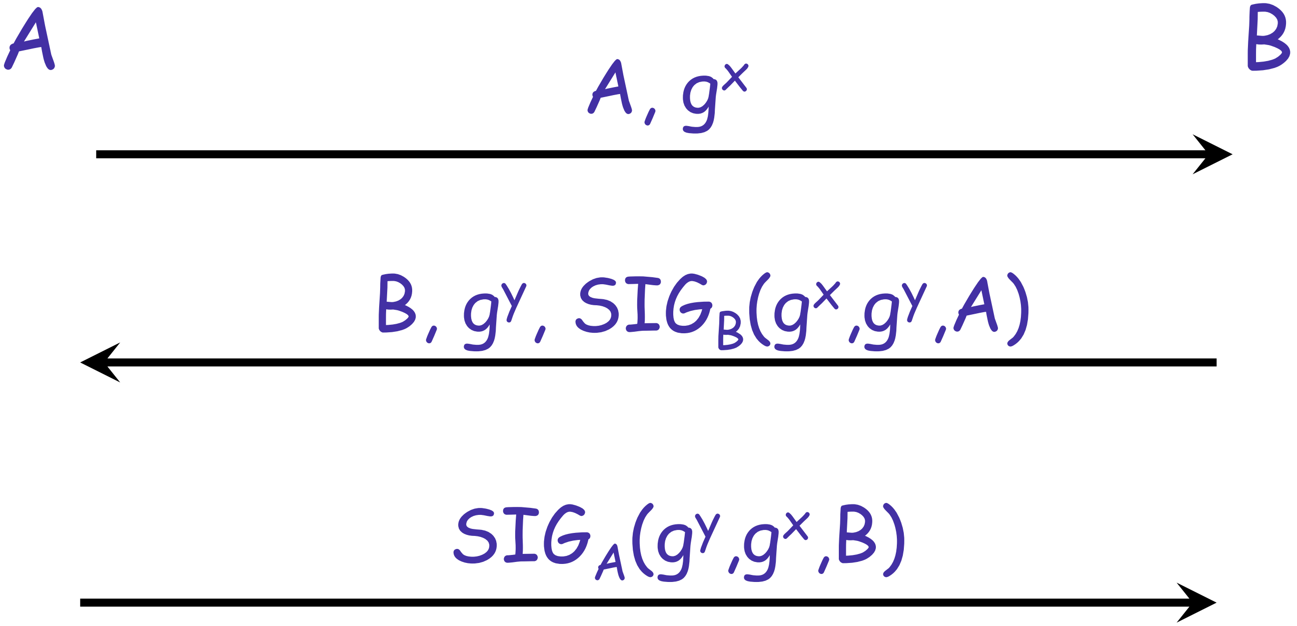
Extreme minimality:

Implicitly Authenticated KE Protocols

A natural Authenticated DH Solution (ISO 9796)



A

B

A, g^x

B, g^y, $SIG_B(g^x,g^y,A)$

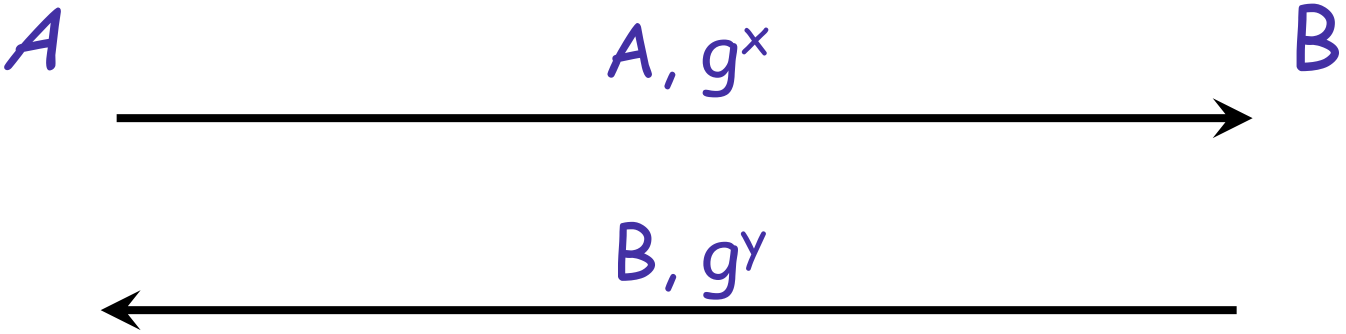
$SIG_A(g^y,g^x,B)$

Simple, but 3 messages plus signatures [and certificates]

The quest for Authenticated DH

- What is the inherent cost of authentication in Diffie-Hellman? In terms of
  - Communication: number of messages, group elements, authentication information, actual message size
  - Computation: algebraic operations and actual speed
  - Security: *What can we prove?*
- How close can we get to the fundamental limits? And still prove security…

Implicitly Authenticated DH



A

B

A, g^x

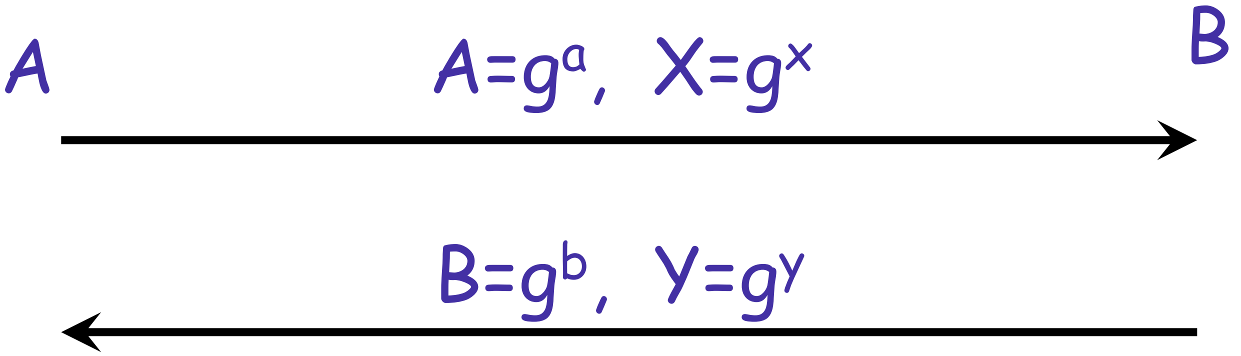
B, g^y

- Authentication via session key computation
  - □ No transmitted signatures, MAC values, etc
  - □ Session key must involve long-term and ephemeral keys:

    $K=F(PK_A,PK_B,SK_A,SK_B ,g^x,g^y,x,y)$
  - □ Ability to compute key ➔ authentication
- The simpler the trickier: many insecure proposals

(Abuse of) Notation

Public key of A (resp. B) denoted $A=g^a$ (resp. $B=g^b$)



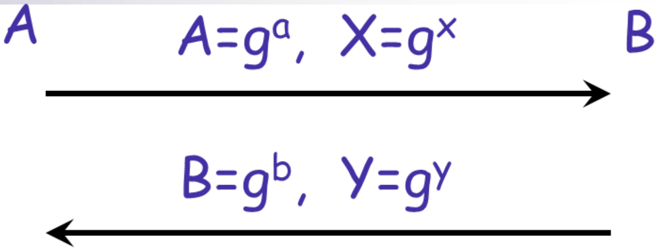
A

B

$A=g^a,\ X=g^x$

$B=g^b,\ Y=g^y$

Some Ideas

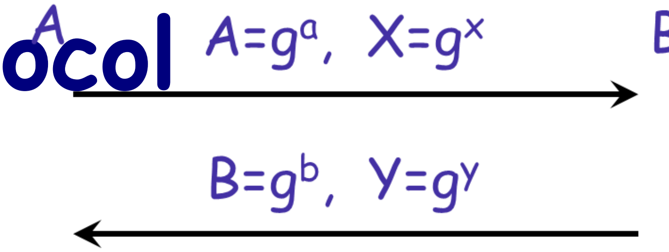


- Can we really have a *non-replayable* 2-msg protocol?
  - □ Remember A→B: g^x, $SIG_A(g^x,B)$, A→B: g^y, $SIG_B(g^y,A)$ insecurity
- Combining A, B, X, Y:
  - □ K=H(gab, gxy): Open to known key and interleaving attacks
  - □ K=H(gab, gxy, gx, gy) works but open to "KCI attacks" (a general weakness of protocols with gab)
- We want that no attack except if learning pair (x,a) or (y,b)
- Idea: $K = g^{(a+x)(b+y)}$ (computed by A as $(BY)^{a+x}$, by B as $(AX)^{b+y}$)
  - □ Doesn't work: Attacker sends $X^*=g^{x^*}/A$, B sends Y, $K=(BY)^{x^*}$ (no need to know A)

MQV

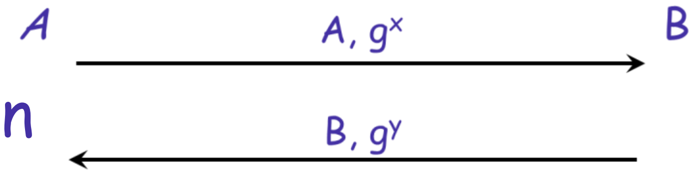
- Idea: set $K = g^{(a+dx)(b+ey)}$ and define d, e so that attacker cannot control e and Y, or d and X
- MQV: d=half bits of X, e=half bits of Y
- Does not quite work
- But a simple variation does

The HMQV Protocol



- Basic DH + special key computation
- Notation: $G=\langle g\rangle$ of prime order q; g in supergroup G' (eg. EC, Z^*_p)
  - □ Alice's PK is $A=g^a$ and Bob's is $B=g^b$ (private keys are a, b, resp.)
  - □ Exchanged ephemeral DH values are $X=g^x$, $Y=g^y$
- Both compute $\sigma=g^{(x+da)(y+eb)}$ as $\sigma = (YB^e)^{x+da} = (XA^d)^{y+eb}$
  - □ $d=H(X,\text{"Bob"})$ $e=H(Y,\text{"Alice"})$ (here H outputs $|q|$ bits)
  - □ Session key $K=H(\sigma)$ (here H outputs $|K|$ bits, say 128)
- Authentication almost for free ($\frac{1}{6}$ exponentiation, no commun'n)

The HMQV Protocol



A

A, g^x

B

B, g^y

- Basic DH + special key computation
- Notation: $G=\langle g\rangle$ of prime order q; g in supergroup G' (eg. EC, Z^*_p)
  - Alice's PK is $A=g^a$ and Bob's is $B=g^b$ (private keys are a, b, resp.)
  - Exchanged ephemeral DH values are $X=g^x$, $Y=g^y$
- Each computes $\boxed{\sigma=g^{(x+da)(y+eb)} \text{ as } \sigma = (YB^e)^{x+da} = (XA^d)^{y+eb}}$
  - $d=H(X,\text{"Bob"})$ $e=H(Y,\text{"Alice"})$ (H outputs $|q|/2$ bits)
- Session key $K=H'(\sigma)$ (H' outputs $|K|$ bits, say 128)
- Almost free authentication: $\frac{1}{6}$ exponentiation, = communic'n

multi-exponentiation

- Input: g_0, g_1, $e_0=(a_0, a_1,\ldots, a_{t-1})$ Output: $g_0^{e0} \cdot g_1^{e1}$
  $e_1 =(b_0, b_1,.., b_{t-1})$
- Pre-computation: $G_0=1$, $G_1=g_0$, $G_2=g_1$, $G_3=g_0 \cdot g_1$, $s(i)=a_i+2b_i$
- Compute: $A:=1$; For $i=0$ to $t-1$: $\{A:=A \cdot A;\ \ A:=A \cdot G_{s(i)}\ \}$
- Ops: t-1 squarings; $\frac{3}{4}$ t multiplies ($\frac{3}{4}$ because $G_{s(0)}=1$)
- Compared to full exponentiation t-1 squares, $\frac{1}{2}$ t mult's

→ $g_0^{e0} \cdot g_1^{e1}$ costs $1\frac{1}{6}$ exponentiations rather than 2

  - □ Works for any number k of bases (extra 2_k-2 mults)

The HMQV Protocol (w/short d,e)

- Both compute $\sigma = g^{(x+da)(y+eb)}$ as $\sigma = (YB^{e})^{x+da} = (XA^{d})^{y+eb}$
  - □ d=H(X,"Bob") e=H(Y,"Alice") **(here H outputs |q|/2 bits)**
- Session key K=H(σ) (here H outputs |K| bits, say 128)
- Authentication for "½ exponentiation" (no multiexp optimiz'n)

- Original formulation and proof (full length d, e simplifies some aspects of proof)

HMQV Explained

- HMQV: basic DH ($X=g^x$, $Y=g^y$), PKs: $A=g^a$, $B=g^b$
  - □ $\sigma=g^{(x+da)(y+eb)}$ as $\sigma = (YB^e)^{x+da} = (XA^d)^{y+eb}$; $K=H(\sigma)$
    - $d=H(X,\text{"Bob"})$ $e=H(Y,\text{"Alice"})$
- No signatures exchanged, authentication achieved via computation of σ (must ensure: only Alice and Bob can compute it)
- Idea: $(YB^e)^{x+da}$ is a sig of Alice on the pair (X, "Bob") and, at the same time, $(XA^d)^{y+eb}$ is a sig of Bob on (Y, "Alice")
  - □ Two signatures by two different parties (different priv/publ keys) on different msgs but with the same signature value!

Underlying Primitive: Challenge-Response Signatures

- Bob is the signer (PK is $B=g^b$), Alice is the verifier (no PK)
  - □ Alice sends a "challenge" ($X=g^x$) and a msg m to Bob, who responds with a "challenge-specific" signature on m (sig depends on b, X, m)
  - □ Alice uses her "challenge trapdoor" (x) to verify the signature
- Alice→Bob: m, $X=g^x$

  Bob→Alice: $Y=g^y$, $\sigma=X^{y+eb}$ where $e=H(Y,m)$

  Alice accepts the signature as valid iff $(YB^e)^x=\sigma$
- Note: Alice could generate the signature by herself! (signature convinces only the challenger – non-transferable -- *bug or feature?*)
- We call this scheme XCR (Xponential Challenge Response)

Security of XCR Signatures

- Theorem: XCR signatures are unforgeable
  - Unforgeability under usual adaptive chosen message attack
  - Only signer and challenger can compute it
  - Assumptions: Computational DH; also H modeled as random oracle
- Idea of proof: "exponential" Schnorr via Fiat-Shamir
  - More later…

Dual XCR (DCR) Signatures

- Alice and Bob act as signers and verifiers simultaneously
- Alice has PK $A=g^a$, Bob has PK $B=g^b$
- Alice and Bob exchange values $X=g^x$, $Y=g^y$ and msgs m_A,m_B
- Bob generates an XCR sig on m_A under challenge XA^d

  Alice generates an XCR sig on m_B under challenge YB^e
- The signature is the same! $\sigma = (YB^e)^{x+da} = (XA^d)^{y+eb}$
- This is exactly HMQV if one puts m_A="Alice", m_B="Bob" (since sig is the same value *it needs not be transmitted*!)

Proof of HMQV

- Reduction from breaking HMQV as KE (in the CK model) to forging DCR
  - Not a trivial step
  - Great at showing the necessity of all elements in the protocol: drop any element and the proof shows you an attack (e.g. MQV)
- Reduction from forging DCR to forging XCR
  - Quite straightforward
- Reduction from forging XCR to solving CDH in RO model
  - I expand on this next

XCR Proof via "Exponential Schnorr"

- Schnorr's protocol (given $B=g^b$, Bob proves knowledge of b)
  - □ Bob→Alice: $Y=g^y$
  - □ Alice→Bob: $e \in_R Z_q$
  - □ Bob→Alice: $s=eb+y$ (Alice checks $YB^e=g^s$)

[FS]: ZK for honest verifier (Alice) → $(Y, s=eb+y)$ w/ $e=H(m,Y)$ is a RO sig on m

- Exponential Schnorr: Bob proves *ability to compute* $()^b$
  - □ Bob→Alice: $Y=g^y$ $\{0,1\}^{|q|/2}$
  - □ Alice→Bob: $e \in_R \cancel{Z_q}$, $X=g^x$
  - □ Bob→Alice: $\sigma=X^{eb+y}$ (Alice checks $(YB^e)^x=\sigma$)

ZK for honest verifier (& any X) → $(Y, \sigma=X^{eb+y})$ w/ $e=H(m,Y)$ is a RO XCR sig on m

Theorem: XCR is strongly CMA-unforgeable (CDH + RO)

Proof: A CDH solver C from XCR forger F

- Input: U, V in $G=\langle g \rangle$ (a CDH instance; goal: compute g^{uv})
- Set $B = V$ $X_0 = U$ (B is signer's PK, X_0 is challenge to forger)
- Run F; for each msg m and challenge X queried by F (*a CMA attack*) *simulate* signature pair (Y,X^s) (random s, e; $Y=g^s/B^e$; $H(Y,m) \leftarrow e$)
- When F outputs forgery (Y_0, m_0, σ): (* (Y_0,m_0) fresh and $H(Y_0,m_0)$ queried *)

  Re-run F with new independent oracle responses to $H(Y_0,m_0)$

- If 2nd run results in forgery (Y_0, m_0, σ') (* same (Y_0,m_0) as before! *) then C outputs $W=(\sigma/\sigma')^{1/c}$ where $c=(e-e')$ mod q .
  (e, e' are the responses to $H(Y_0,m_0)$ in 1st and 2nd run, respectively)

Lemma: with non-negligible probability W=DH(U,V)

Proof: [PS] + $W= (\sigma/\sigma')^{1/c} = ((Y_0B^e)^{x0} / (Y_0B^{e'})^{x0})^{1/c} = ((B^c)^{x0})^{1/c} = B^{x0}$

Implications for HMQV (* X → XA^d *)

- We used $W = (\sigma/\sigma')^{1/c} = ((Y_0B^e)^{x0} / (Y_0B^{e'})^{x0})^{1/c}$

  But can we divide by $Y_0B^{e'}$? Yes if B and Y_0 in G (have inverses)

- B in G always true (chosen by honest signer) but what about Y_0 which is chosen by forger?
  - □ Do we need to check that Y_0 in G? (An extra exponentiation?)
  - □ No. If $G \subset R$, then enough to check Y_0 has inverse in R
    - E.g: $G = G_q = \langle g \rangle \subset Z_p^*$; $R = Z_p$; simply check Y in Z_p and $Y \neq 0$

➔ HMQV needs no prime order verification! (later: only if exponent leak)

- Forger can query arbitrary msgs with arbitrary challenges X (even challenges not in group G) → No need for PoP or PK test in HMQV!

  (X becomes XA^d and we do not need to check X nor A!)

➔ **Robust security of HMQV without extra complexity (no extra exponentiations, PoP's, PK validation, etc.)**

More on Security of HMQV

- Note that each party can start the protocol (no initiator/responder roles) even simultaneously
- Protocol is not secure against leakage of both {a,x} or {b,y} but secure against any other pair in {a,x,b,y}
  - Secure against disclosure of {a,b} is equivalent to PFS
  - But does HMQV really achieve PFS?

PFS in HMQV

- PFS *achieved only against passive attackers*
- *Impossibility fact*: If the messages sent in the protocol are computed without knowledge of the long-term keys of the sender then PFS fails to active attackers
  - □ The attacker chooses the message in the name of *A* (e.g. it chooses x and sends g^x) ; later it learns the long-term key of *A*, hence can compute the session key.
  - □ Thus, authentication specific information must be transmitted to achieve PFS (e.g., via a third "key confirmation" message)

A fundamental question:

- Can one obtain a FULLY authenticated DH protocol with
  - □ A single message per party (2 message total)
  - □ A single group element per message
  - □ NO certificates
  - □ Minimal computational overhead for authentication
  - □ AND PFS AGAINST FULLY ACTIVE ATTACKERS

?????????????

A surprising answer: YES!

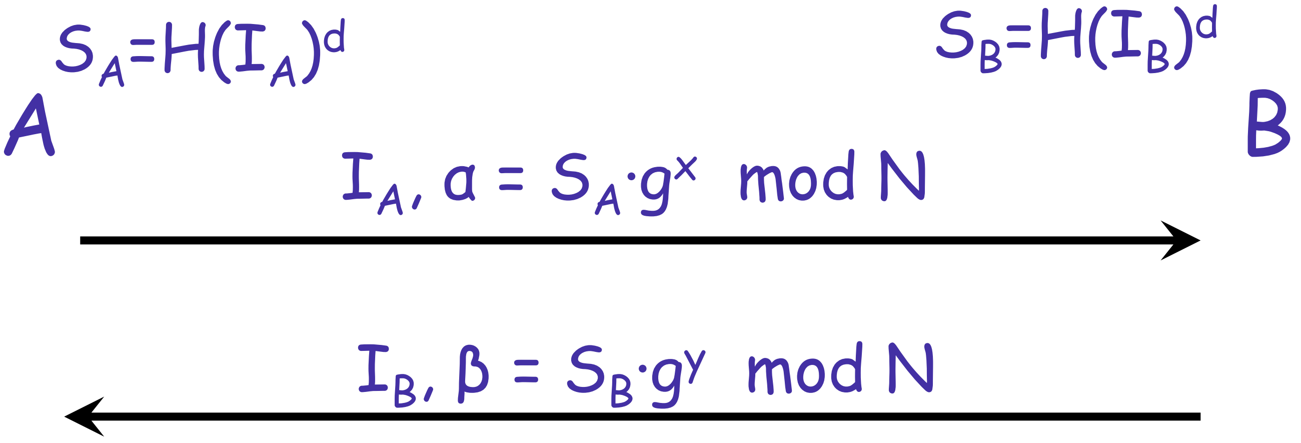
- By working over cyclic groups modulo a composite (we will refer to the bit size of elements later)
- Resorting to classical Okamoto-Tanaka protocol (1987)
- With simple modifications required for security
- With full proof of security, *including proof of PFS against active attackers!!*

Modified Okamoto-Tanaka (mOT)

Modified Okamoto-Tanaka Protocol

- Identity-based setting: Key Generation Center (KGC)
  - □ Chooses safe primes p, q (p=2p'-1, q=2q'-1) and RSA exponents e, d for N=pq
  - □ Chooses generator g of QR_N (set of quadratic residues), a cyclic group of order p'q'
  - □ Publishes N, g, e (e.g. e=3)
- Secret key for party I is $S_I=H(I)^d$ mod N (computed by KGC)
- Ephemeral session values: g^x mod N for x of length twice the security parameter (e.g., between 160-256 bits)

Modified Okamoto-Tanaka (mOT)



$S_A=H(I_A)^d$

A

$S_B=H(I_B)^d$

B

$I_A,\ \alpha = S_A\cdot g^x \mod N$

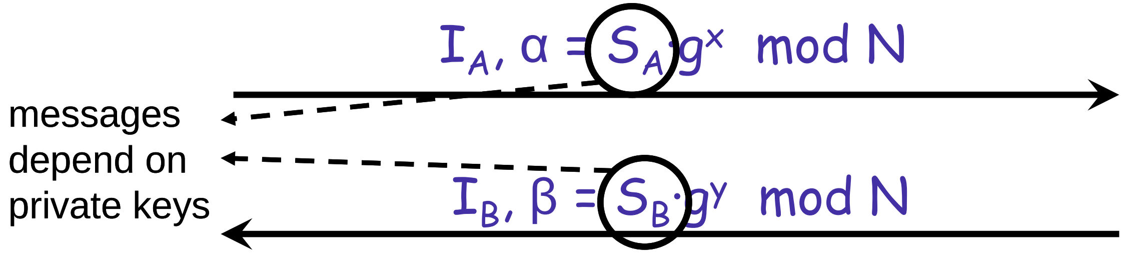
$I_B,\ \beta = S_B\cdot g^y \mod N$

$$K=H(\beta^e/H(I_B))^{2x} \quad = g^{2xye} = \quad K=H(\alpha^e/H(I_A))^{2y}$$

- Two msgs, single group element, no certificate
- Computation: 1 off-line + 1 on-line expon’n (= basic DH) + e-exponentiation (= 2 multiplications) and 1 squaring
- Just 3 mult’s more than a basic DH over composite N !!!

mOT: Minimal Overhead, But is it secure?

- YES!
- With proof of security in the Canetti-Krawczyk model
  - □ RSA assumption + Random Oracle Model (passive PFS only)
- And *proof of PFS* security *against active attackers*
  - □ With additional “knowledge-of-exponent” type assumptions
- Note: mOT avoids the “implicit-authenticated PFS impossibility” since messages α, β depend on the private key of the sender



$I_A, \alpha = S_A g^x \bmod N$

$I_B, \beta = S_B \cdot g^y \bmod N$

messages
depend on
private keys

On the Proof

- The basic case: CK model, weak PFS, under plain RSA in ROM follows more or less standard arguments
- The challenge is in proving full PFS (against active attacks and without additional messages/communication)
- Good news: we can do it!
  - In particular, security of past communication if KGC compromise
- Less good news: non-standard assumptions
  - But a STRONG indication of security!

KEA-type Assumptions

- KEA-DH (a.k.a KEA1): "Computing g^{xy} from g, g^x, g^y can only be done if one *knows* x or y"
- KEA-DL:
  - □ Given $y=g^x$ want to compute x with the help of a Dlog oracle D that accepts *any input but y*
  - □ Obvious strategy: query D with $y \cdot g$
    More generally: Can query D with $z=y^i g^j$ for any known i,j
    (and recover x from the oracle's response ix+j)
  - □ KEA-DL states that this is the most general strategy, i.e., if you find x by querying D(z) then you know i, j such that z=ix+j

Real-World Performance

☺ Complexity is essentially the same as the basic unauthenticated DH…

☹ … but it runs over Z_N for composite N

☺ … with short exponents (assumes dlog hard w/ such exponent)

☺ … no certificate transmission and processing

In all, comparable performance to HMQV/ECC for security levels under 2048 bits (for RSA)

The really important point, however, is theoretical:
Testing the limits of what is *possible*

Conclusions and Open Problems

- Conclusions
  - It is amazing how little one may need to pay over the basic DH for a fully authenticated exchange *with full PFS*:
    Over QR groups the ONLY overhead is JUST 3 multiplications!
    (no communication penalty, not even certificates)
- Open questions
  - Achieve the same performance properties with full PFS over other Dlog groups (e.g. Elliptic Curves)
  - Get rid of special assumptions for PFS proof
  - Reduce reliance on secrecy of the ephemeral g^x and g^y

One-Pass HMQV and Asymmetric Key-Wrapping

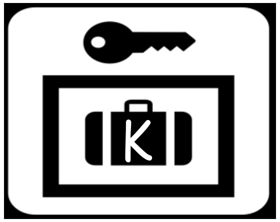
Motivation

- Key wrapping as a basic functionality (e.g. storage systems)
- A good example of:

  Optimizing cryptography via *"Proof-driven design"*
- Proof tells us precisely what elements in the design are essential and which can be avoided
  - □ Avoid unnecessary safety margins
  - □ Better performance and functionality
  - □ A great protocol debugging tool

Key Wrapping

- *Key-wrapping or key encapsulation*: Server wraps a symmetric key for transporting it to a client
  - □ Think of wrapping as a *key encryption mechanism*
  - □ Encrypting key may be symmetric or asymmetric (AES, RSA)
  - □ Wrapped key may be a fresh key, or a previously generated one, sometimes bound together with associated data
  - □ Wrapping typically done off-line and non-interactively.



Example: Encrypted Backup Tapes

- Tape encryption:
  - Tape sent to KMM (key management module)
  - KMM encrypts tape with tape-specific key K

    wraps K (under another key) and stores wrapped key with tape
- Tape decryption
  - wrapped key sent to KMM* who unwraps (decrypts) K
  - K is sent back to tape holder for tape decryption
- Notes: Decryption may happen many years after encryption

  KMM and KMM* may not be the same (KMM* holds de-wrapping key)

Key Wrapping and Standards

- *Major key management tool:*
  - storage, hardware security modules, secure co-processors, ATM machines, clouds, etc.
  - Complex: long-lived keys & systems, backwards compatibility,…
- *Standards are important:* server and client typically run different systems (and by different vendors)
  - Industry standards: storage systems, financial, HSMs, etc.
- Currently deployed: mainly DES/AES and RSA
- **Searching for ECC-based key wrapping techniques**

Main Candidate: DHIES Encryption

- Elgamal encryption + RO-based key derivation + Enc/Mac
- G=<g>: prime-order q H: hash function (RO)
  Enc: symmetric encryption Mac: message auth code
  - □ Receiver's PK: $A=g^a$, message to be encrypted: M
  - □ Sender chooses $y \in Z_q$, sends: (Y, C, T) where
    1. $Y = g^y$ $\sigma = A^y$ $K = H(\sigma)$ (2 exp)
    2. $K \rightarrow K1, K2$ $C = Enc_{K1}(M)$ $T = Mac_{K2}(C)$
  - □ Decryption: $\sigma = Y^a$, $K = H(\sigma)$, etc. (1 exp)
- [ABR01]: Scheme is CCA-secure in the ROM

DHIES as Key Wrapping

- DHIES instantiates the KEM/DEM paradigm: (Y,C,T)
  - □ Key Encapsulation: $Y=g^y$ encapsulates key $K=H(A^y)$ under PK A
  - □ Data Encapsulation: (C,T) CCA-encrypts data under K
- Simple, efficient, functional
  - □ KEM: Can be used to transmit a random fresh key K
  - □ DEM: Can be used to transport a previously defined key (and possible associated data)
    - The message M (under C) is the transported key and assoc'd data
- **Missing: Sender's authentication**

Authenticated Key Wrapping

- DHIES implicitly authenticates the receiver
  - Only intended receiver can read the key/data
  - This is the case for most key wrapping techniques
- But how about sender's authentication?
  - Who encrypted the tape? Who can it be decrypted for?
- *Authenticated key wrapping:* Key wrapping with sender's authentication (➔ mutual authentication)

Authenticating Key Wrapping

- Solution: Add sender's signature on wrapper $Sign_S(Y,C,T)$
- But, is it necessary (performance)?
- Is it sufficient?
- No. Needs to bind signer to key, not just to the wrapper
  - □ For example, Bob encrypts tape, sends wrapper with signature
  - □ Charlie strips Bob's signature and generates its own
  - □ Alice believes the key is owned by Charlie
  - □ Thus, she may later decrypt the tape for Charlie

Similar to UKS (or identity-misbinding) attacks on KE protocols

Authenticated Wrapping: Equivalent Notions

- Requirements are essentially of a key exchange protocol (w/replay)
  - Alice will never associate with Charlie a key created by Bob (assuming Bob and Alice are honest)
- Considering just KEM part of key wrapping (fresh key) with sender authentication, the following are equivalent:
  - Authenticated Key Wrapping, Authenticated KEM, One-Pass AKE
- With the DEM part (a "message" encrypted with the KEM key) one obtains a notion of "authenticated encryption" or its equivalent
  - UC-secure message transmission (w/replay) [Gjosteen, Krakmo]
  - Secure signcryption [Gorantla et al, Dent]

Authenticated Wrapping & One-Pass KE

- We can use any one-pass AKE to instantiate authenticated KEM ➔ authenticated key wrapping
  - □ Want something as simple and as close as possible to DHIES
  - □ More secure and more efficient than adding sender's signature

➔ HOMQV (a One-Pass HMQV KE protocol)

Group G=<g>, hash function H, sym encryption Enc, msg auth Mac

DHIES*

- Receiver's PK: $A=g^a$
- Sender chooses y,
  sends (Y,C,T) where

  1. $Y = g^y$ $\quad \sigma = A^y$

     $K = H(\sigma)$

     (2 expon's)

  2. K → K1, K2

     $C= Enc_{K1}(M)$ $\quad T= Mac_{K2}(C)$

- Decryption: $\sigma=Y^a$, etc
  (1 expon.)

Authenticated DHIES*

- R's PK: $A=g^a$; Sender's PK $B=g^b$
- Sender chooses y,
  sends (Y,C,T) where

  HOMQV

  1. $Y = g^y$ $\quad \sigma = A^{y+be}$ $\quad e = H_{½}(Y,id_R)$

     $K = H(\sigma, id_S, id_R, Y)$

     (2 expon's)

  2. K → K1, K2

     $C= Enc_{K1}(M)$ $\quad T= Mac_{K2}(C)$

- Decryption: $\sigma= (YB^e)^a$, etc
  (1.5 expon.)

[*] Group membership tests or cofactor exponentiation omitted (more later…)

HOMQV (Hashed One-pass MQV)

- Functionally optimal
  - Minimal performance overhead: Just extra ½ exp for receiver. Free for sender. No extra communication
  - Backwards compatibility with DHIES: Set B=1 b=0
- How about security?
- We prove security of HOMQV as one-pass key-exchange (➔ authenticated key wrapping)

HOMQV

- Sender id_S has public key $B=g^b$
- Receiver id_R has public key $A=g^a$
- $S \rightarrow R$: $Y = g^y$
- S computes $\sigma = A^{y+be}$
- R computes $\sigma = (YB^e)^a$
- Both set $K = H(\sigma, id_S, id_R, Y)$

$e = H_{½}(Y, id_R)$

Theorem

- Under Gap-DH in the ROM, HOMQV is a secure one-pass key-exchange protocol
  - Security of one-pass protocol: Canetti-Krawczyk relaxed to allow for key-replays
  - Guarantees mutual authentication in a strong adversarial model
  - Proof: Reduction to XCR signatures (defined in [HMQV])
- Some important leakage-resilience properties
  - Sender's Forward Security
  - Resistance to leakage of ephemeral Diffie-Hellman exponents (y-security)

Leakage-resilience Properties

- Sender forward security (disclosure of sender's secret key b does not compromise past keys and messages)
  - □ Weak FS: For sessions where attacker was passive
  - □ For full FS: Add a "key confirmation" $Mac_{K^*}(1)$ to sender's message (in particular, satisfied by the DEM part of DHIES)
- *y-security* : The disclosure of ephemeral secret y does not compromise *any* keys or messages
  - □ Not even the key/msg transported using $Y=g^y$
  - □ Moreover: the disclosure of both y <u>and</u> b reveals the msg sent using y but no other msgs sent by b's owner

On the Proof

- Too technical… for a short presentation
- but amazingly precise: The proof tells exactly what the role of each element in the protocol is
- and what the consequences of leakage are for each such element (a, b, y, σ and their combinations)

➔ Better security, better efficiency: Proof-driven design

- □ Get rid of safety margins
- □ Compare DHIES+signature vs HOMQV
- □ Would you buy it without a proof?

Additional checks

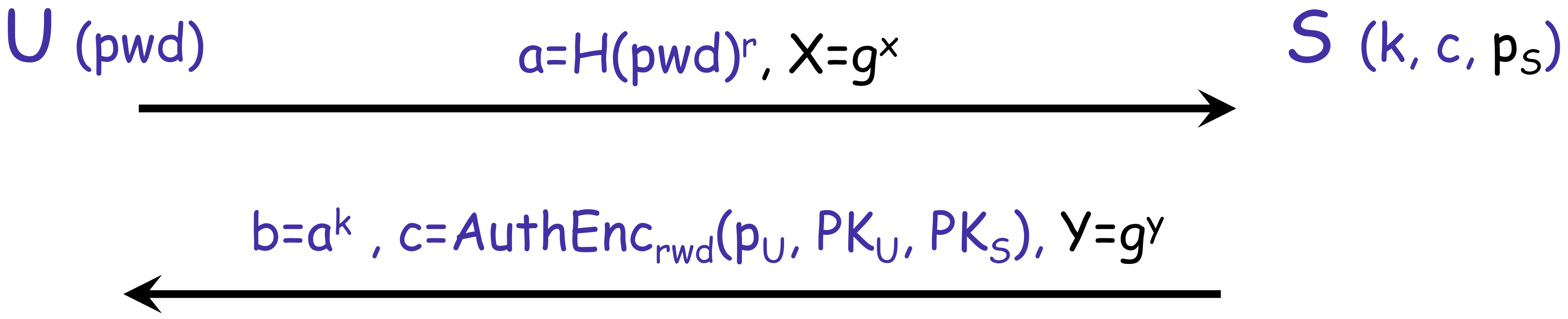
- Proof tells us exactly what properties of incoming values (Y, B, A, etc.) each party needs to check
- Need to assure YB^e is of order q (no need for separate Y,B test)
- Can implement more efficiently over elliptic curve by *cofactor exponentiation*
  - □ s = A^{fy} instead of A^y or $A^{f\cdot(y+be)}$ instead of $A^{(y+be)}$ where f=|G'|/ord(g) and G' a supergroup containing g (e.g. G' = ell. curve)
- Note: Same needed for DHIES (Y test), hence ½ expon advantage remains

For Fun

HMQV application to PAKEs

- What's a PAKE (Password Authenticated Key Exchange)
  - □ Peers share a password as the only means of authentication (same as pre-shared key but a low-entropy key)
  - □ As long as attacker does not guess password, security in full
  - □ Only attack option: online guessing (one passwd per connection)
- Asymmetric PAKE: user has pwd, server stores H(pwd)
  - □ Above security requirements PLUS: If server is broken into, finding pwd requires a full *offline dictionary attack*

OPAQUE: Application of HMQV to aPAKE (the beauty of minimality)



U (pwd)

S (k, c, p_S)

$a=H(pwd)^r$, $X=g^x$

$b=a^k$, $c=AuthEnc_{rwd}(p_U, PK_U, PK_S)$, $Y=g^y$

- $rwd=H(pwd)^k \leftarrow H(b^{1/r})$
- $p_U, PK_U, PK_S \leftarrow AuthDec_{rwd}(c)$
- $SK = HMQV(x,p_U,Y,PK_S)$ $SK = HMQV(y,p_S,X,PK_U)$