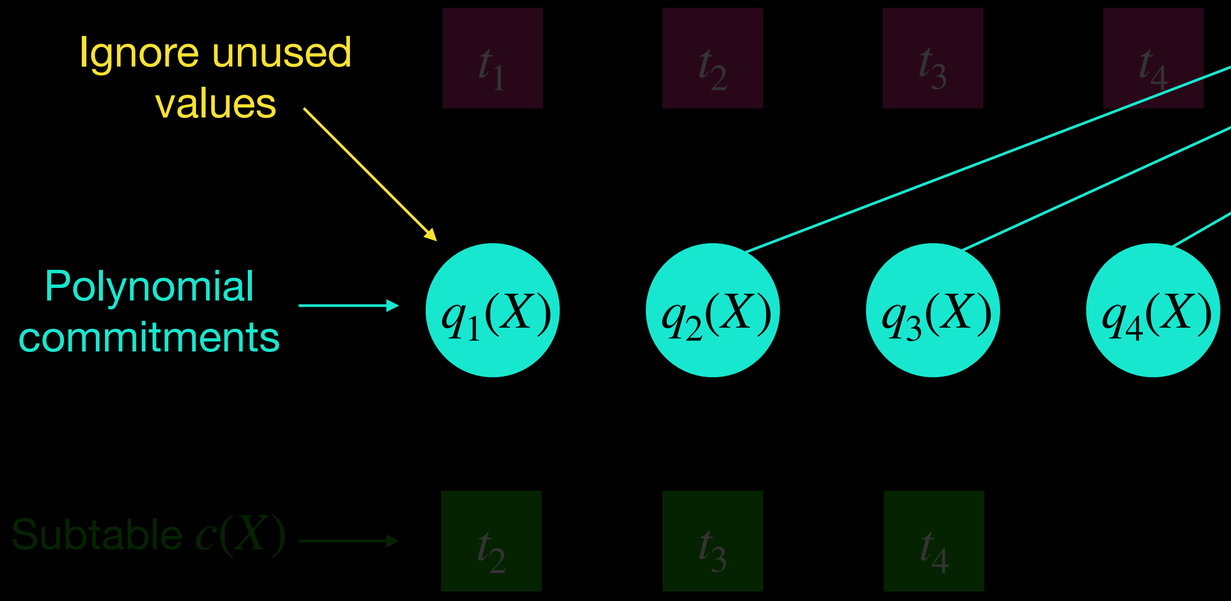

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Can quickly compute

$$q(X) = f(\ q_2(X),\ q_3(X),\ q_4(X)\ )$$

Once we have a subtable how do we use it?

- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$

The Baloo Strategy

Prove lookup over hidden domain

Ba*loo*: Nearly Optimal Lookup Arguments

Arantxa Zapico*, Ariel Gabizon[3], Dmitry Khovratovich[1], Mary Maller[1], and Carla Ràfols[2]

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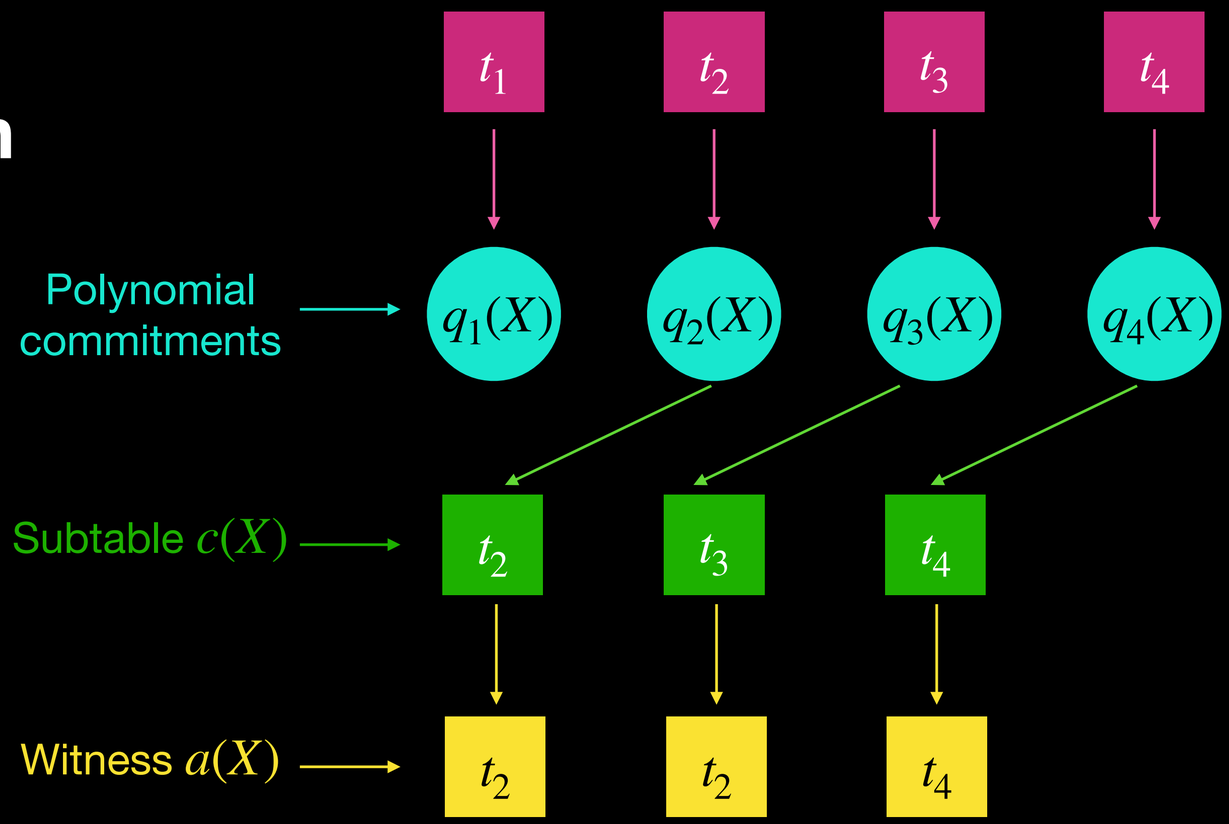

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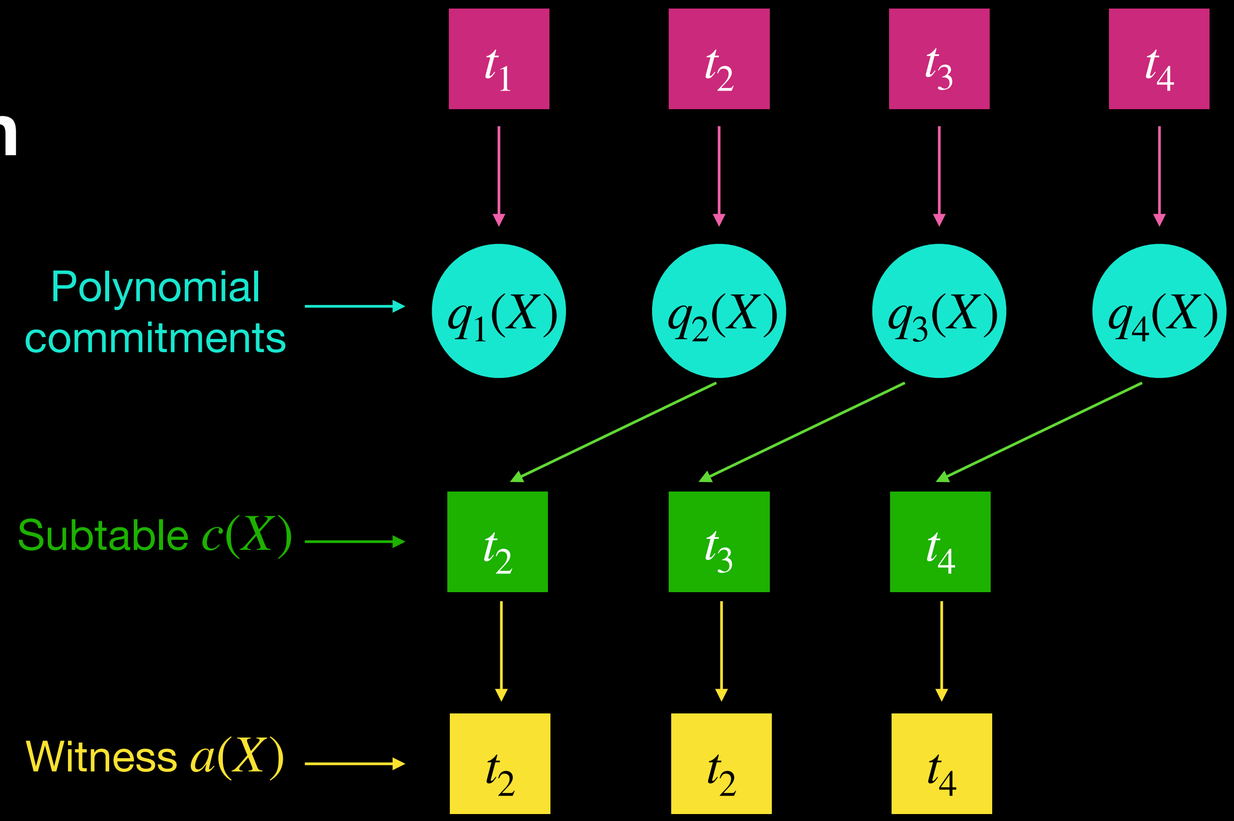
The Baloo Strategy

Prove lookup over hidden domain

$$\sum_{j \in I} e_{i,j} c(j) = a_i$$

Lincheck Argument



Aurora: Transparent Succinct Arguments for R1CS

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May 8, 2019

- Polynomial commitments $q_i(X)$ can be precomputed quickly.
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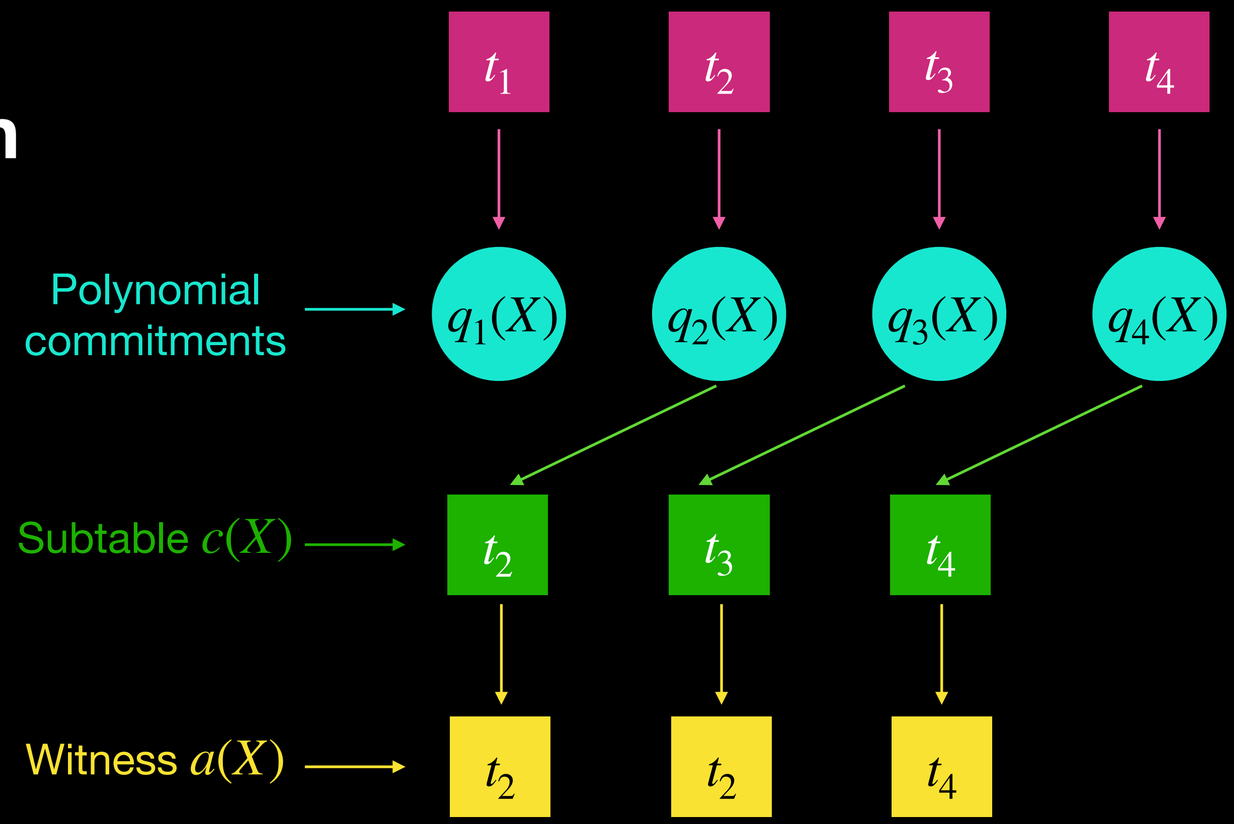
The Baloo Strategy

Prove lookup over hidden domain

$$\sum_{j \in I} e_{i,j} c(j) = a_i$$

Lincheck Argument over unit matrix

$$\mathbf{e}_i \in \{(1,0,0), (0,1,0), (0,0,1)\}$$



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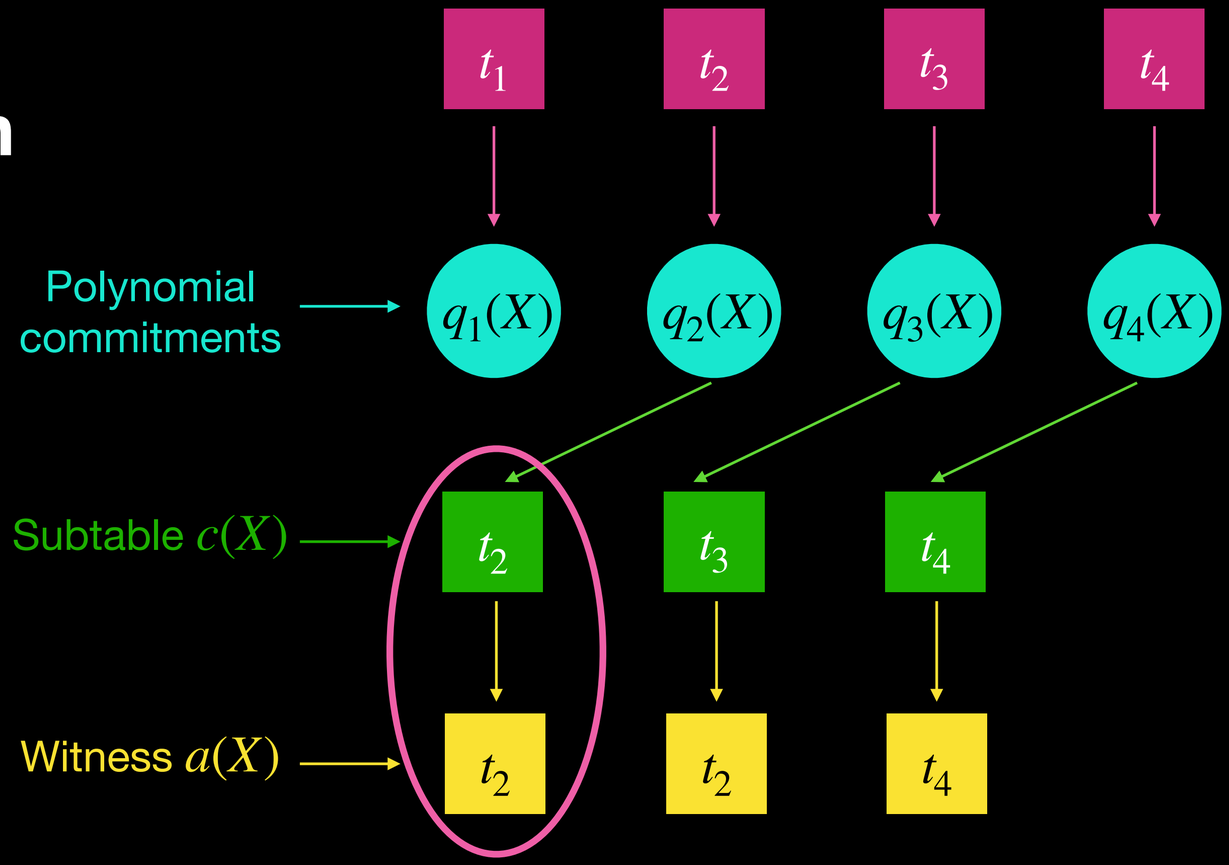

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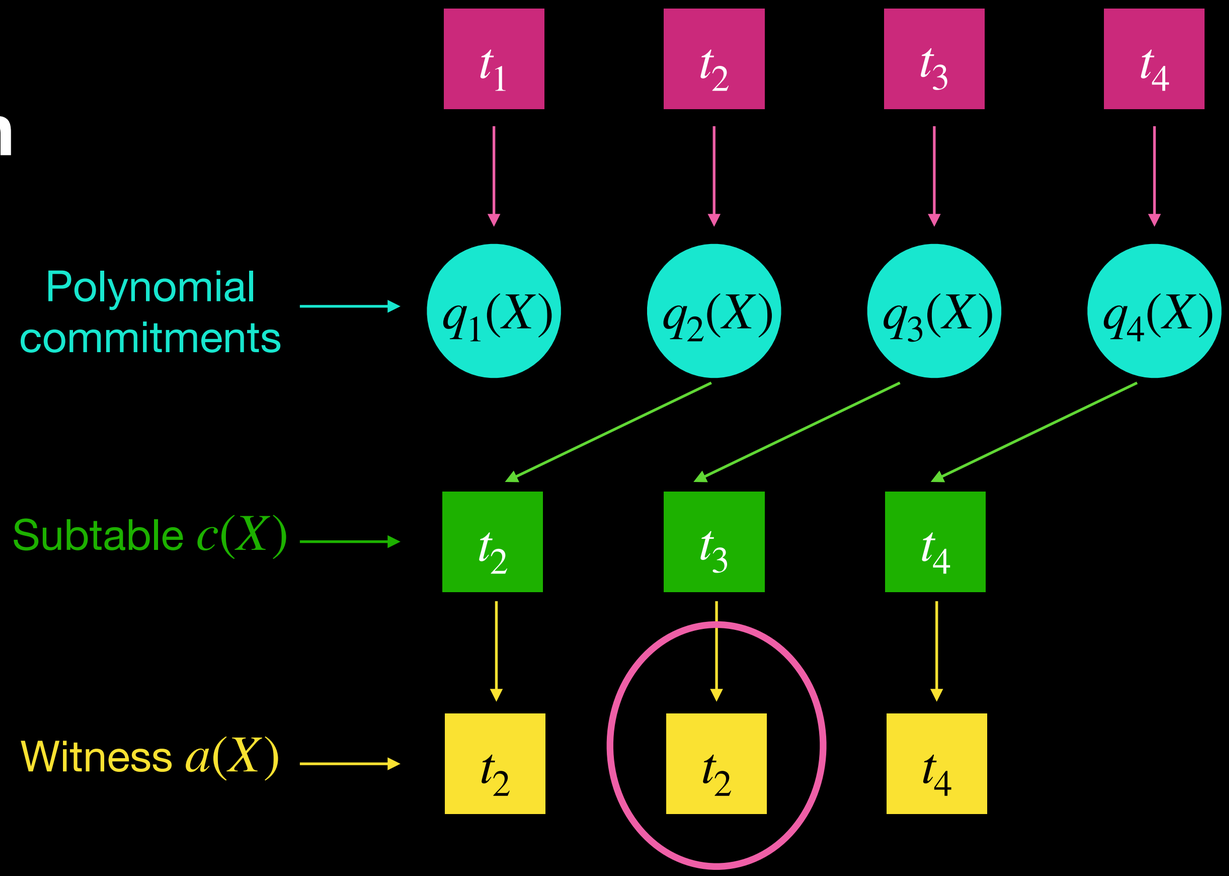
$$\mathbf{e}_2 = (1,0,0)$$



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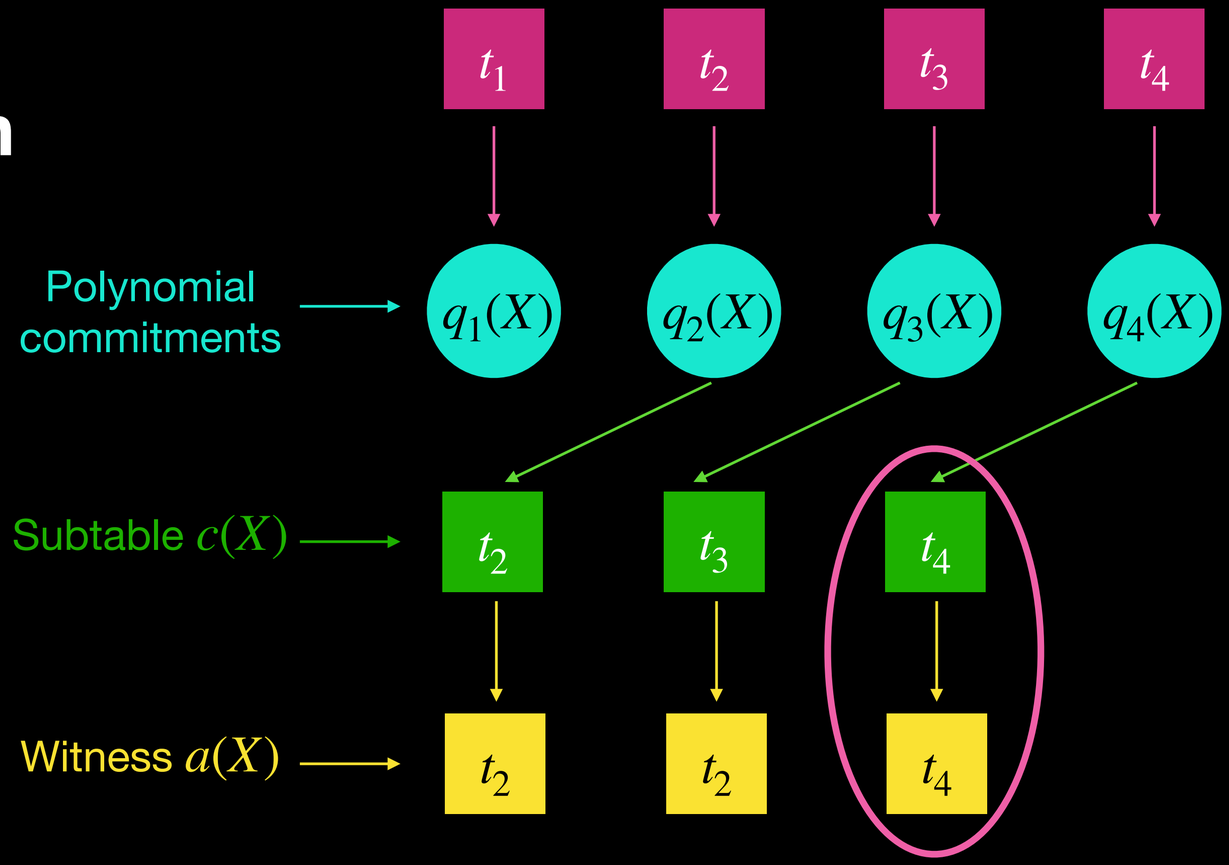

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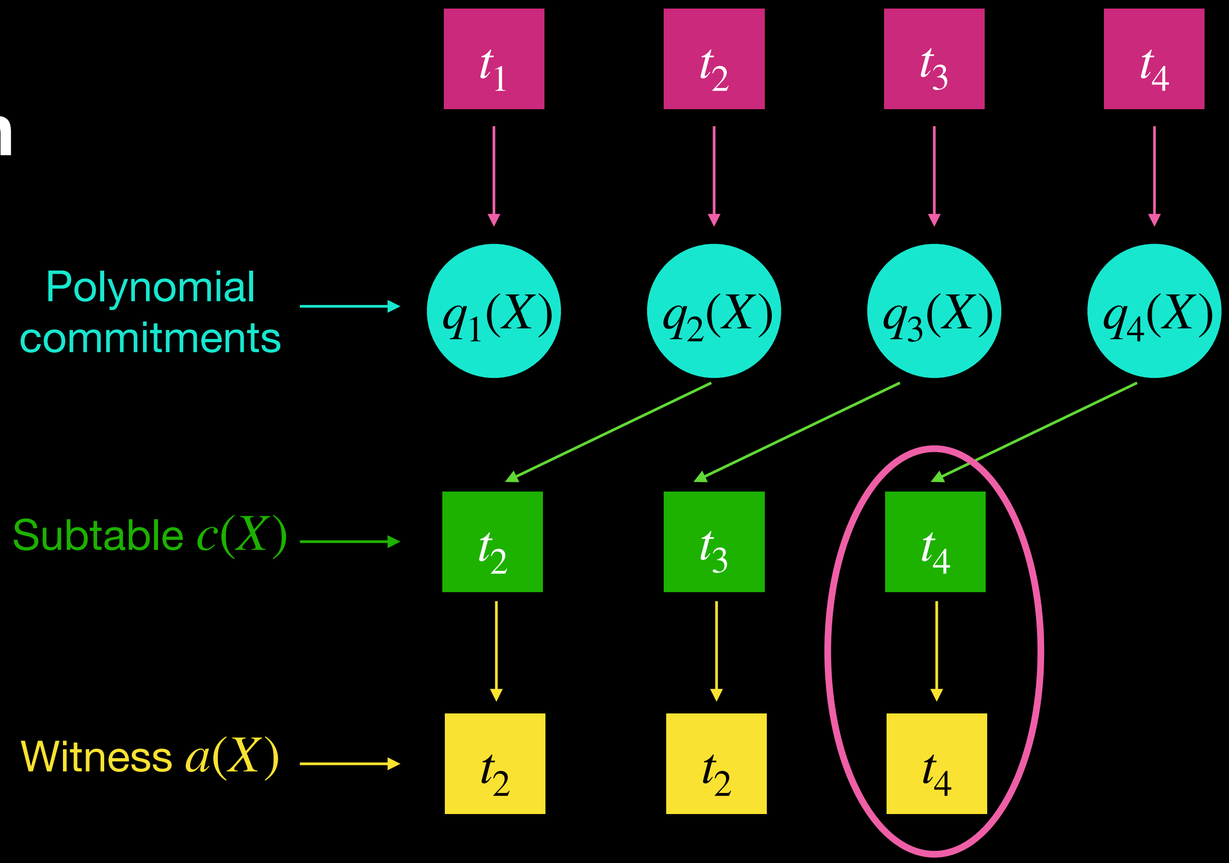
Repeated units allowed



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The Baloo Strategy

Prove lookup over hidden domain

$$\sum_{j \in I} e_{i,j} c(j) = a_i$$

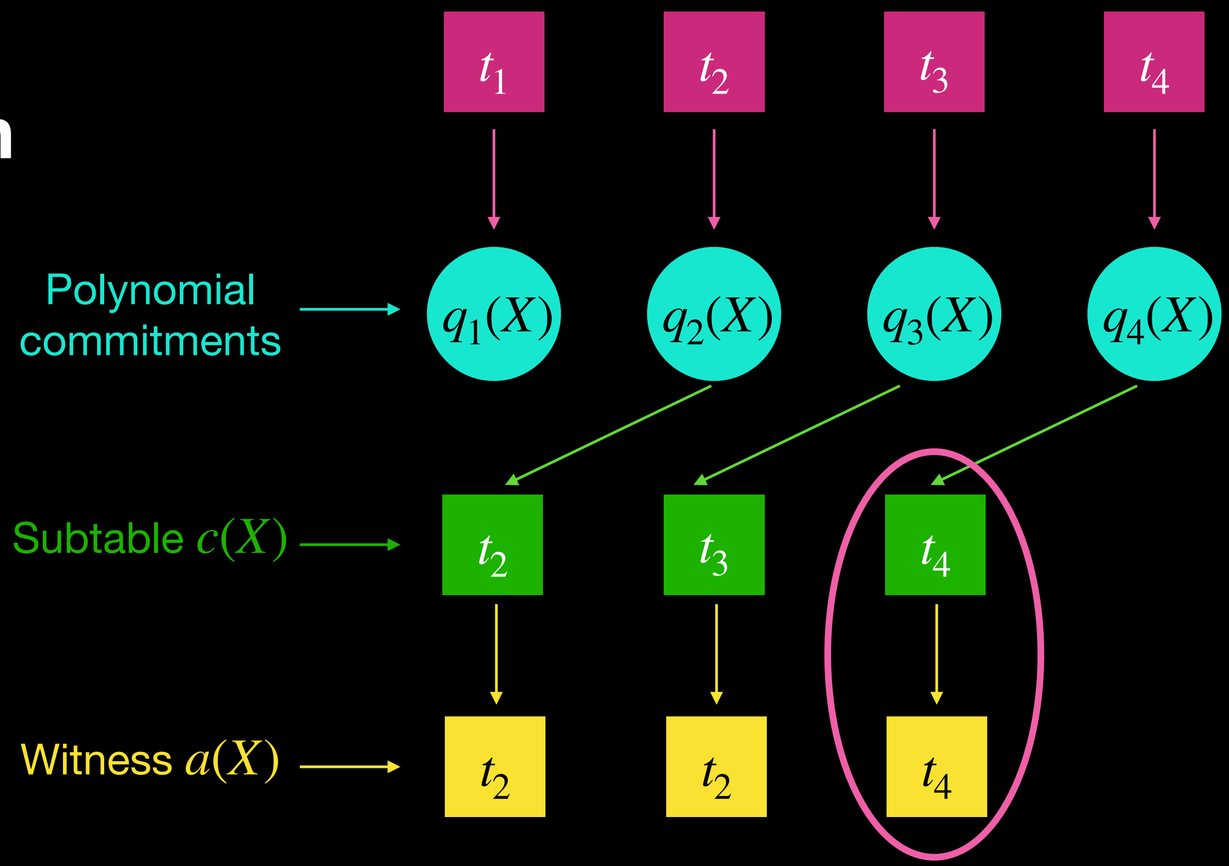
Lincheck Argument over unit matrix

$$\mathbf{e}_i \in \{(1,0,0), (0,1,0), (0,0,1)\}$$

$$\mathbf{e}_1 = (1,0,0)$$
$$\mathbf{e}_2 = (1,0,0)$$
$$\mathbf{e}_3 = (0,0,1)$$

Repeated units allowed

Lagrange polynomials are the unit vectors of the evaluation domain.



- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$

The Baloo Strategy

Recall Lagrange Polynomials

$$L_i(X) = \begin{cases} 1 \text{ if } X = i \\ 0 \text{ if } X \neq i \end{cases}$$

$$L_i(X) = \frac{\prod_{j \in I, j \neq i}(X - j)}{\prod_{j \in I, j \neq i}(i - j)}$$

Lagrange polynomials are the unit vectors of the evaluation domain.

The Baloo Strategy

Recall Lagrange Polynomials

$$L_i(X) = \begin{cases} 1 \text{ if } X = i \\ 0 \text{ if } X \neq i \end{cases}$$

Lagrange polynomials are the unit vectors of the evaluation domain.

$$L_i(X) = \frac{\prod_{j \in I, j \neq i} (X - j)}{\prod_{j \in I, j \neq i} (i - j)}$$

If $\tau_i(X) = \prod_{j \in I, j \neq i} (X - j)$ then $L_i(X) = \frac{\tau_i(X)}{\tau_i(i)}$

The Baloo Strategy

Recall Lagrange Polynomials

$$L_i(X) = \begin{cases} 1 \text{ if } X = i \\ 0 \text{ if } X \neq i \end{cases}$$

Lagrange polynomials are the unit vectors of the evaluation domain.

$$L_i(X) = \frac{\prod_{j \in I, j \neq i}(X - j)}{\prod_{j \in I, j \neq i}(i - j)}$$

For $z_I(X)$ the vanishing polynomial over I

If $\tau_i(X) = \prod_{j \in I, j \neq i}(X - j) = \frac{z_I(X)}{X - i}$ then $L_i(X) = \frac{\tau_i(X)}{\tau_i(i)}$

The Baloo Strategy

Prove lookup over hidden domain

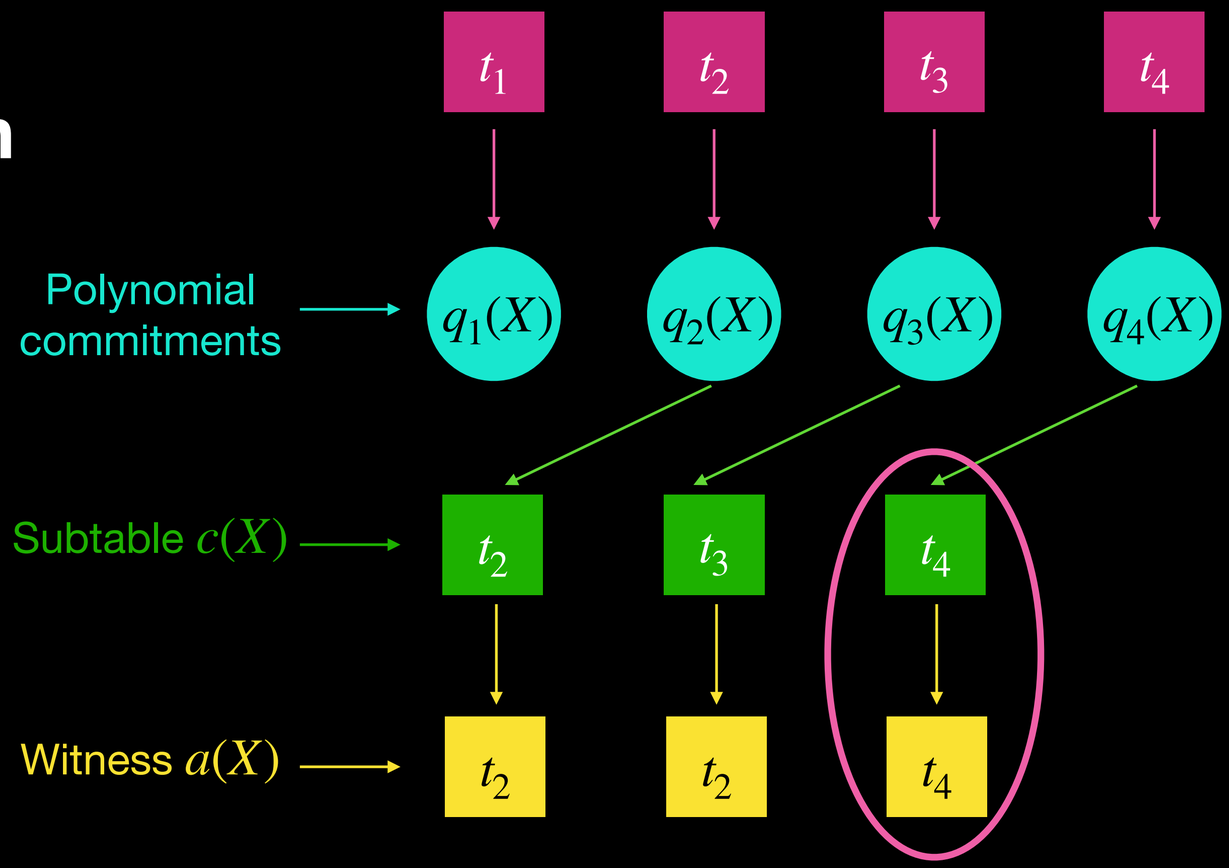
$$\sum_{j \in I} \frac{\tau_i(j)}{\tau_i(v_i)} c(j) = a_i$$

Lincheck Argument over unit matrix

$$\mathbf{e}_i \in \{(1,0,0), (0,1,0), (0,0,1)\}$$

$$\mathbf{e}_1 = (1,0,0)$$
$$\mathbf{e}_2 = (1,0,0)$$
$$\mathbf{e}_3 = (0,0,1)$$

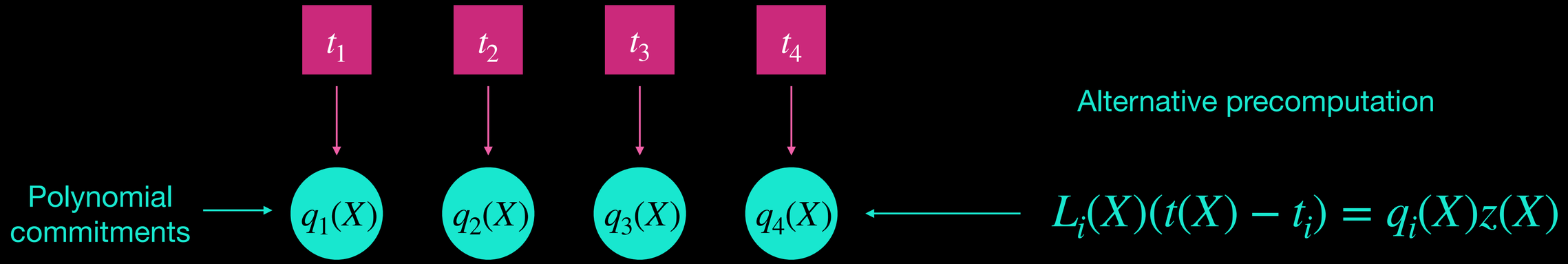
Unit vectors $e_i(X) = \frac{\tau_j(X)}{\tau_j(v)}$ and $\tau_j(X) = \frac{z_I(X)}{X - v}$ for some v



- Polynomial commitments $q_i(X)$ can be precomputed quickly.
- Subtable $c(X)$ contains entries in $t(X)$ over domain $z_I(X)$
- Define unit vectors with respect to $z_I(X)$

The CQ Strategy

Cached Quotients



Cached quotients for fast lookups

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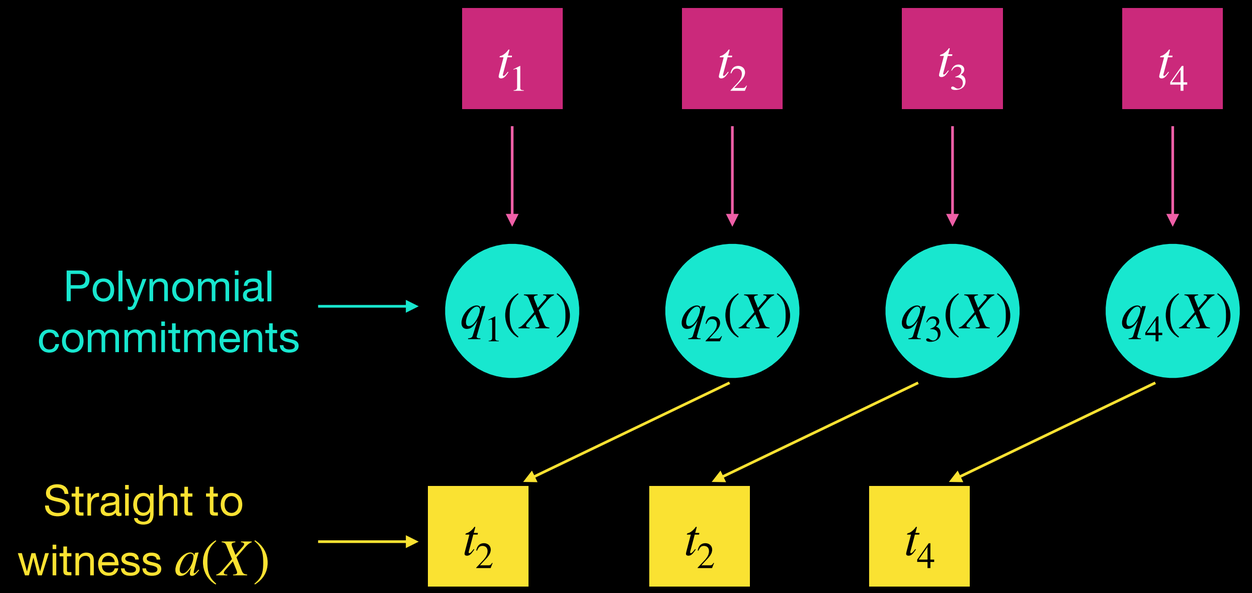
Zeta Function Technologies

January 8, 2023

The CQ Strategy

Cached Quotients



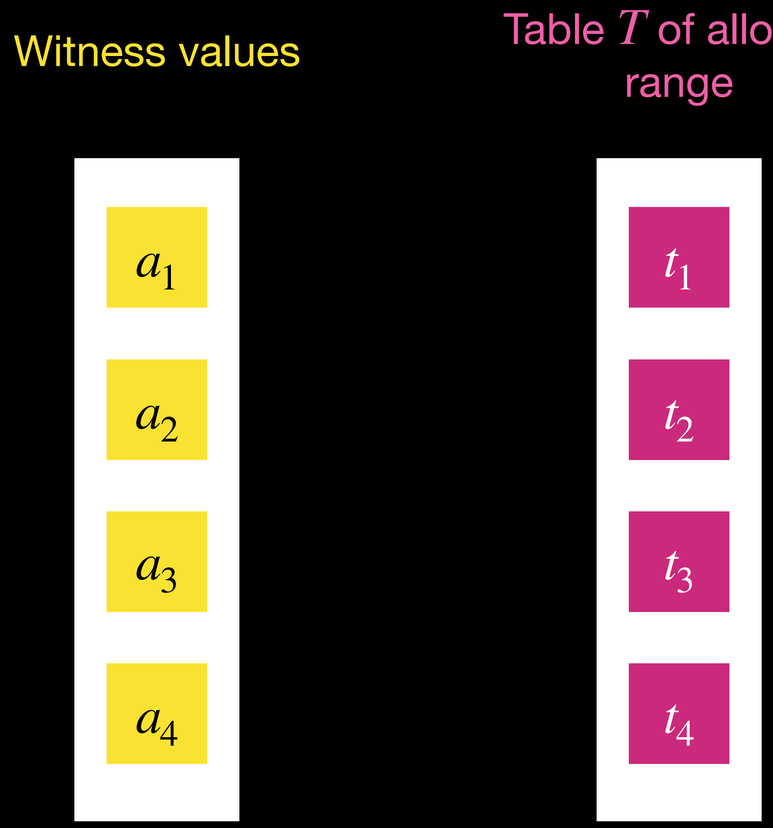
There exists $\mathbf{m}$ such that

$$\sum_{\text{all } t_j} \frac{m_j}{Y + t_j} = \sum_{\text{all } a_i} \frac{1}{Y + a_i}$$

Prover time
$O(m \log(m)),\ m = \deg(a(X))$ ***is independent from*** $t(X)$

Lookup Constraints in Plonkish

Function Tables



Lookup Constraints in Plonkish

Function Tables

$$a_1 \times b_1 \times c_1 - d_1 - e_1 = 0$$
$$a_2 + b_2 - c_2 = 0$$
$$a_3 \times b_3 \times c_3 - d_3 - e_3 = 0$$
$$a_4 + b_4 - c_4 = 0$$

$$a_1 = b_1 = c_1 = d_3 = a_4$$
$$x_1 = a_2 = d_1$$
$$y_1 = e_1$$
$$1 = b_2$$
$$a_3 = b_3 = c_3$$
$$e_3 = c_2$$
$$2 = b_4$$

u1	v1	w1	t1	r1	s1
u2	v2	w2	t2	r2	s2
u3	v3	w3	t3	r3	s3
u4	v4	w4	t4	r4	s4
u5	v5	w5	t5	r5	s5
u6	v6	w6	t6	r6	s6
u7	v7	w7	t7	r7	s7
u8	v8	w8	t8	r8	s8

output

Plonkish uses multiple columns

Lookup Constraints in Plonkish

Function Tables

Witness values

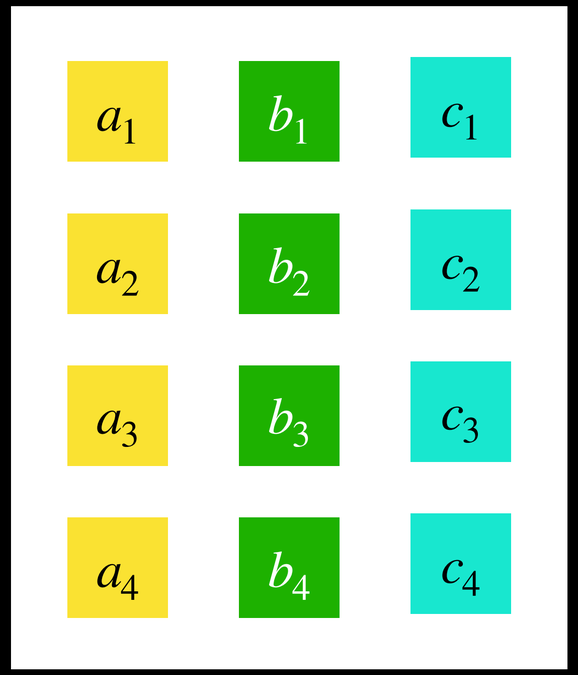
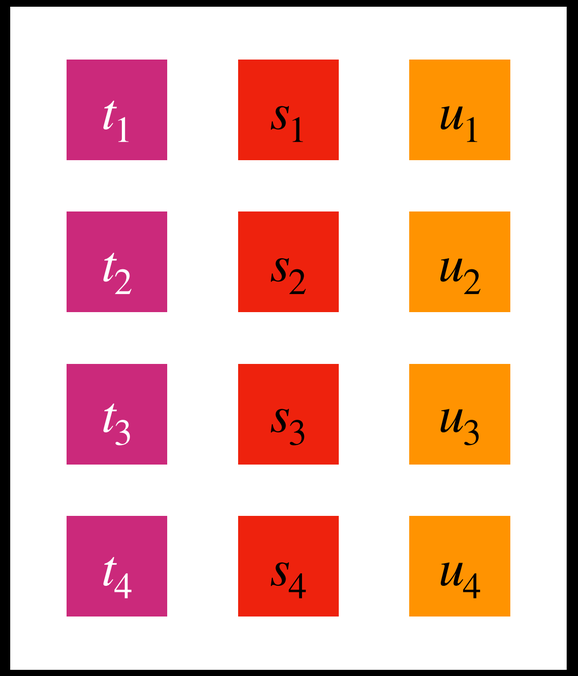


Table T of allowed range



Plonkish uses multiple columns

Lookup Constraints in Plonkish

Function Tables

Can lookup function tables e.g.

$$a_i \vee b_i = c_i$$

Witness values

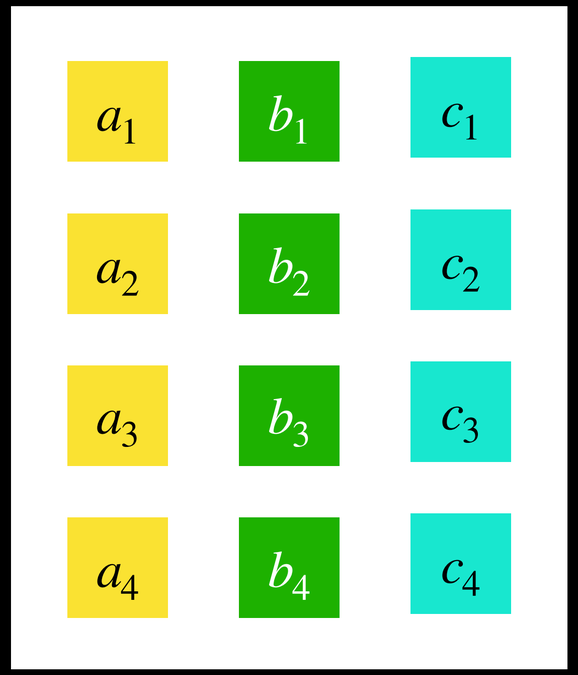
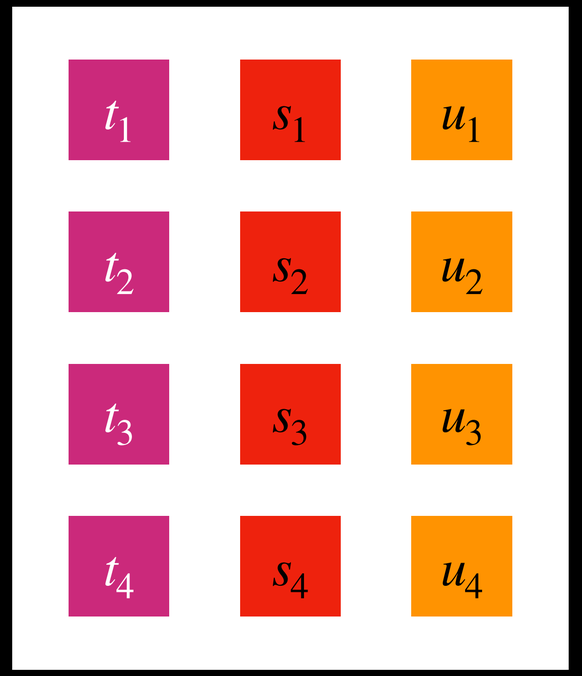


Table T of allowed range



Plonkish uses multiple columns

Lookup Constraints in Plonkish

Function Tables

Can lookup function tables e.g.

$$a_i \vee b_i = c_i$$

Witness values

a_1	b_1	c_1
a_2	b_2	c_2
a_3	b_3	c_3
a_4	b_4	c_4

All possible inputs and outputs.

Table T of allowed range

0	$\vee$	0	=	0
0	$\vee$	1	=	1
1	$\vee$	0	=	1
1	$\vee$	1	=	1

Plonkish uses multiple columns

Lookup Constraints in Plonkish

Function Tables

Can lookup function tables e.g.

$$a_i \vee b_i = c_i$$

Witness values

a_1	b_1	c_1
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a_3	b_3	c_3
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All possible inputs and outputs.

Table T of allowed range

0	$\vee$	0	=	0
0	$\vee$	1	=	1
1	$\vee$	0	=	1
1	$\vee$	1	=	1

Prove that $(a_i, b_i, c_i) \in (t_j, u_j, v_j)$

Lookup Constraints in Plonkish

Function Tables

Witness values

$$a_1 + b_1X + c_1X^2$$

$$a_2 + b_2X + c_2X^2$$

$$a_3 + b_3X + c_3X^2$$

$$a_4 + b_4X + c_4X^2$$

Table T of allowed range

$$t_1 + s_1X + u_1X^2$$

$$t_2 + s_2X + u_2X^2$$

$$t_3 + s_3X + u_3X^2$$

$$t_4 + s_4X + u_4X^2$$

- **Prove that** $(a_i, b_i, c_i) \in (t_j, u_j, v_j)$
- **Prove that** $a_i + b_iX + c_iX^2 \in (t_j + s_jX + u_jX^2)$

Lookup Constraints in Plonkish

Function Tables

Witness values

$$a_1 + b_1X + c_1X^2$$

$$a_2 + b_2X + c_2X^2$$

$$a_3 + b_3X + c_3X^2$$

$$a_4 + b_4X + c_4X^2$$

Table T of allowed range

$$t_1 + s_1X + u_1X^2$$

$$t_2 + s_2X + u_2X^2$$

$$t_3 + s_3X + u_3X^2$$

$$t_4 + s_4X + u_4X^2$$

- **Prove that** $(a_i, b_i, c_i) \in (t_j, u_j, v_j)$
- **Prove that** $a_i + b_iX + c_iX^2 \in (t_j + s_jX + u_jX^2)$

Random linear combination

Because random oracles exist..

Final Thoughts

- All sublinear time lookup arguments use pairings.
- Pairings are not post-quantum secure and require special, non-standardised, elliptic curves.
- Open question: can we design a practical, sublinear time lookup argument (in the size of the table) without pairings?

Final Thoughts

- All sublinear time lookup arguments use pairings.
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- Open question: can we design a practical, sublinear time lookup argument (in the size of the table) without pairings?

Thanks for listening!