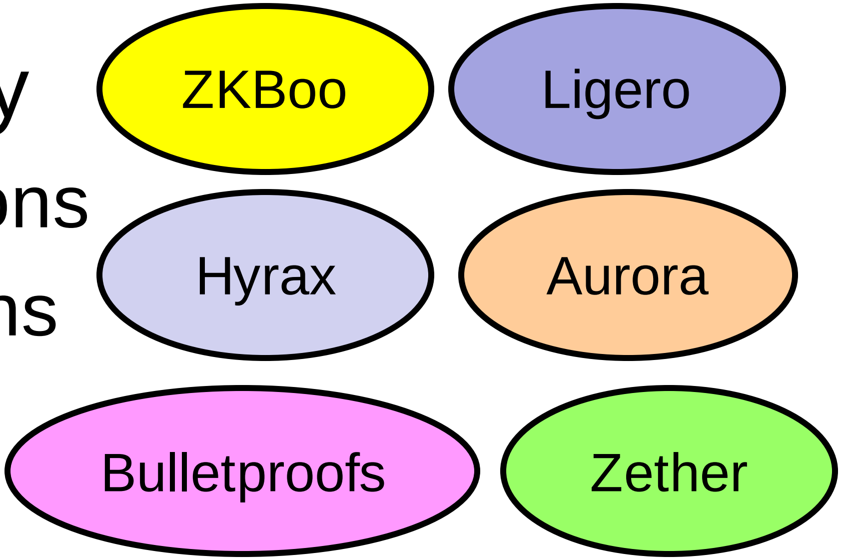
Compilers for Zero-Knowledge: An Overview

Yuval Ishai
Technion

9th BIU Winter School on Cryptography

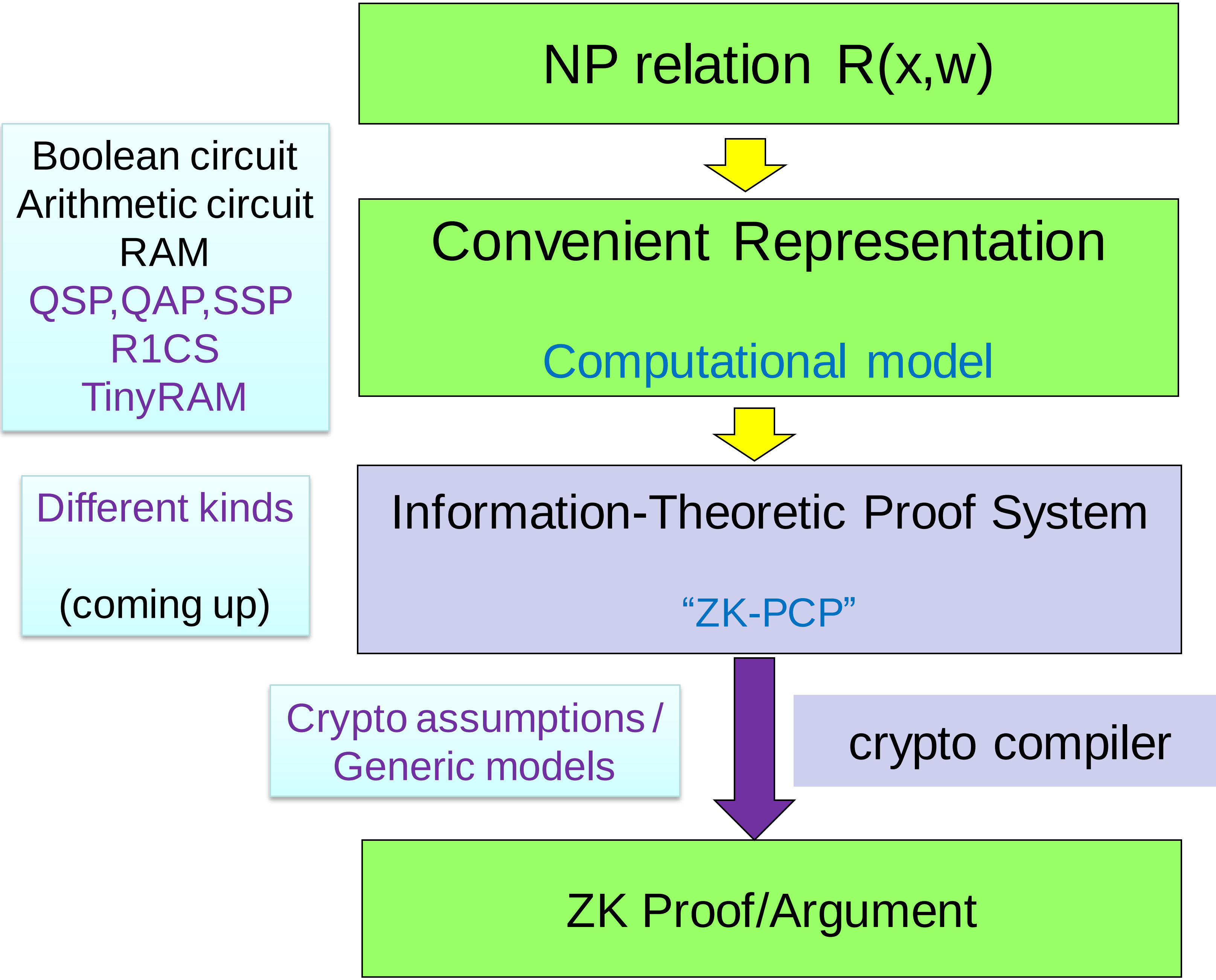
Broad Motivation

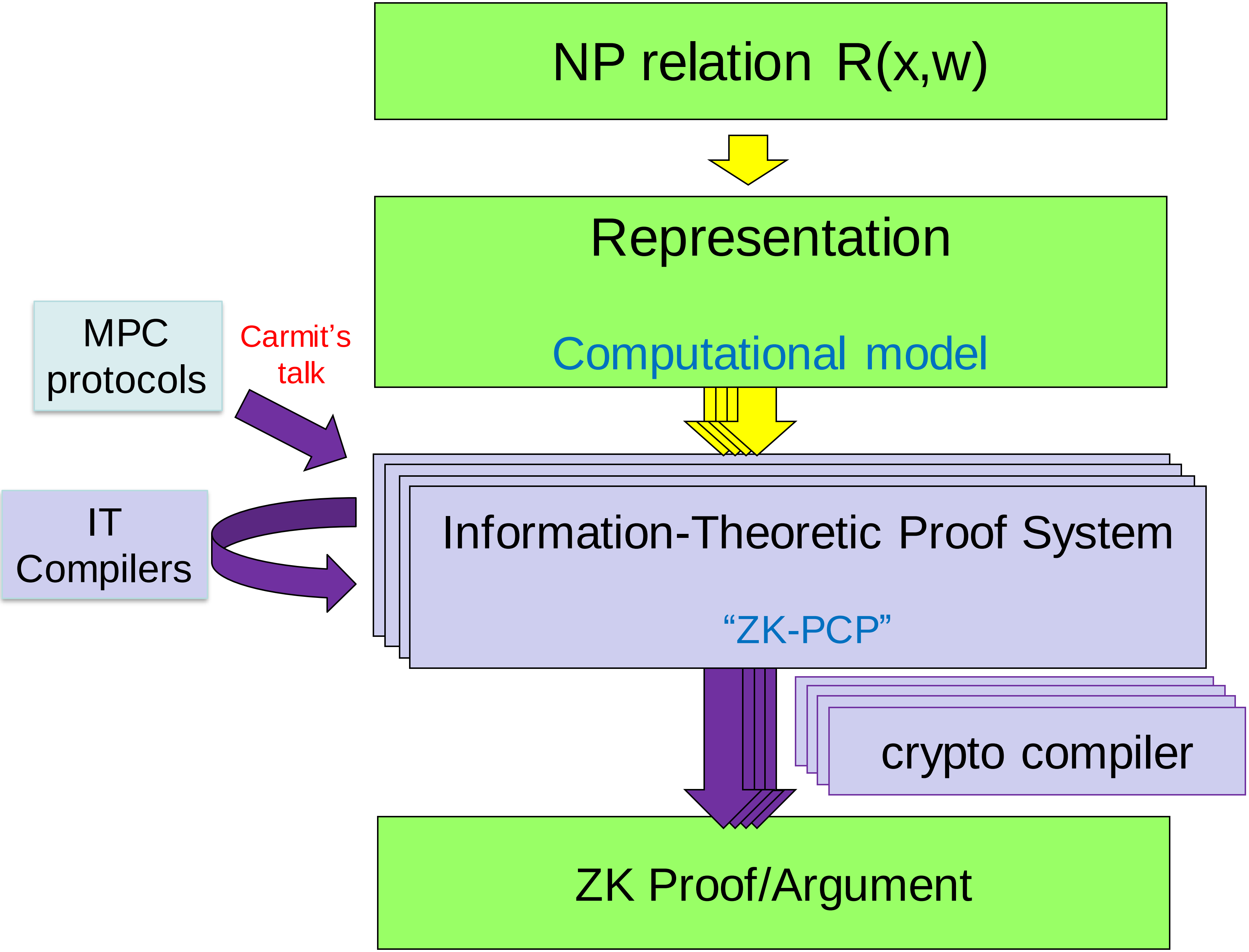
- ZK research is a big party
  - Many motivating applications
  - Many challenging questions
  - Many exciting results



- Big party → Big mess ?

- This talk: advocating a modular approach
  - Separate "information-theoretic" and "crypto" parts
  - General cryptographic compilers (IT → crypto)
  - General information-theoretic compilers (IT → IT)


NP relation R(x,w)
Convenient Representation
Computational model
Boolean circuit
Arithmetic circuit
RAM
QSP,QAP,SSP
R1CS
TinyRAM
Information-Theoretic Proof System
"ZK-PCP"
Different kinds
(coming up)
Crypto assumptions / Generic models
crypto compiler
ZK Proof/Argument


NP relation R(x,w)
Representation
Computational model
MPC protocols
Carmit's talk
IT Compilers
Information-Theoretic Proof System
"ZK-PCP"
crypto compiler
ZK Proof/Argument

Why?

- Simplicity
  - Break complex tasks into simpler components
  - Easier to analyze and optimize
  - Potential for proving lower bounds
- Generality
  - Apply same constructions in different settings
  - Research deduplication, less papers to read/write
- Efficiency
  - Port efficiency improvements between settings
  - Mix & match different components
  - Systematic exploration of design space

ZK Zoo

(ignoring assumptions for now…)

Qualitative features

- Interactive?
- Succinct?
- Fast verification?
- Public verification?
- Public input?
- NP vs. P?
- Trusted setup?
- Symmetric crypto only?
- Post quantum?

Quantitative features

- Communication
- Prover complexity
- Verifier complexity

Major commercialization efforts

Standardization process
zkproof.org
2nd workshop: April 10-12

Optimal ZKP protocol?

Food for thought…

- Which verifier is better?
  - V1: SHA256 hash
  - V2: PKE decryption
- V2 can be more obfuscation-friendly! [BISW17]
  - Relevant complexity measure: branching program size
  - Motivated "lattice-based" designated-verifier SNARKs
  - Promising avenue for practical general-purpose obfuscation
- Similar: MPC-friendly prover, etc.

Back to 20th Century

Theorem [GMW86]:
Bit-commitment → ZKP for all of NP

Theorem [GMW86+Naor89+HILL99]:
One-way function → ZKP for all of NP

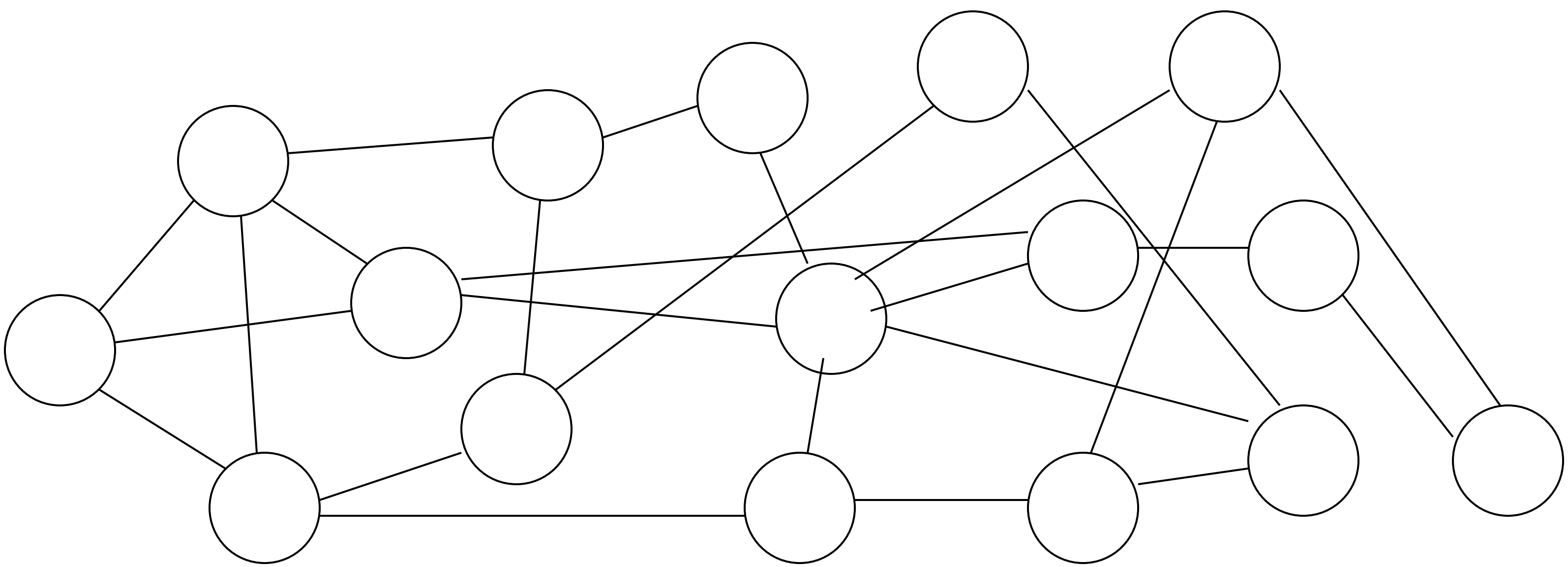
Theorem [OW93]:
ZKP for “hard on average” L in NP → i.o. one-way function

Are we done?

ZKP for 3-Colorability

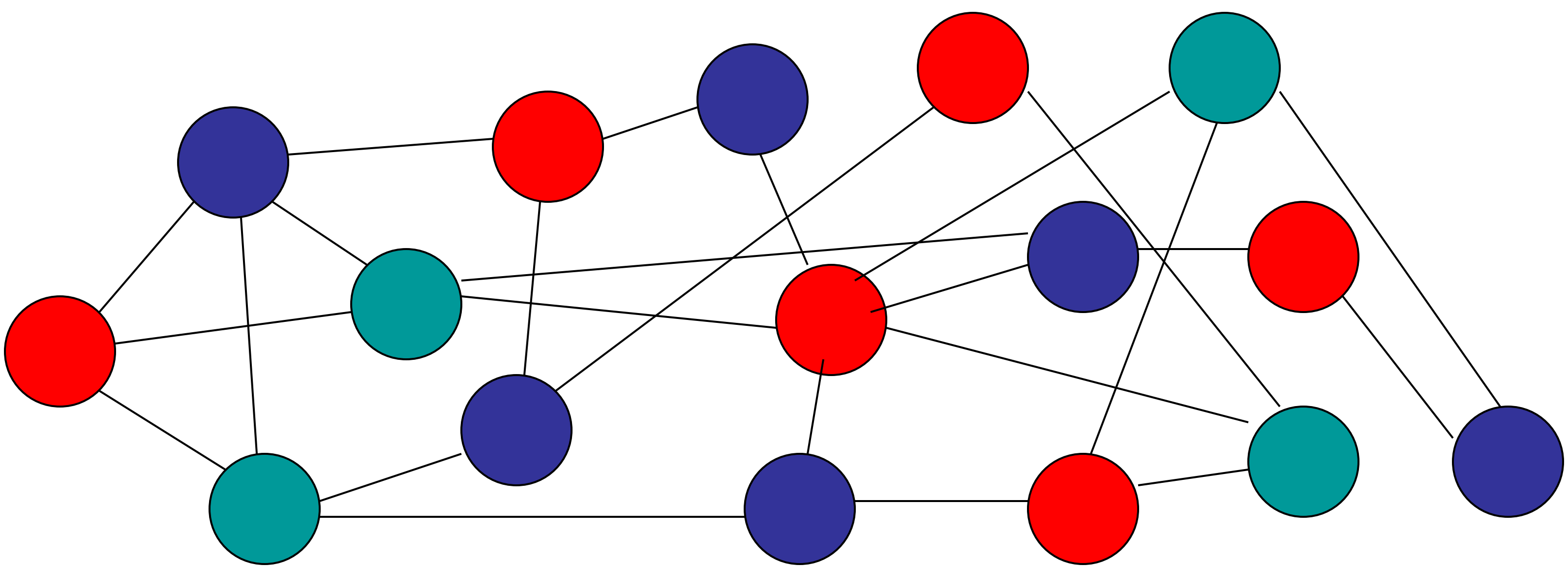
[GMW86]

- Prover wants to prove that a given graph is 3-colorable

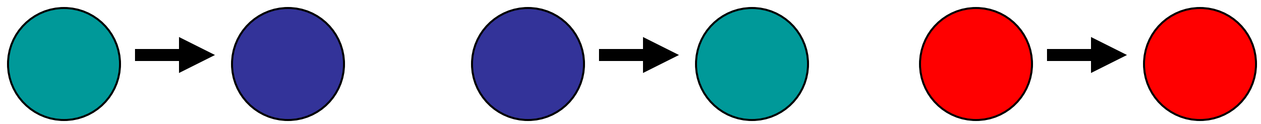


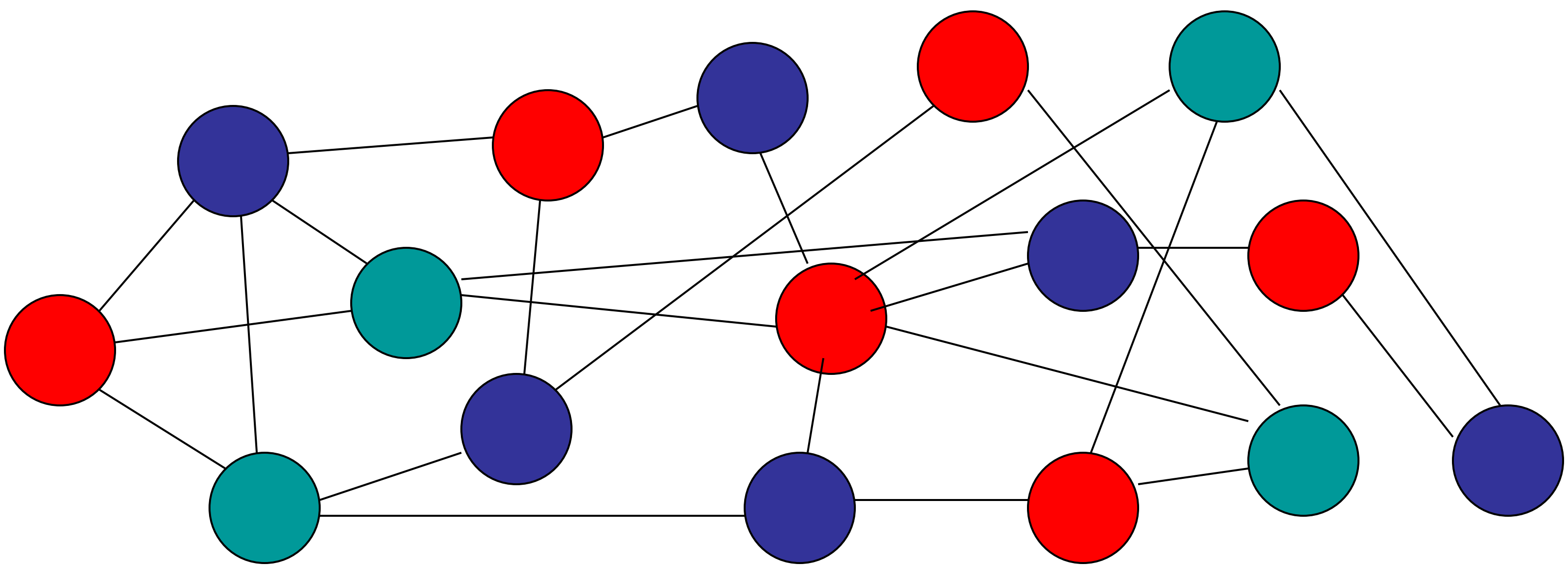
ZKP for 3-Colorability

- Prover wants to prove that a given graph is 3-colorable
  - x=graph w=coloring

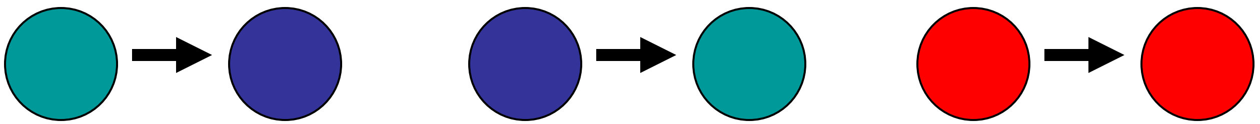


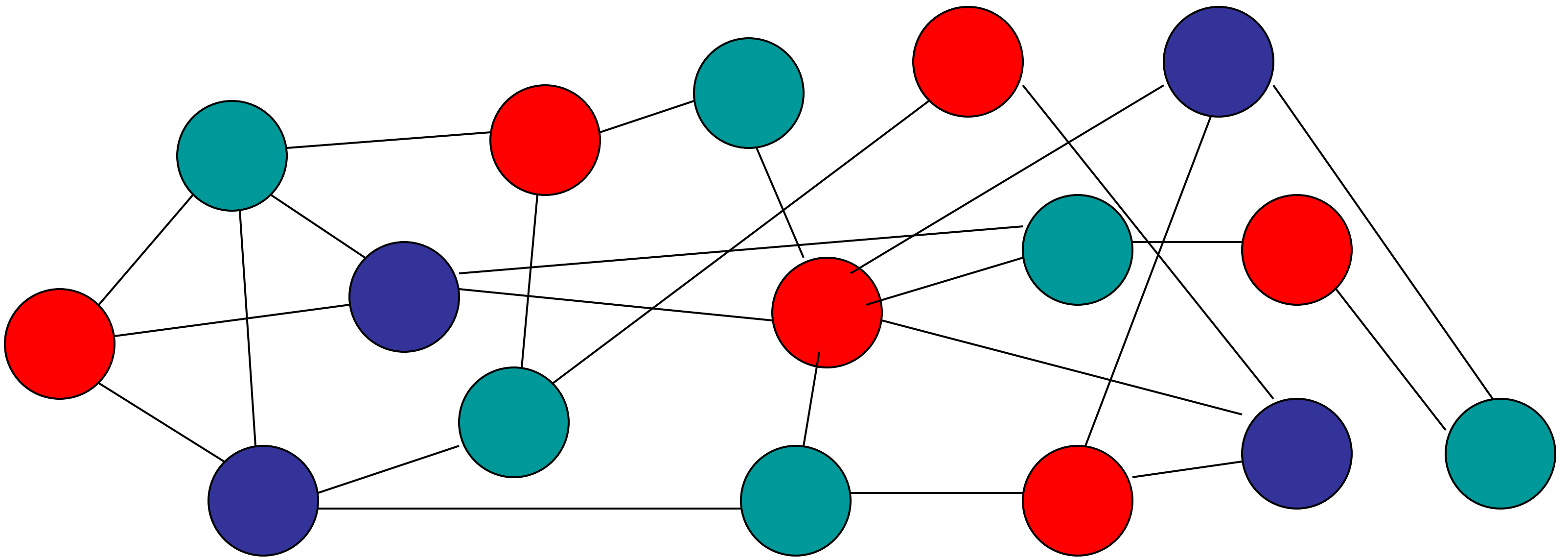
ZKP for 3-Colorability

- Prover randomly permutes the 3 colors (6 possibilities)
  - Say, 



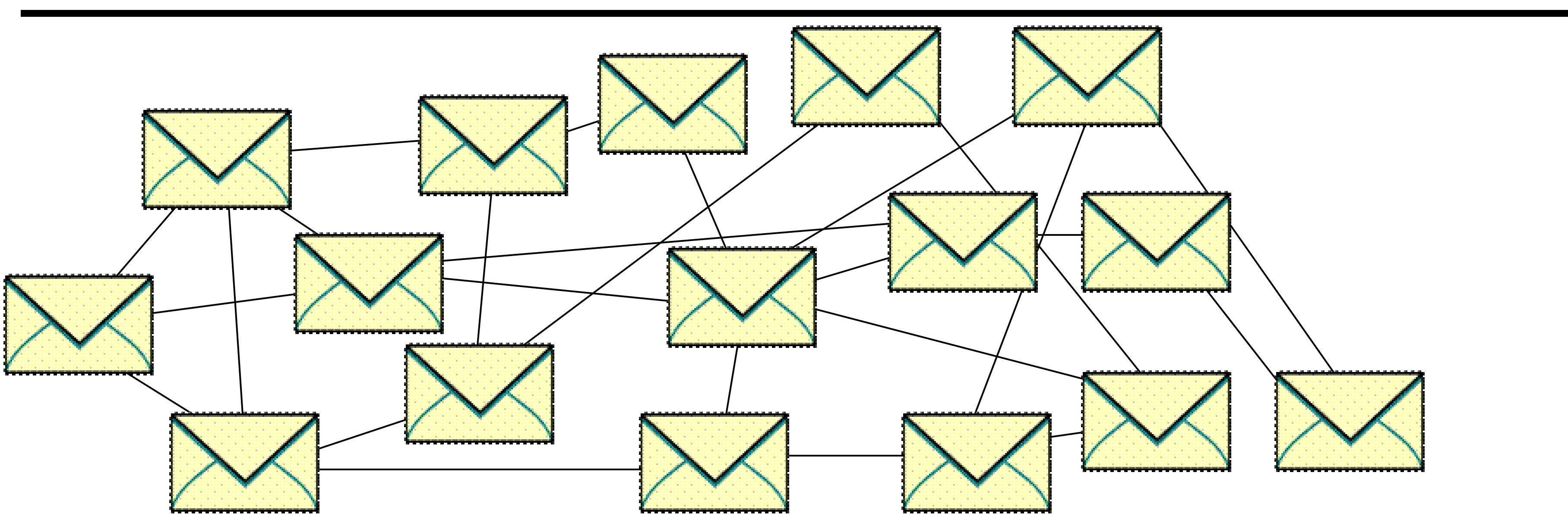
ZKP for 3-Colorability

- Prover randomly permutes the 3 colors (6 possibilities)
  - Say, 



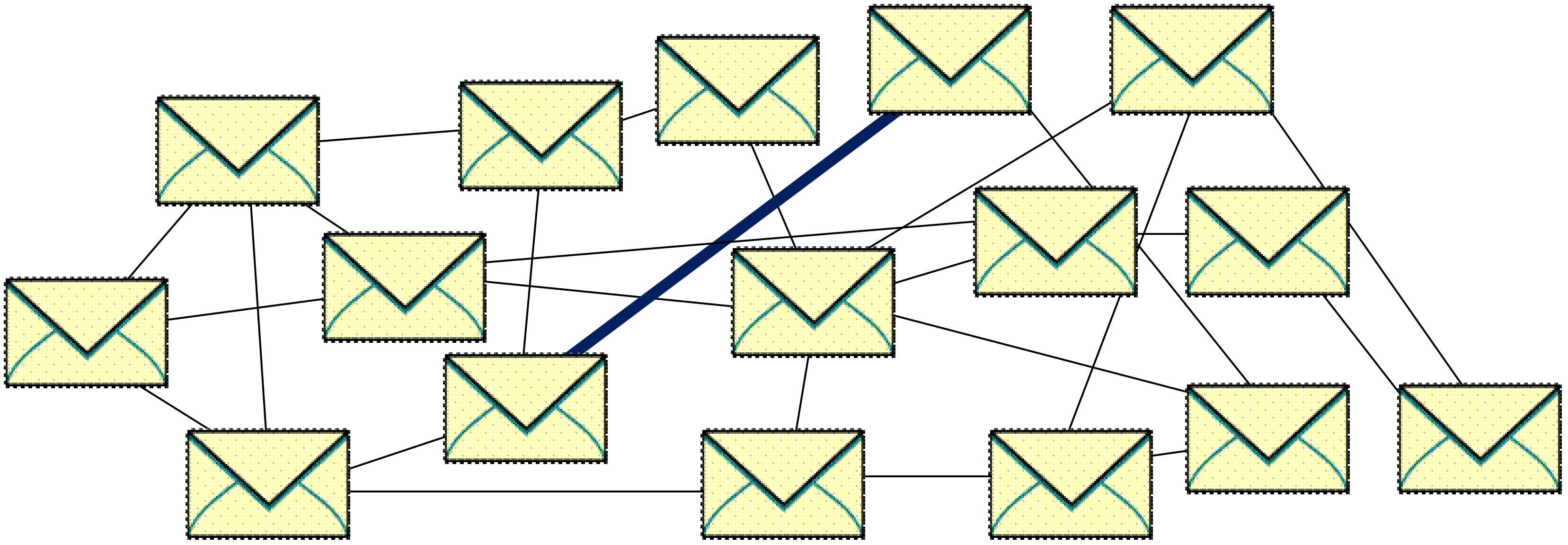
ZKP for 3-Colorability

- Prover separately commits to color of each node and sends commitments to Verifier



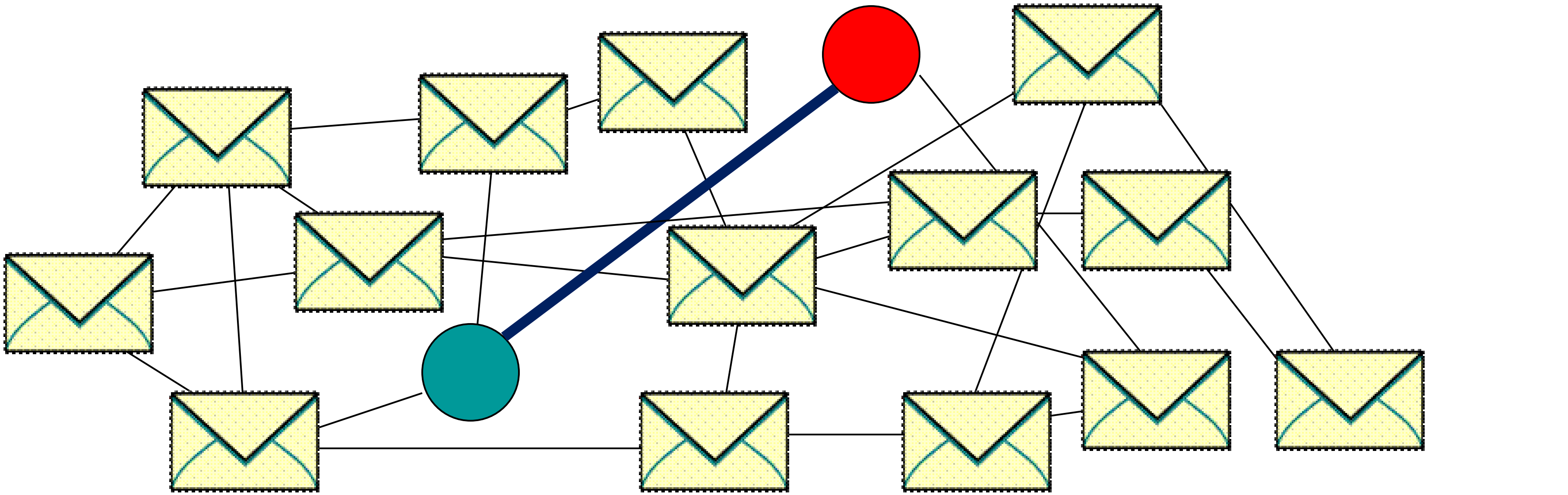
ZKP for 3-Colorability

- Verifier challenges Prover by selecting a random edge



ZKP for 3-Colorability

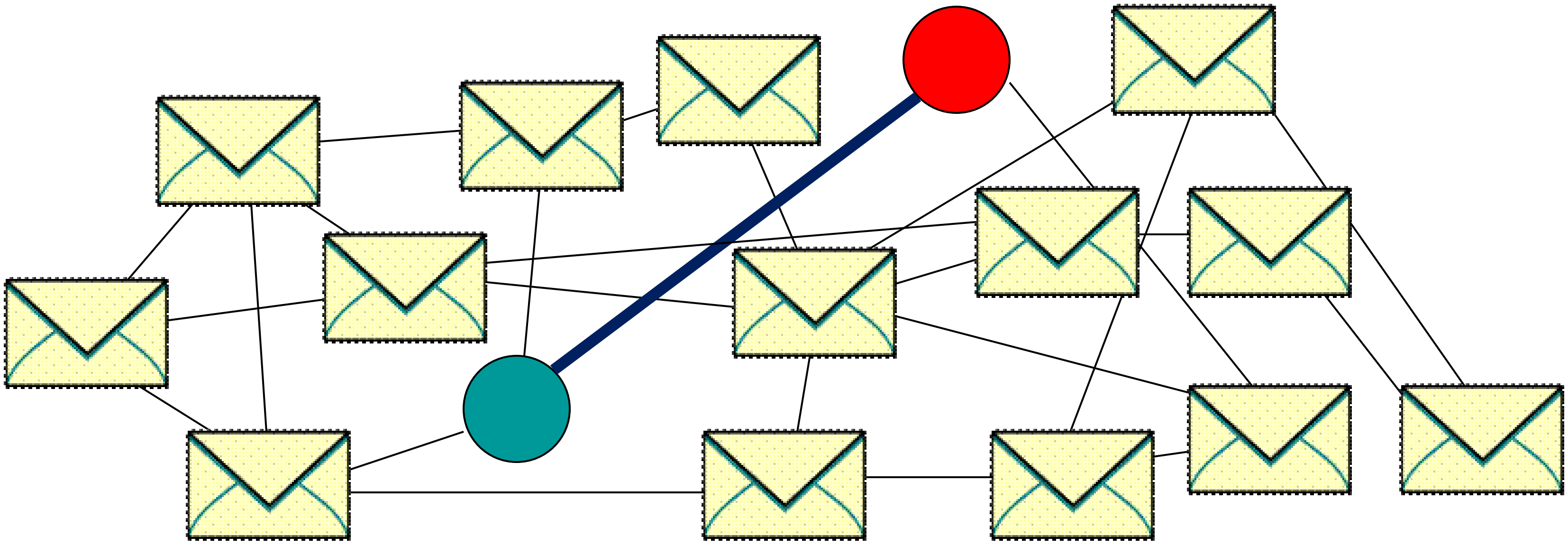
- Prover sends decommitments for opening the colors of the two nodes



ZKP for 3-Colorability

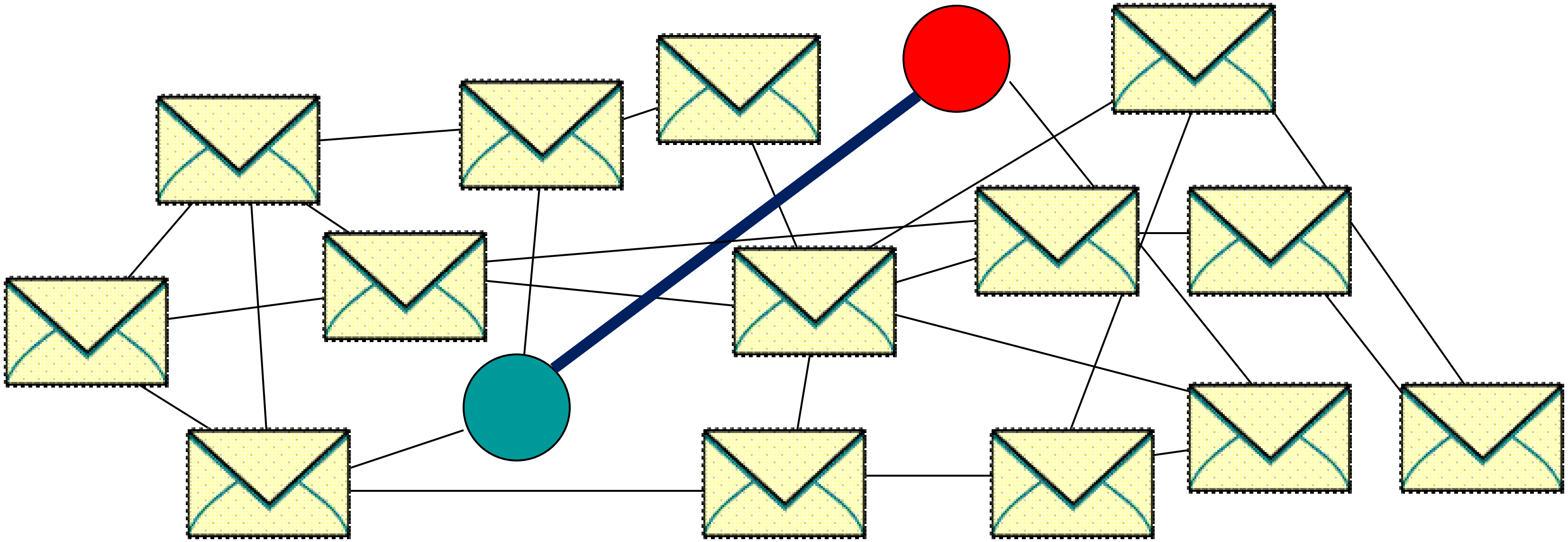
- Verifier accepts if both colors are valid and are distinct (otherwise it rejects).

- Repeat O(|E|) times to amplify soundness

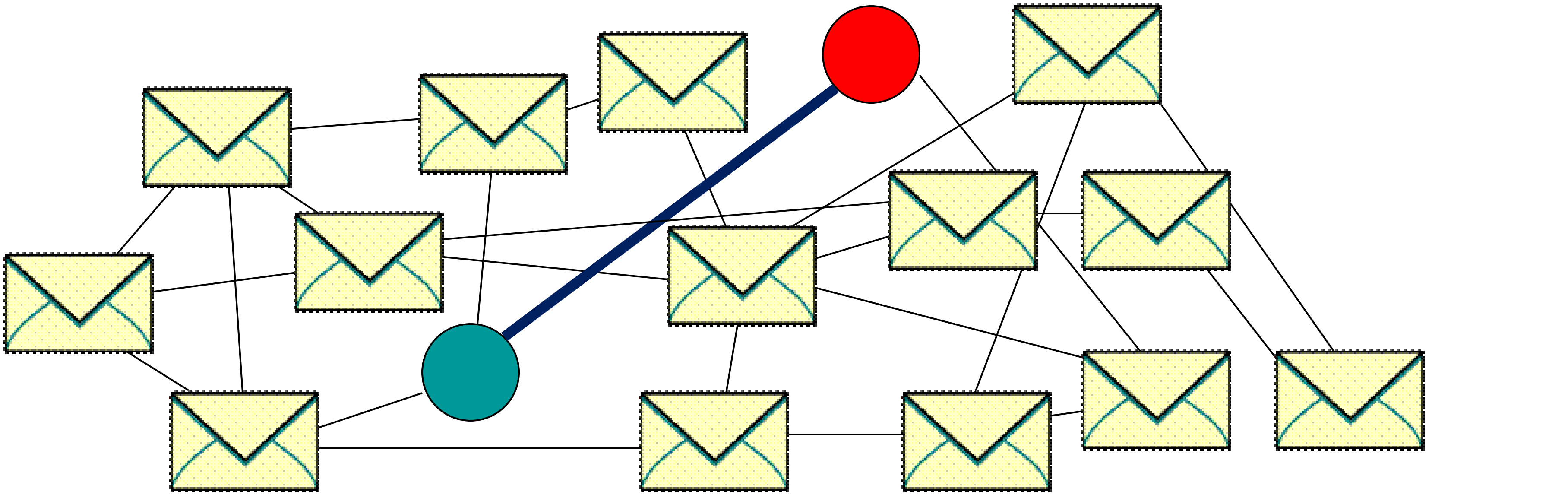


Issues

- Security proof more subtle than it may seem
  - Need to redo analysis of Hamiltonicity-based ZK?
- Two sources of inefficiency
  - Karp reduction
  - Soundness amplification (+ many rounds)



Abstraction to the rescue…

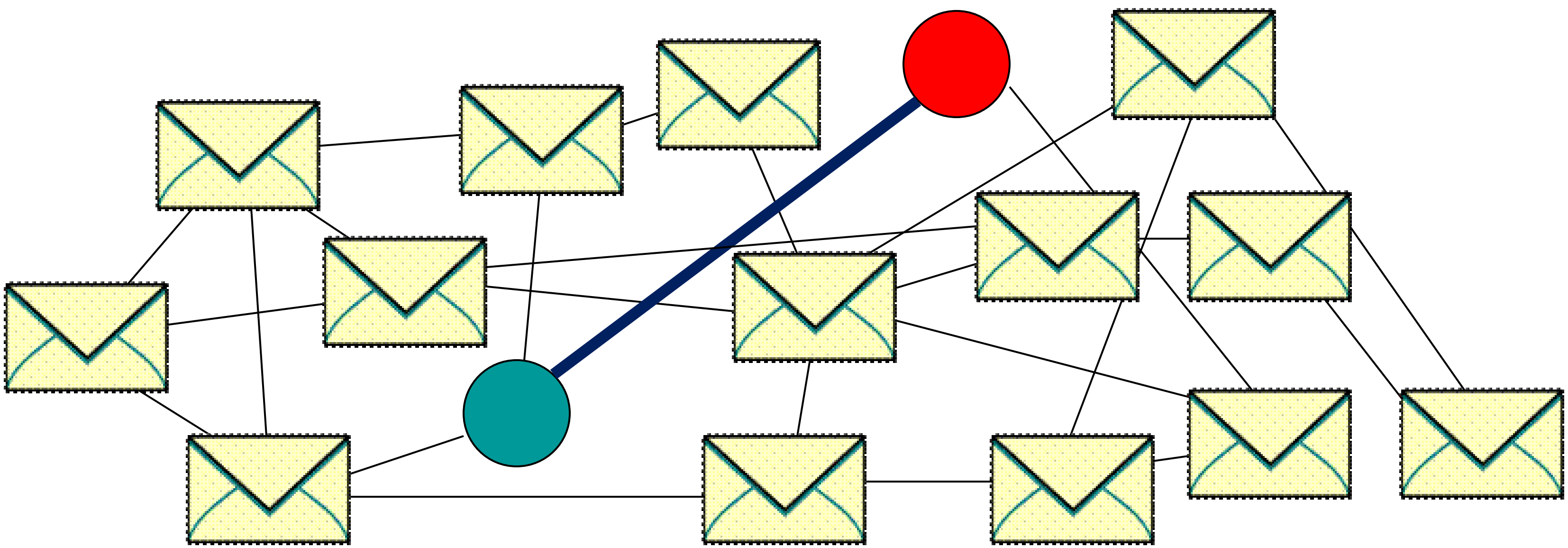


Information-Theoretic Proof System: ZK-PCP

Prover: (x,w) $\rightarrow \pi$

$\pi =$ | 1 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 | 1 | 3 | 1 | 3 | 1 | 2 | 1 |

Verifier



Information-Theoretic Proof System: ZK-PCP

Prover: (x,w) $\rightarrow \pi$

$$\pi = \boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{1}\boxed{3}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}$$

Verifier

- Simple security definition
  - Completeness
  - Perfect (public-coin) ZK
  - Soundness error ϵ
    (amplified via parallel repetition)

- Clean efficiency measures
  - Alphabet size
  - Query complexity
  - Prover computation
  - Verifier computation

Information-Theoretic Proof System: ZK-PCP

Prover: (x,w) $\rightarrow \pi$

$\pi =$ | 1 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 | 1 | 3 | 1 | 3 | 1 | 2 | 1 |

Verifier

Crypto compilers

+Stat-binding commit [GMW86, PW99]

+Stat-hiding commit [GK96]

ZK in plain model

+Random oracle [FS86,Mic00]

NIZK in ROM

Information-Theoretic Proof System: ZK-PCP

Prover: (x,w) → π

π = | 1 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 | 1 | 3 | 1 | 3 | 1 | 2 | 1 |

Verifier

Ron's talk:

NIZK in
Hidden Bits Model

Crypto compilers

+Stat-binding commit

[GMW86, PW99]

+Stat-hiding commit

[GK96]

+Trapdoor permutation

[FLS90]

ZK in plain model

NIZK in CRS model

Information-Theoretic Proof System: ZK-PCP

Prover: (x,w) $\rightarrow \pi$

$\pi =$ | 1 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 | 1 | 3 | 1 | 3 | 1 | 2 | 1 |

Verifier

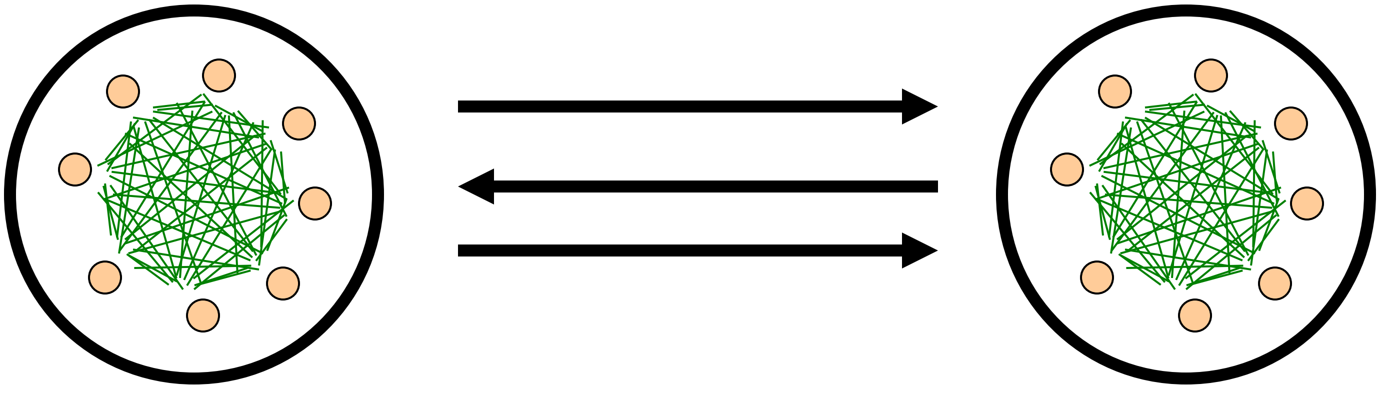
Better parameters?

Simpler?

Less “magical”?

IT Compilers:

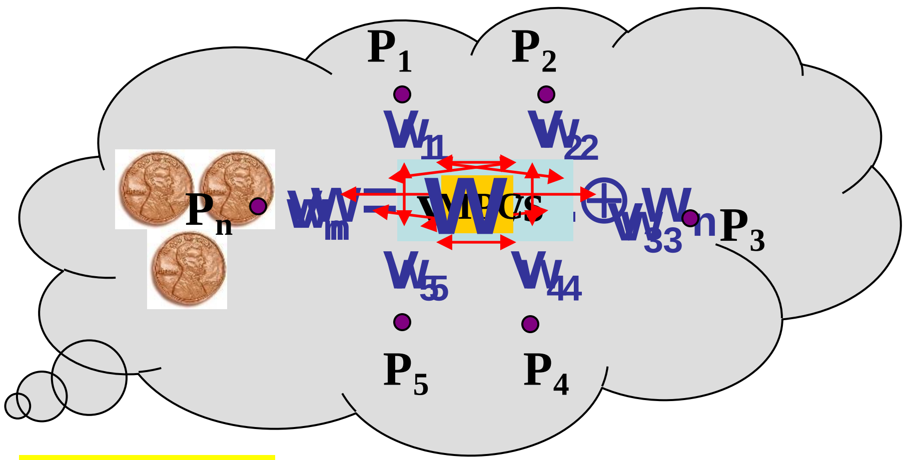
MPC $\rightarrow$ ZK-PCP



MPC ➔ ZK-PCP

[IKOS07]

Given MPC protocol for $\color{red}{f}(\color{blue}{w_1},\ldots,\color{blue}{w_n}) = \color{red}{R}(\color{blue}{w_1}\oplus\ldots\oplus \color{blue}{w_n})$



Prover

Verifier

π

V_1

V_2

V_3

V_4

…

V_n

i

j

accept iff output=1

&

V_i,V_j are consistent

Applications

- Simple ZK proofs using:
  - (2,5) or (1,3) semi-honest MPC [BGW88,CCD88,Maurer02]
  - (2,3) or (1,2) semi-honest MPC^{OT} [Yao86,GMW87,GV87,GHY87]
  - Practical! [GMO16,CDG+17,KKW18] ➔ post-quantum signatures!

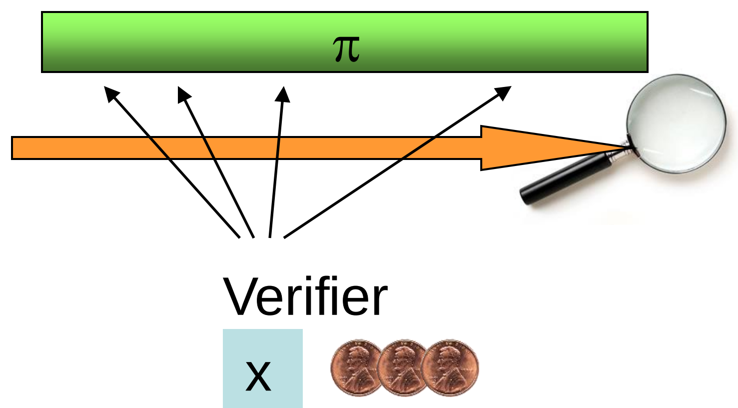
- ZK proofs with O(|R|)+poly(k) communication
  - MPC from AG codes [CC05,DI05]

- Many good ZK protocols implied by MPC literature
  - MPC for linear algebra [CD01,…]
  - MPC over rings [CFIK03] or groups [DPSW07,CDI+13]

- Going (somewhat) sublinear! [AHIV17] – Carmit's talk

Going fully sublinear

Traditional PCPs



- $x \in L$ ➔ $\exists \pi$ Pr[Verifier accepts π] =1
- $x \notin L$ ➔ $\forall \pi^*$ Pr[Verifier accepts π^*]≤1/2

- PCP Theorem [AS,ALMSS,Dinur]:
  NP statements have polynomial-size PCPs in which the verifier reads only O(1) bits.
  - Can be made ZK with small overhead [KPT97,IW04]

Still need crypto compiler…

Verifier

Prover

Input x

ZK-PCP $\pi \in \{0,1\}^{poly(|x,w|)}$

$\xrightarrow{q1,q2,q3}$

$\xleftarrow{\pi_{q1}, \pi_{q2}, \pi_{q3}}$

ACC/REJ

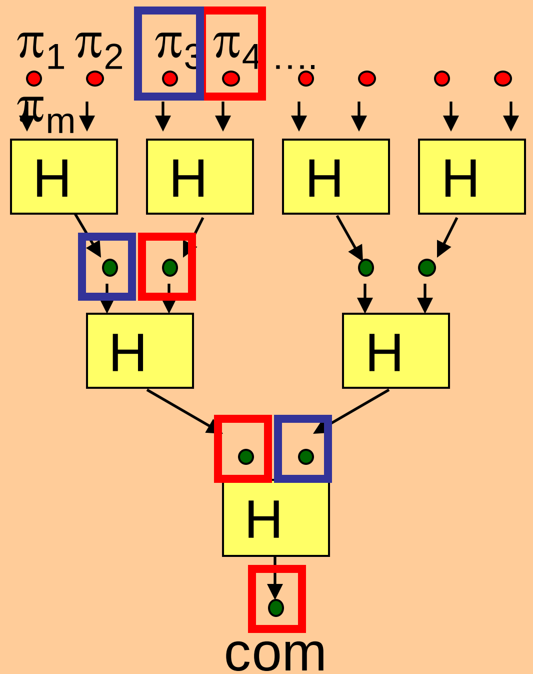
Crypto Compiler

[Kil93,Mic94]

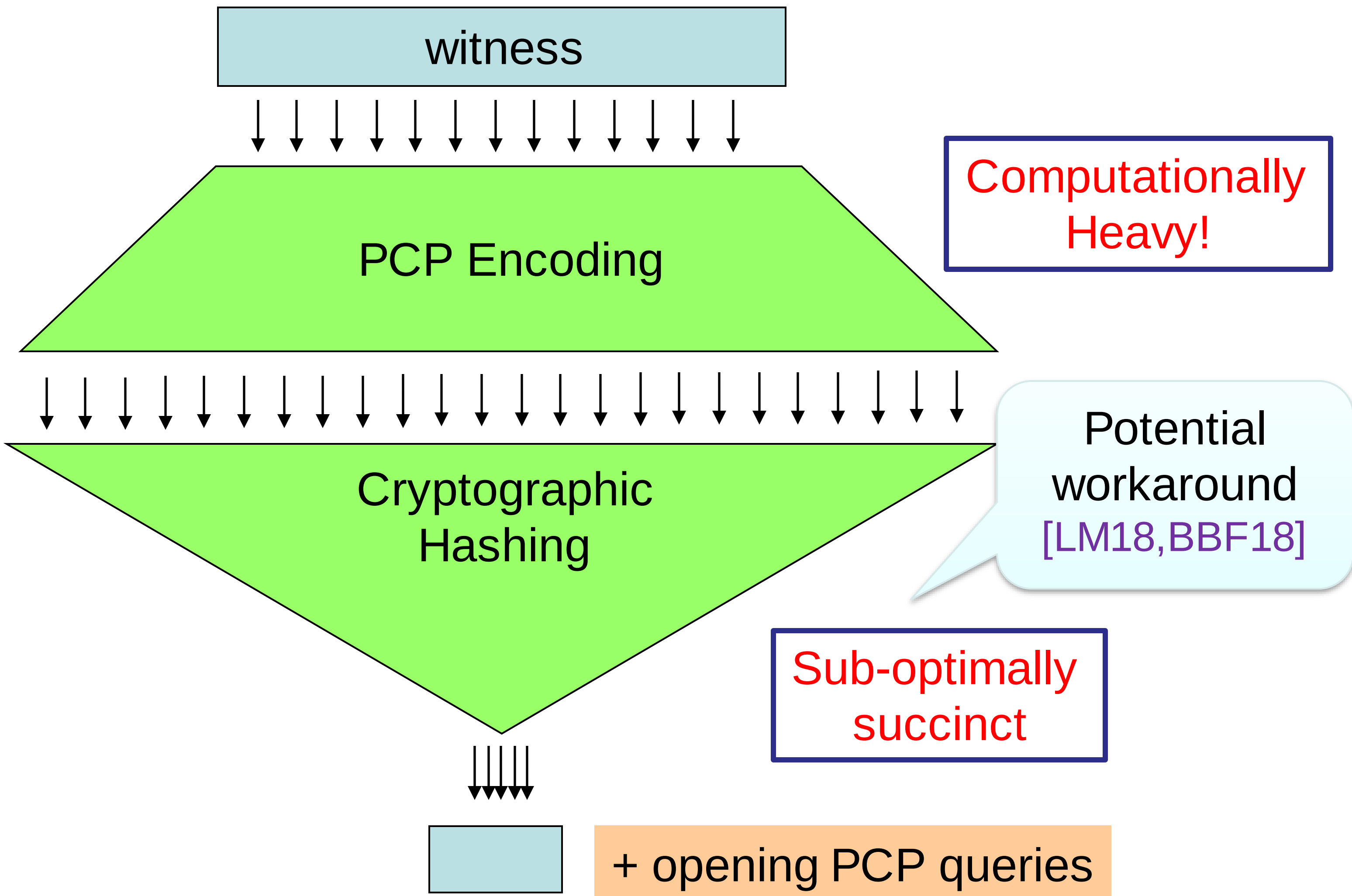
Merkle Tree construction

H = collision resistant hash function

$H:\{0,1\}^* \rightarrow \{0,1\}^k$



Limitations



Relaxing PCP model 1: Interaction

Prover

$$\pi_1 = \boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{1}\boxed{3}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}$$

Verifier

Challenge

$$\pi_2 = \boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}\boxed{1}\boxed{3}\boxed{1}\boxed{3}\boxed{1}\boxed{2}\boxed{1}$$

Interactive PCP [KR08,GIMS10]
IOP [BCS16,RRR16]

Verifier

Challenge

Relaxing PCP model 2: Linear PCP

[ALMSS98,IKO07,BCIOP13]

over a (large) finite field F

Prover

$\pi =$	4	3	1	2	8	3	1	2	1	9	3	1	6	1	2	1

inner product

$q_1 =$	5	3	6	2	1	3	1	2	1	1	6	1	3	1	8	1
$q_2 =$	7	3	1	2	4	3	1	2	7	1	3	1	7	1	2	1
$q_3 =$	1	2	1	2	1	9	1	2	5	1	4	1	3	1	3	1

Verifier

a_1

a_2

a_3

x

ACC / REJ

Advantages of Linear PCPs

- Simple!
  - Hadamard PCP: π = (W, W x W)
- Short, efficiently computable
  - $O(|C|)$-size, quasi-linear time via QSP/QAP [GGPR13, …]
- Negligible soundness error with O(1) queries
  - Reusable soundness

    Pr[π^* is accepted] is either 1 or O(1/|F|)
  - Maximal succinctness
  - In fact, 1 query is enough! [BCIOP13]

Crypto Compilers for Linear PCPs

- First generation [IKO07,GI10,Gro10,SMBW12,…]
  - Standard assumptions
    - Linearly homomorphic encryption, discrete log
  - Interactive, one-way-succinct/somewhat succinct
  - Idea: use succinct vector-commitment with linear opening
- Second generation [Gro10, Lip12,GGPR13, BCIOP13,…]
  - Strong "knowledge" or "targeted malleability" assumptions
  - Non-interactive using a (long, structured) CRS
  - Publicly verifiable via pairings
  - Idea: include "encrypted queries" in CRS

Crypto Compiler: First Attempt

Prover

$\pi =$	4	3	1	2	8	3	1	2	1	9	3	1	6	1	2	1

$q_1 =$	5	3	6	2	1	3	1	2	1	1	6	1	3	1	8	1
$q_2 =$	7	3	1	2	4	3	1	2	7	1	3	1	7	1	2	1
$q_3 =$	1	2	1	2	1	9	1	2	5	1	4	1	3	1	3	1

Verifier

a_1

a_2

a_3

x

$\Rightarrow$ ACC / REJ

Crypto Compiler: First Attempt

CRS

$q_1 =$	5	3	6	2	1	3	1	2	1	1	6	1	3	1	8	1
$q_2 =$	7	3	1	2	4	3	1	2	7	1	3	1	7	1	2	1
$q_3 =$	1	2	1	2	1	9	1	2	5	1	4	1	3	1	3	1

Prover

$\pi =$	4	3	1	2	8	3	1	2	1	9	3	1	6	1	2	1

Verifier

a_1

a_2

a_3

x

ACC / REJ

Crypto Compiler: First Attempt

CRS

$q_1 =$

$q_2 =$ linearly homomorphic encryption

$q_3 =$

Prover

$\pi =$

4	3	1	2	8	3	1	2	1	9	3	1	6	1	2	1

Verifier

a_1

a_2

a_3

x

ACC / REJ

Crypto Compiler: First Attempt

CRS

$q_1 =$

$q_2 =$ linearly homomorphic encryption

$q_3 =$

Prover

$\pi =$ 4 3 1 2 8 ... 2 1 9 3 1 6 1 2 1

a$_1$

a$_2$

a$_3$

Problem 1: May allow more than just linear functions!

Solution 1: Assume it away: "linear-only encryption"

- A natural instance of targeted malleability [BSW12]
- Plausible for most natural public-key encryption schemes
  … including post-quantum ones [Reg05,BISW17]
- Win-win flavor

Crypto Compiler

CRS

$q_1 =$

$q_2 =$ linearly homomorphic encryption

$q_3 =$

Prover

$\pi =$ 4 | 3 | 1 | 3 | 1 | 2 | 1 | 9 | 3 | 1 | 6 | 1 | 2 | 1

Problem 2: Prover can apply different π_i to each q_i or even combine q_i

Solution 2: Compile LPCP into a proof system that resists this attack

- Linear Interactive Proof (LIP): 2-message IP with “linear-bounded” Prover
- IT compiler: LPCP → LIP via a random consistency check [BCIOP13]

Crypto Compiler

CRS

$q_1 =$

$q_2 =$ linearly homomorphic encryption

$q_3 =$

Prover

$\pi =$

4	3	1		8	3	1	2	1	9	3	1	6	1	2	1

Problem 3: Only works in a designated-verifier setting

Solutions 3:

- Look for designated verifiers around your neighborhood
- LPCP with deg-2 decision + "bilinear groups" → public verification [Gro00,BCIOP03]

Combining the Two Relaxations: Linear IOP

Variant: ILC model
[BCGGHJ17]

Prover

$\pi_1 =$	1	3	1	2	1	3	1	2	1	1	3	1	3	1	2	1
$q_1 =$	5	3	6	2	1	3	1	2	1	1	6	1	3	1	8	1

Verifier

Challenge

$\pi_2 =$	1	3	1	2	1	3	1	2	1	1	3	1	3	1	2	1
$q_2 =$	7	3	1	2	4	3	1	2	7	1	3	1	7	1	2	1

Challenge

Implicit in interactive proofs for P
[GKR08,RRR16]

Fully Linear PCP/IOP
[BBCGI19]

- Suppose statement x is known to prover but is
  - Secret-shared between two or more verifiers
  - Partitioned between two or more verifiers

- Goal: strong ZK, hiding x as well

- Tool: fully linear ZK proof systems
  - Only allow linear access to x: q_i applies jointly to (x, π)
  - Can be naturally compiled to ZK in above settings
    - Also with linearly encrypted or committed input
    - Implicitly used in previous systems [BGI16,CB17]

Fully Linear PCP/IOP [BBCGI19]

- Constructions: NP languages
  - Standard LPCPs for NP are fully linear, but big proofs
  - Meaningful also for “simple” languages in P!

- Sublinear-size proofs for “simple” languages
  - Implicit in interactive proofs [GKR08,RRR16,NPY18]
  - New constructions for low-degree polynomials
    - E.g., test that $x \in F^n$ is in $\{0,1\}^n$

Conclusions

- Modular approach to efficient ZKP design
  - Information-theoretic ZK-PCP + crypto compiler
    - point queries vs. linear queries
    - non-interactive vs. interactive
- Applies to most efficient ZKP from the literature
  - In a sense inherent to "black-box" constructions [RV09]
  - but not to non-bb constructions [Val09,BCCT13,BCTV14]
- Lots of room for further progress
  - Better PCPs (and lower bounds)
  - Better crypto compilers
  - Better IT compilers

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