HKDF: Key Derivation and Extraction in Practice

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Main reference: https://eprint.iacr.org/2010/264

(more references at the end)

Key Derivation Functions (plan)

- KDFs: What? Why? How?
- Extract-then-Expand approach
- HKDF (new KDF standard)
  - WhatsApp, Facebook Messenger, Google QUIC and Allo, Signal, TLS 1.3, NIST, …
- HKDF design and rationale
- Sample results
- Applications

Key Derivation Functions (KDF)

- A truly <u>fundamental primitive</u> in applied cryptography
  - A process producing cryptographic keys out of some initial input
  - A somewhat overlooked crucial component of key exchange
- Zillion applications (over-charged notion):
  - Key expansion, key extraction, key hierarchies
  - Key-exchange protocols, Hybrid encryption, Key wrapping, Physical RNGs, System PRNGs, Password-derived keys
- So what is it, really?
- Can we have a *single* scheme for *all* these uses?

Surprisingly Little Formal Work

- Research: Surprisingly little literature
- Practice: Plagued by multiple schemes, almost all ad-hoc, little or naïve rationale
- Dominated by hash-based schemes that treat hash as perfect function ("random oracle")
- Needed: Widely accepted *multi-purpose* *standard* mechanism

The Challenge

- A practical but theoretically well-founded KDF scheme
  - But we do not even have definitions (or a full understanding of the extensive meaning/requirements of KDFs)
- Prudent use of hash functions: Minimize as much as possible assumptions on underlying hash scheme
  - Different uses → different requirements
- Single scheme, simple, efficient, hash-based
- Suitable for industry-wide standard

KDF: Two Main Functionalities

- **Key Extraction**: Derive a cryptographically strong key from an *"imperfect source of key material"*
  - Imperfect RNG, system entropy sources, Diffie-Hellman (KE), ...
- **Key Expansion**: Given a cryptographically strong key derive more keys
- Two fundamentally different functionalities
- Often mixed/confused in ad-hoc KDF schemes (a recipe for weaknesses and pitfalls)

Keys = Hash(s || "1") || Hash(s || "2") || …

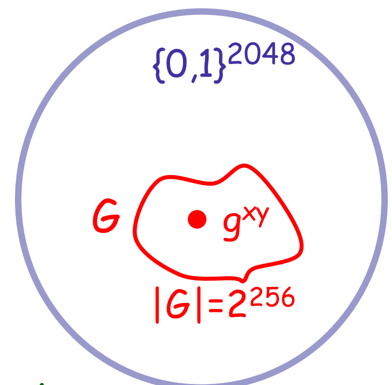
Example of Sources of Key Material

- A uniform random and secret master key (say, 256 bits)
  - The key expansion case
- Imperfect physical RNG (random number generators)
  - e.g., bit 0 with ~0.45 probability
- Software PRNG
  - Entropy source: e.g. sampled events, user's key strokes, etc
  - Attacker has partial knowledge, can even influence source, yet conditional entropy (attacker's uncertainty) assumed to be significant
- A Diffie-Hellman value g^{xy} output by a key exchange
  - restricted/computational entropy

DH as a source of randomness

- Diffie-Hellman key exchange outputs g^{xy} in a group G from which one needs to "extract" a cryptographic key.
  - We treat g^{xy} as a source of "imperfect randomness"
- DDH: g^{xy} indistinguishable from random element in G
  - Example. G over Z_p^* of order q, |q|=256, |p|=2048

    → g^{xy} has 256 bits of entropy "trapped" in a 2048 long number
  - Very non-uniform in Z_p^* but *sufficient entropy* (256-bit) to extract key



- Sufficient entropy? Statistical entropy of g^{xy} is 0 (attacker knows g^x,g^y)

  But *computationally* (by DDH) attacker has no information on g^{xy}

  ➔ sufficient *computational entropy* for extracting a key

See [Gennaro-K-Rabin, Eurocrypt 2004]

The DH Example (cont.)

- What if DDH does not hold, or protocol does not guarantee indistinguishability from uniform?
- Can only rely on CDH: g^{xy} hard to guess but not necessarily indistinguishable from uniform
  - Need to extract keys based on unpredictability of g^{xy}
  - Hard-core function as extractor (can use dedicated functions, e.g lsb's, or cryp'c hash functions under suitable assumptions)
- Other considerations: Independence of samples (g^{xy} vs $g^{x(y+1)}$), (independence of samples an issue for all extractor applications)

Imperfect Source of Randomness (source key material)

- Imperfect: non-uniform, partial knowledge by attacker
- But substantial *conditional entropy*, e.g. 160 bits, though not necessarily uniform
  - Entropy is ***conditioned*** on knowledge by attacker
  - **Entropy can be computational** (e.g. Diffie-Hellman)
    - Computational hardness as a source of randomness (uncertainty)
    - HILL entropy (indistinguishable from a high-entropy source, DDH)
    - Unpredictability entropy (one-wayness, e.g. CDH)

Source Entropy: min-entropy

- Large Shannon entropy of source not sufficient to guarantee close-to-uniform output
  - Can have a high-probability element in the source which implies a high-probability value in the output, i.e. far from uniform.
- Need *min-entropy:* No input assigned too high probability
  - A probability distribution X has min-entropy m if for all x, $Prob_X(x) \leq 2^{-m}$ (i.e. m = $-\log_2$ of highest probability)
- In our applications, *computational* min-entropy suffices
  - Source is computationally indistinguishable from a distribution that has that amount of true min-entropy

Module I: Key Extraction

- Key Extraction: Derive a cryptographically strong key from a given *source of keying material*
  - imperfect source but with *sufficient* min-entropy
- Process: Source--> Sample --> Extract --> Key
  - Output key used to bootstrap the key expansion stage

Module II: Key Expansion

- Given a first strong key derive more keys
  - K → K1, K2, K3 (e.g. keys for MAC, encryption, etc)
  - Requirement: pseudo-randomness (even given partial knowledge) (pseudorandom = computationally indistinguishable from uniform)
  - Standard implementation via PRG/PRF
- Usually additional "context parameter" (→ need for PRF)
  - For example: Ki = PRF_K (i, "context")
  - "context" could be a functionality ("mac"), a protocol name ("ssl"), a session or user identity, etc. (a.k.a. domain separation)

Extract-then-Expand

- Two well differentiated modules, for the two well differentiated functionalities
- Basis for design and analysis
  - modules are orthogonal and replaceable
  - can implement both with same underlying cryptographic primitive (hash functions or block ciphers)
  - HKDF: a specific hash-based design, uses HMAC for both
- **First, we need some definitions**

Formalizing KDFs

- KDF: A transformation from a (weak) source of keying material to a pseudorandom key. But
  - Attacker has full knowledge of source distribution and partial knowledge on specific sample
  - Attacker can influence output by choosing context information (e.g. user identities, nonces, etc.)

- I am skipping formal definitions for this class
  - See next hidden slides and HKDF paper

Extract-then-Expand

- "Extract-then-expand" paradigm

K_{prf} = Extract(salt*, skm)   skm= source key material

Keys = Expand(K_{prf} , Keys-length, ctxt_info)

Binds key to the application "context"

- salt: practice jargon for "a random *non-secret* quantity" ; in our setting it works as an ***extractor seed***   (→ strong extractor)

Instantiating Extract-then-Expand

- Expand: Just a PRF (with variable input/output length)
- Extract: (strong) randomness extractors
- Limitations of info-theoretic/combinatorial extractors
  - practical schemes require large salt (~ |input|)
  - entropy loss* (e.g. 256-bit DH → 160-bit SHA: security of 2^{-48})
  - unsuited for extraction-from-unpredictability (e.g. only CDH) or deterministic extraction ("hard-core functions")
  - some crypto scheme proven only with RO-derived keys
  - cases where independence of samples is not ensured

Idea: Use a PRF for both Expand and Extract

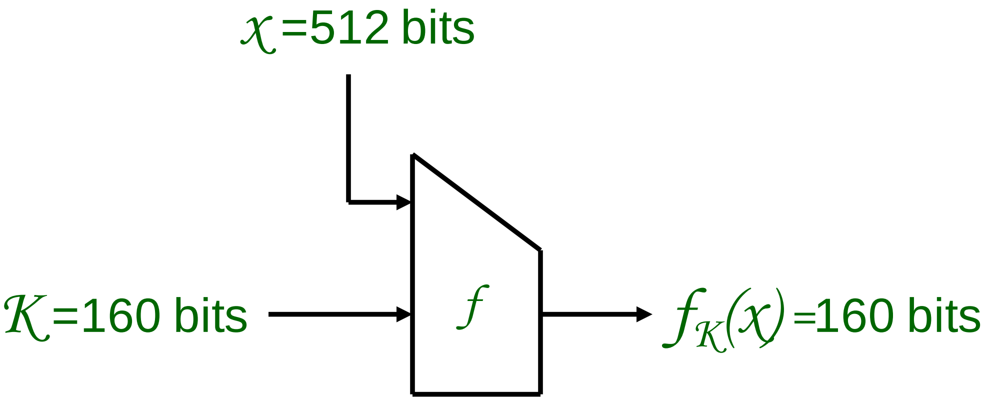
- We need a PRF for expand, can we use it for extract?
- Replace PRF's key with a random, but known, seed (*salt*)
  - Extract(salt, sample) = PRF_{salt}(sample)
- Unfortunately, a PRF w/ a known key has no guarantee
  - Counter-examples use artificial (PK-based) constructions
  - Maybe practical hash-based PRFs do work (somehow)?
  - HMAC: The standard hash-based PRF
- We'll see: HMAC enjoys good extraction properties

→ HKDF

NMAC
A 2-slide ~~HMAC~~ Primer

Merkle-Damgard Hash Functions

- Compression function



Merkle-Damgard Hash Functions

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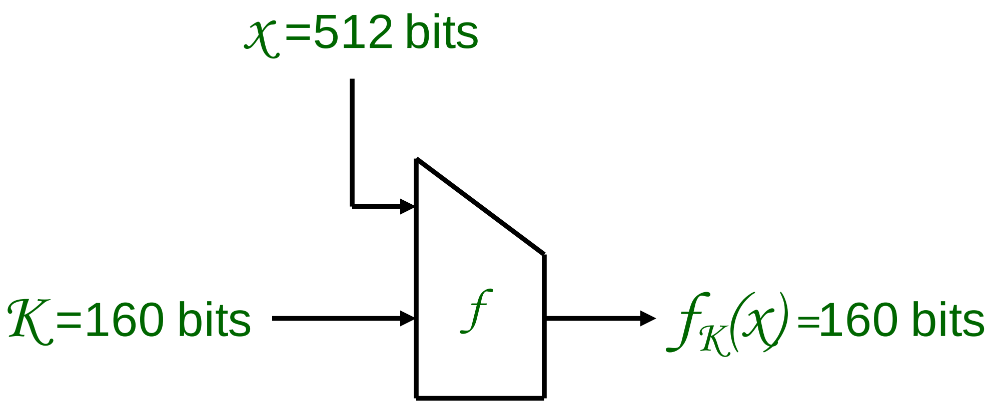
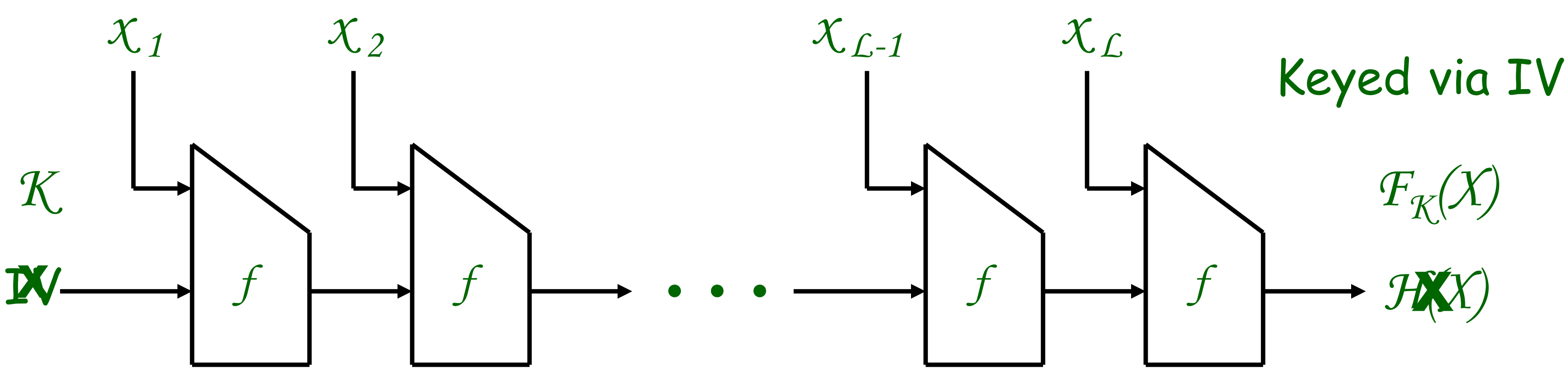
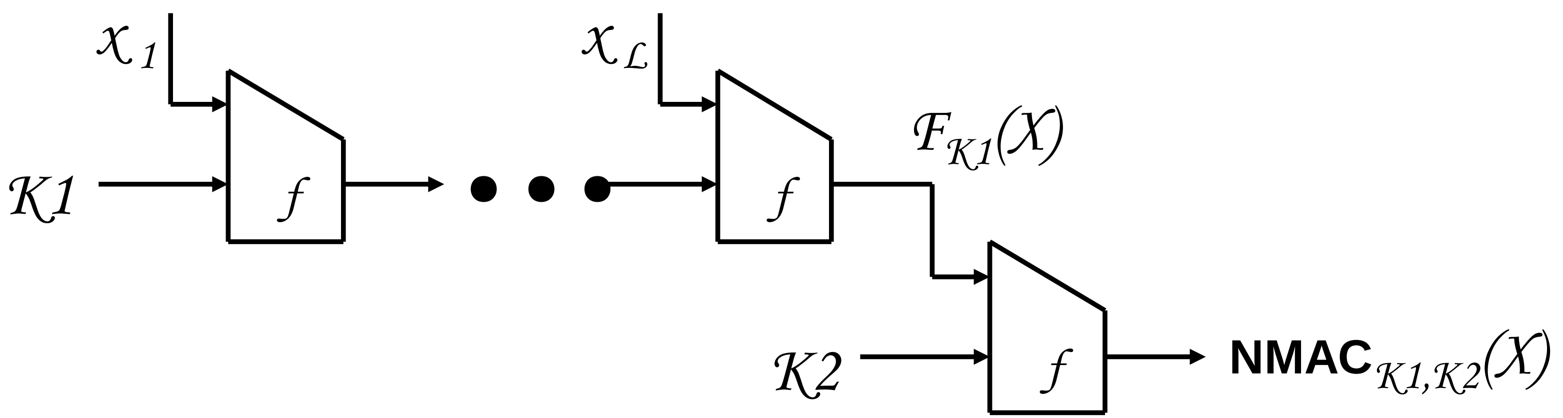


- (Unkeyed) Merkle-Damgard iterated hash



NMAC: PRF mode for Merkle-Damgard

- $NMAC_{K1,K2}(x) = f_{K2}(F_{K1}(x))$
  - f= comp. function, F= keyed M-D



- Provable PRF if compression function is PRF
- HMAC = Same with K1, K2 derived from a single K (and black box use of hash function)

HKDF: HMAC-based KDF

(HMAC as extractor and PRF)

K_{prf} = HMAC(salt, skm)   skm= source key material

Keys = HMAC*(K_{prf} , keys_length, ctxt_info)

where Keys = K_1 || K_2 || . . .

K_{i+1} = HMAC(K_{prf}, K_i || ctxt_info || i)   Feedback mode

Note use of a PRF with salt, a random but non-secret "key"

(sometimes we'll set salt = 0)

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(sometimes we'll set salt = 0)

Properties of HMAC to support HKDF

- Results that back HMAC in a variety of relevant applications:
  - Single function (hash, random oracle)
  - Family of functions with secret or public keys
  - Functionalities: PRF, extractor, random oracle, collision resistance
- Results in the form of: *If compression function has property A then HMAC has property A'*
  - Examples: PRF, delta-AU, extractor, RO
  - Note: NMAC vs HMAC

PRF and RO-based results

- If compression function f is PRF then NMAC is a PRF
- If f is a RO family then HMAC is indifferentiable from RO ("indifferentiable" = indistinguishability for ideal objects)
- Corollary: If f is RO, HMAC is a good extractor and a good hard-core (on distributions that are independent from f)
  - Useful in restricted cases: CDH-only, small gap, no salt, …
- $f(H_K(x))$ is a good extractor if f is RO and H_K is δ-AU
  - δ-AU is implied by collision resistance (design goal for hash f'n)

Non-idealized Assumptions

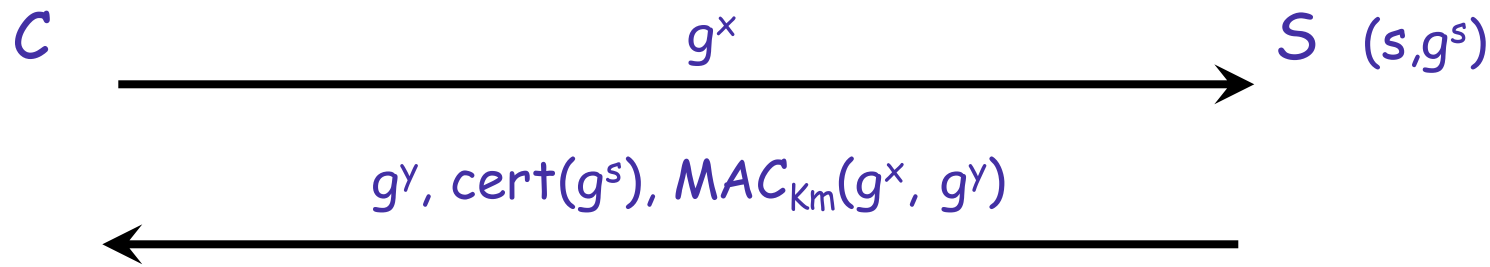
- If $\{f_k\}$ is a good extractor family and also a PRF then NMAC is a good k-bit extractor on any distribution w/ blockwise entropy k
  - Application to IKE/DH with safe primes

- If $\{f_k\}$ is strongly universal and $\{H_k\}$ is coll. resistant against linear-size circuits, then NMAC *truncated by c bits* is $(n2^{-c/2})$-statistically close to unif.
  - Application: HKDF with SHA-512 for extraction, SHA-256 for PRF → 128-bit security under very mild assumptions

(versatile) application of HKDF

- IKE (IPsec Key Exchange)
  - SK = HKDF(nonces, g^{xy}) - (nonces exch'd and auth'd during KE)
  - Dual use of HKDF:
    - cleartext nonces → HKDF as extractor (nonces = salt)
    - Secret nonces → HKDF as PRF (PKE mode of IKE)
- TLS 1.3 with shared key K (e.g. resumption)
  - SK = HKDF(K, g^{xy})
  - If K revealed, K acts as salt (random) and HKDF as extractor (PFS)
  - If K secret and g^{xy} revealed, HKDF acts as PRF.

Application Example (OPTLS KDF)



- SK ← derived from g^{xs} (static) and g^{xy} (ephemeral/PFS) via HKDF
  - K_{xs} = HKDF(0, g^{xs})
  - K_{xy} = HKDF(0, g^{xy})
- SK = HKDF(K_{xs}, K_{xy}): Secure as long as one of g^{xs}, g^{xy} not exposed

Application Example (OPTLS KDF)

C → g^x → S (s, g^s)

C ← g^y, cert(g^s), $MAC_{Km}(g^x, g^y)$ ← S

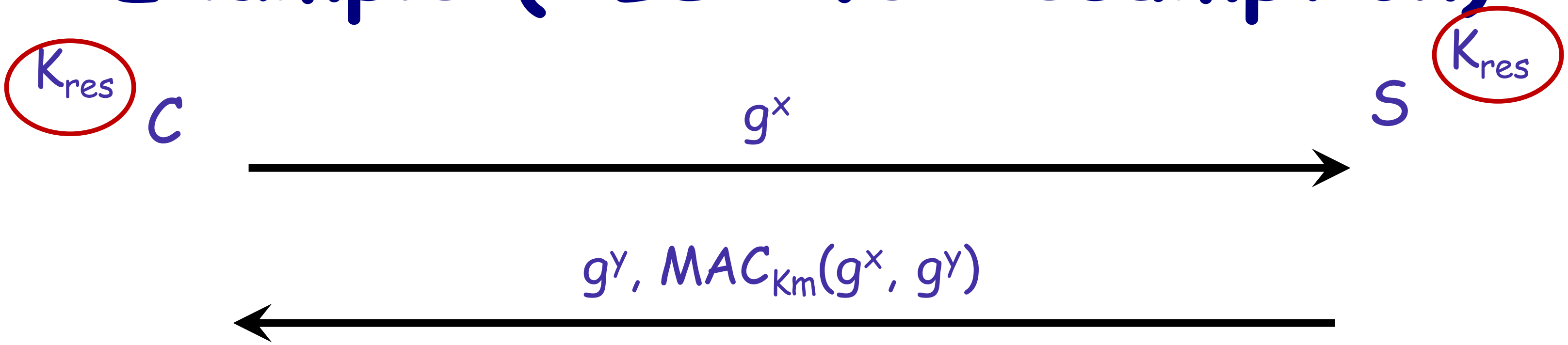
- SK ← derived from g^{xs} (static) <u>and</u> g^{xy} (ephemeral/PFS) via HKDF
  - K_{xs} = HKDF(0, g^{xs}): Implements RO(g^{xs}) for CCA security (~DHIES)
  - K_{xy} = HKDF(0, g^{xy}): Implements Extract(g^{xy}) with salt=0
- SK = HKDF(K_{xs}, K_{xy}): Secure as long as one of g^{xs}, g^{xy} not exposed
  - If g^{xs} not compromised then HKDF(K_{xs}, …) a PRF
  - If g^{xs} eventually compromised (the forward secrecy case) then HKDF(K_{xs}, …) works as extractor w/ random but public salt K_{xs}
    - K_{xs} was generated by honest parties, hence uniform

Minimize use of ROM

Note: Why salt=0 in K_{xy} and K_{xs} ?

- Because we don't have authenticated randomess to use as extractor seed
- Unauthenticated seed can be chosen by attacker and break source-seed independence or chosen as "weak seed" (e.g. DRST'13)
  - Contrast IKE where salt = ($nonce_A$,$nonce_B$) which are signed before use
    - Note: KE guarantees security of a key only with honest peer

Example (TLS 1.3 Resumption)



- SK ← derived from K_{res} (static) <u>and</u> g^{xy} (ephemeral/PFS) via HKDF
  - K_{xs} = HKDF(0, K_{res}): Implements RO(K_{res}) if K_{res} is low entropy, e.g pwd
  - K_{xy} = HKDF(0, g^{xy}): Implements Extract(g^{xy}) with salt=0
- SK = HKDF(K_{res}, K_{xy}): Secure as long as one of g^{xs}, g^{xy} not exposed
  - If K_{res} not compromised then HKDF(K_{res}, …) a PRF
  - If K_{res} eventually compromised (the forward secrecy case) then HKDF(K_{res}, …) works as extractor w/ random but public salt K_{res}
    - K_{res} was generated by honest parties, hence uniform

HKDF as Collision Resistant

- TLS 1.3: Simultaneous RO, PRF, Extractor,… **CRHF**
- Use case: Binding resumption key to original HS session
  - bind(C,S, session-id), Mac_{Km}(bind(…), …)
  - bind can be CRHF(C, S, session-id) but allows traceability
  - Instead: K_{bind} = HKDF(g^{xy}, C, S, session-id) at orig session
  - During resumption use K_{bind} as a key to create a *one-time* bind value MAC_{Kbind} (…)
- Crucial point: Derivation of K_{bind} requires *CR key deriv.*
  → *Another HKDF goodie* (derives from underlying hash)

Standards and Deployments

- Becoming the industry-wide standard for KDF
- IETF (RFC 5869): Already 18 RFC's use it + many internet drafts (incl. TLS 1.3)
- NIST: NIST SP 800-56C (Recommendation for Key Derivation through *Extraction-then-Expansion*)
- Industry implementations: TLS 1.3, Google QUIC, WhatsApp, Facebook Messenger, "Snowden's" Signal, …
- Bonus: "extract" made it into IETF jargon/notion…

Theory and Practice

- Theory: understanding requirements, formalizing, weaknesses in existing solutions, generalization, design, analysis, minimize RO
- Practice: Engineering considerations, minimize compromise, conservative design
  - minimize RO, "bad adviser"
- Combination: Proof-driven design®