Zero-knowledge and multi-party (quantum) computation in the quantum world

Alex Bredariol Grilo

Primitives

Public-key encryption

Functional encryption

Oblivious transfer

indistinguishable Obfuscation

Secret-key encryption

Two-party computation

Witness encryption

Multi-party computation

One-way functions

Pseudo-random number generators

Zero-knowledge proof systems

How to propose implementations and prove their security?

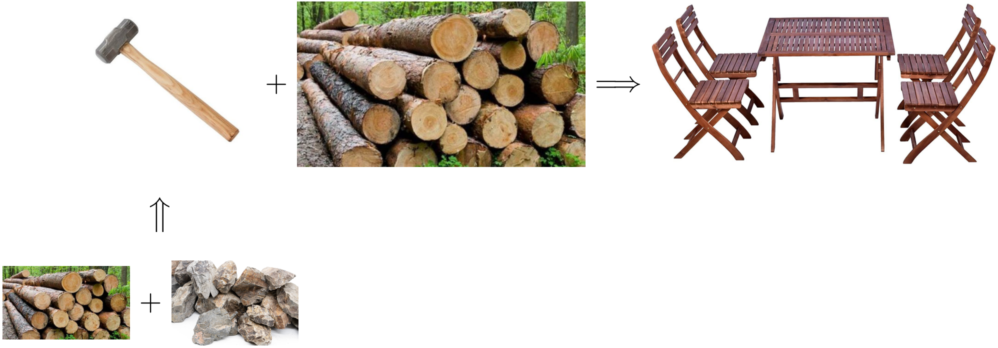
Reductions

Reductions

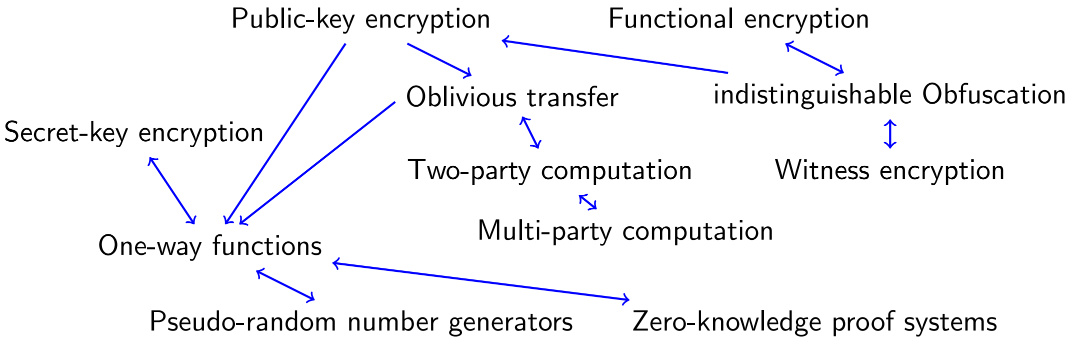
$+$

$\Longrightarrow$

Reductions



Primitives



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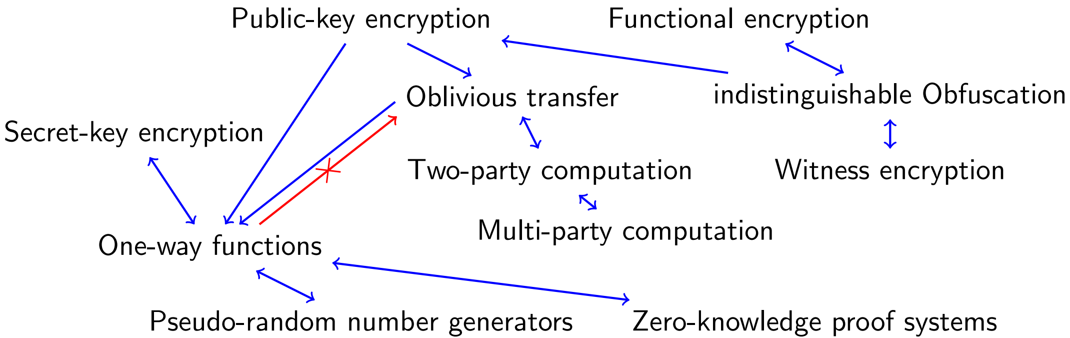

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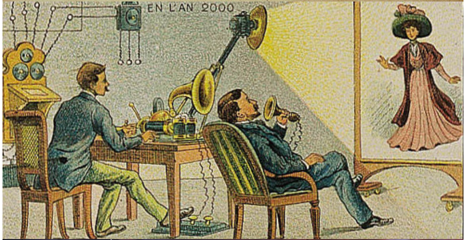
Minicrypt: OWFs exist



Cryptomania: PKE schemes exist

Obfutopia: iO exists

… if crypto is possible

Algorithmica(+Heuristica): We can solve NP (in practice)

Pessiland: We cannot solve NP and OWFs do not exist

How do quantum resources affect these reductions/worlds?

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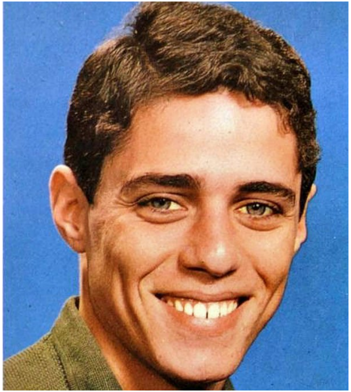

Quantum helps honest parties

How do quantum resources affect these reductions/worlds?

Quantum helps honest parties

Quantum helps malicious parties

How do quantum resources affect these reductions/worlds?

Quantum helps honest parties

Quantum helps malicious parties

What are the minimal assumptions for quantum functionalities?

ZK and MPC in the quantum world

ZK and MPC in the quantum world

Zero-knowledge proofs

Central tool in crypto toolbox

ZK and MPC in the quantum world

Zero-knowledge proofs
Central tool in crypto toolbox

Multi-party computation
Most-general functionality (modulo #rounds)

ZK and MPC in the quantum world

Zero-knowledge proofs

Central tool in crypto toolbox

1. ZK for NP in MiniCrypt
2. ZK against quantum adversaries
3. ZK for QMA (“quantum NP”)

Multi-party computation

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ZK and MPC in the quantum world

Zero-knowledge proofs

Central tool in crypto toolbox

1. ZK for NP in MiniCrypt
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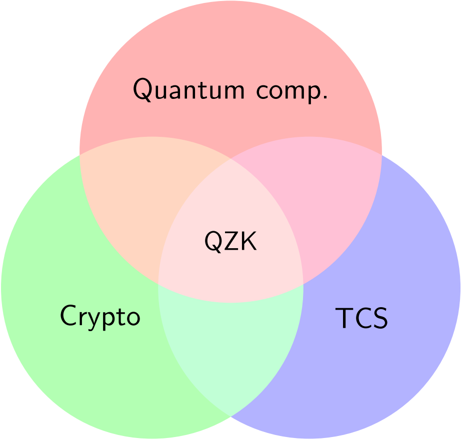
Multi-party computation

Most-general functionality (modulo #rounds)

1. MPC from Oblivious transfer
2. OT is in Mini**Q**Crypt
3. Multi-party quantum computation

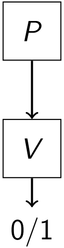
Zero-knowledge in the quantum world


Quantum comp.
QZK
Crypto
TCS

Interactive proofs

Interactive proofs

$L \in \mathrm{NP}$



P

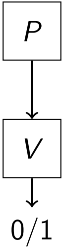
V

$0/1$

for $x \in L$, $\exists P$
V accepts
for $x \notin L$, $\forall P$
V rejects

Interactive proofs

$L \in \mathrm{NP}$



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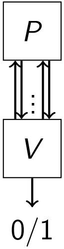
$L \in \mathrm{IP}$



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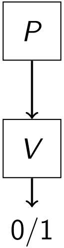

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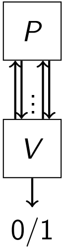
$L \in \mathrm{IP} = \mathrm{PSPACE}$



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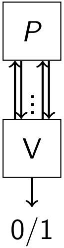
Zero-knowledge



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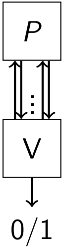

$L \in \mathsf{ZK}$

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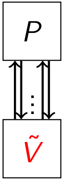
Zero-knowledge: V "learns nothing" when $x \in L$

Zero-knowledge



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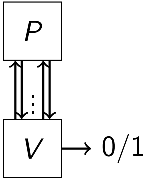

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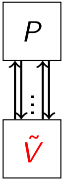

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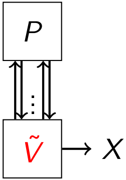

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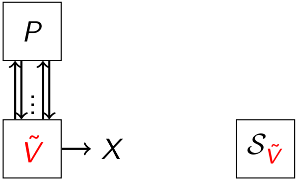

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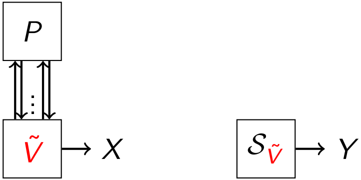

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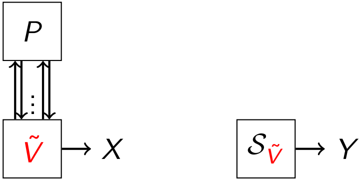
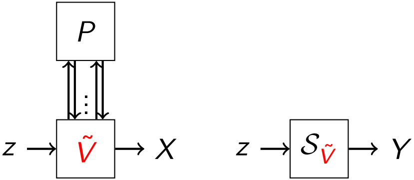


Zero-knowledge property: X is indistinguishable from Y

(**Computational**) ZK: No **efficient distinguishers** for the distributions

$$\forall \text{ poly-time } \mathcal{A} : |Pr_{x \sim X}[\mathcal{A}(x) = 1] - Pr_{y \sim Y}[\mathcal{A}(y) = 1]| \leq negl(n)$$

Zero-knowledge



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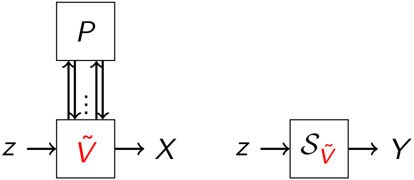

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Statistical ZK: $\forall z$, Distribution X is **statistically close** to distribution Y
Perfect ZK: $\forall z$, Distribution X **=** distribution Y

ZK: bread-and-butter of cryptography

- **Applications:** authentication schemes, building block of several cryptographic compilers, blockchains,...

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- Example:

x A B

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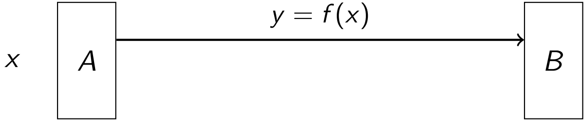

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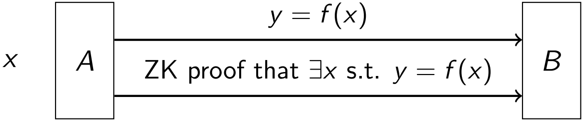

x

A

$y = f(x)$

ZK proof that $\exists x$ s.t. $y = f(x)$

B

ZK: bread-and-butter of cryptography

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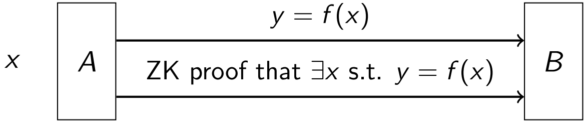

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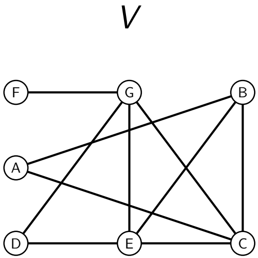
B

- Zero-knowledge protocols for problems in NP
  - ZK proof of 3-coloring

ZK proof for 3-coloring: attempt 1



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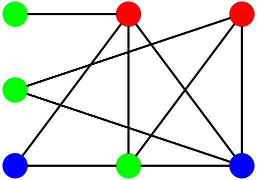
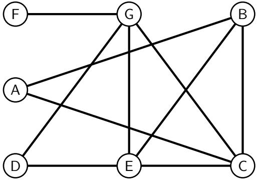


V



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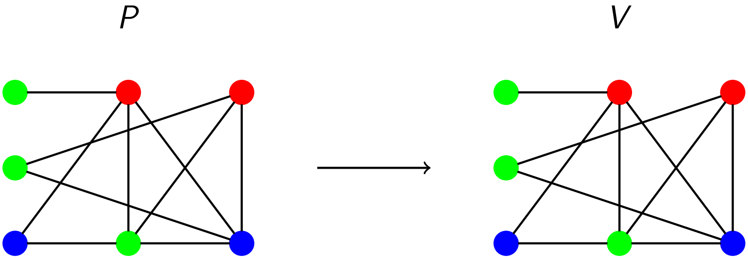

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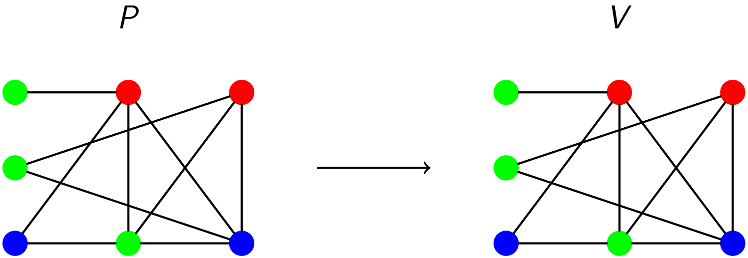

Completeness ✓ Soundness ✓ ZK ✗

Bit-commitment

"Cryptographic safe"

C

R

Bit-commitment

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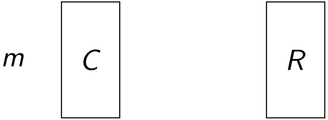

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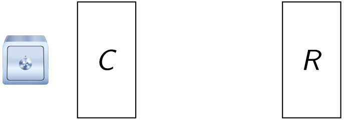

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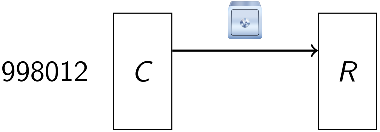

998012

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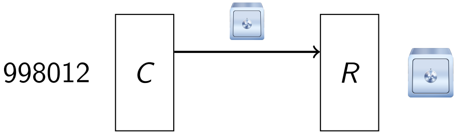

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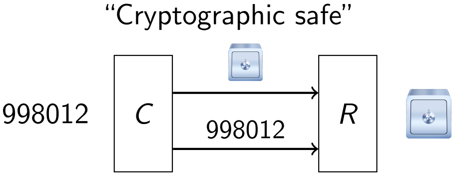

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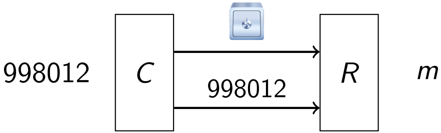

998012

C

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m

Bit-commitment

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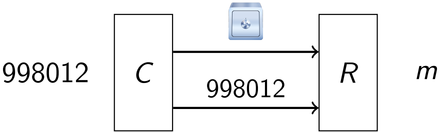
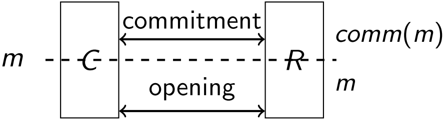


More concretely...



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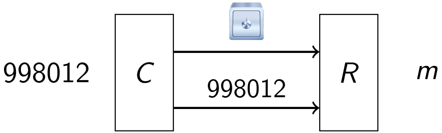

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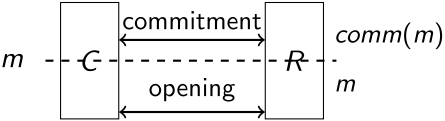


m

C

commitment

opening

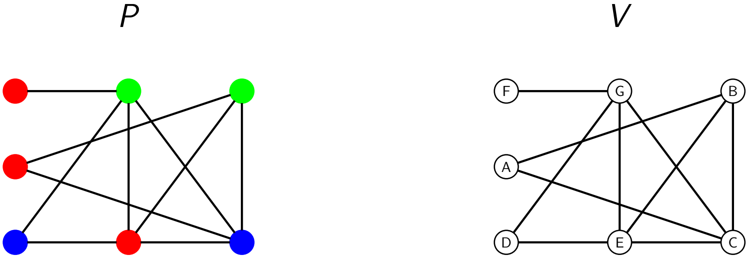
R

$comm(m)$

m

Hiding: R cannot learn m from $comm(m)$
Binding: C cannot successfully open $comm(m)$ to a message $m' \neq m$

ZK proof for 3-coloring: GMW'91



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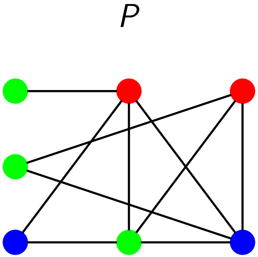

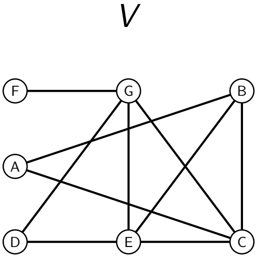

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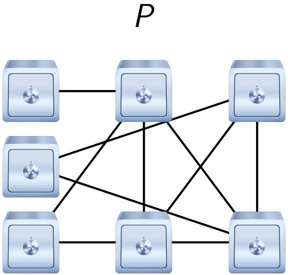

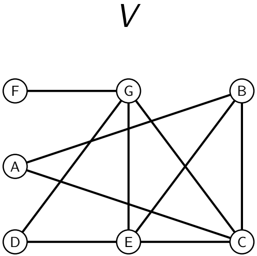

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P

$A \to 564651$
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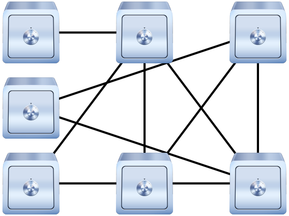

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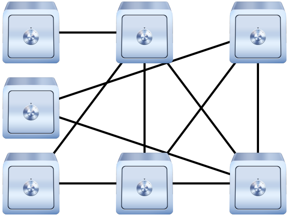

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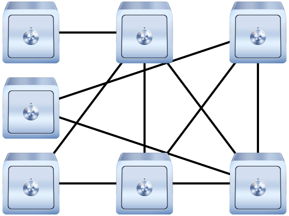

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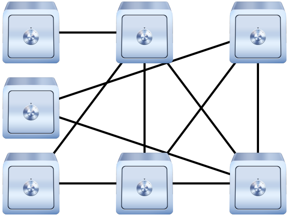

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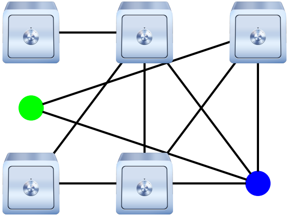

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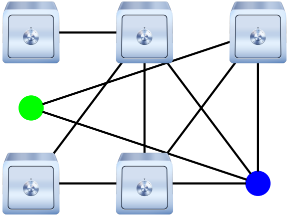
$\xrightarrow[564651,\, 984565]{}$

V



Completeness ✓ Soundness ✓ CZK

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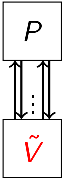
What happens if $\tilde{V}$ is quantum?

Classical zero-knowledge against quantum adversaries



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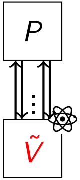

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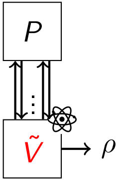

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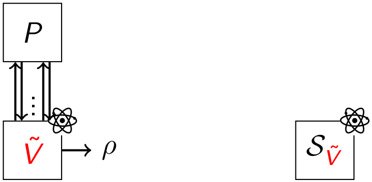

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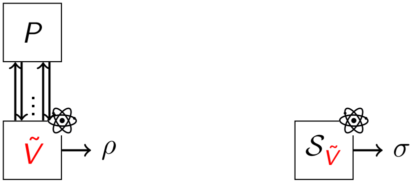

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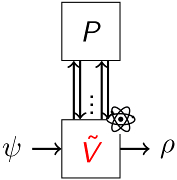

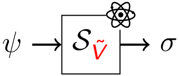

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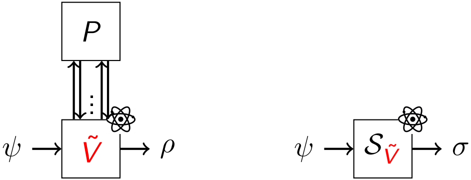

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$$\forall \text{ quantum poly-time } \mathcal{A} : |Pr[\mathcal{A}(\rho) = 1] - Pr[\mathcal{A}(\sigma) = 1]| \leq negl(n)$$

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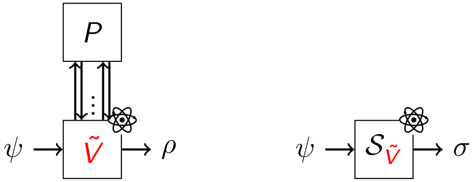

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Sim measures register M and gets e' w.p. $|\alpha_{e'}|^2$ and post-meas. state is $|e'\rangle|\gamma'_e\rangle$:

$e' = e$: all is good

$e' \neq e$: rewinding does not work

$$V^\dagger |e'\rangle|\gamma'_e\rangle \text{ vs. } V^\dagger|\phi\rangle$$

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We cannot sequentially repeat this protocol!

Watrous's rewinding

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$$Q|\psi\rangle|0\rangle = \sqrt{p}|0\rangle|\phi_0(\psi)\rangle + \sqrt{1-p}|1\rangle|\phi_1(\psi)\rangle$$

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- Similar statement holds for the non-exact case

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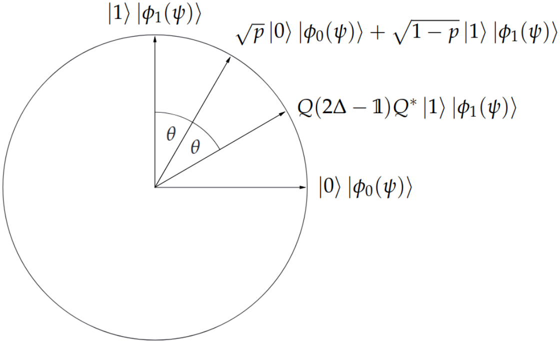

Figure from Watrous'09

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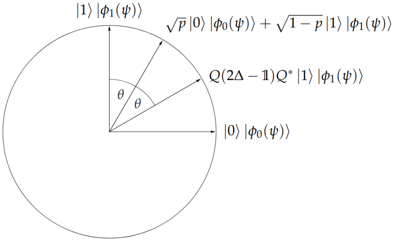

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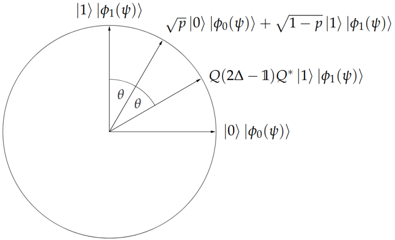

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Classical ZK - wrap up

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Theorem

Assuming post-quantum commitment schemes, GMW'91 is secure against quantum adversaries.

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Zero-knowledge proofs for NP is in MiniQCrypt

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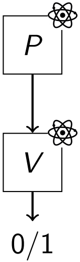
Zero-knowledge proofs for NP is in $MiniQCrypt$

Can we have (simple) zero-knowledge protocols for quantum proofs?

Quantum proofs

Quantum proofs

$L \in \mathrm{QMA}$



P

V

$0/1$

for $x \in L$, $\exists P$
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Quantum proofs

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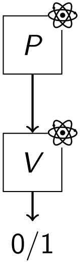

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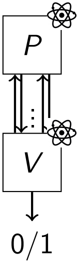

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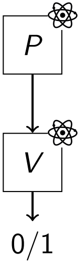

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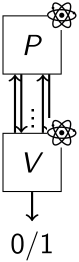

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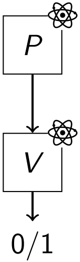

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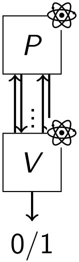

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Expected: $\mathrm{NP} \subsetneq \mathrm{QMA} \subsetneq \mathrm{IP} = \mathrm{QIP} = \mathrm{PSPACE}$

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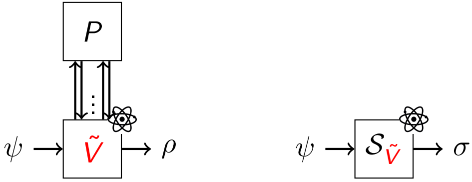

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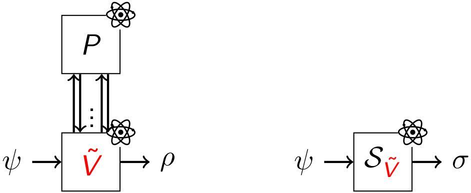

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Option 1: ZK from generic problem in QMA.

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Quantum ZK protocols for QMA

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Option 4: **ZK from Consistency of Local density matrices**

Consistency of local density matrices problem

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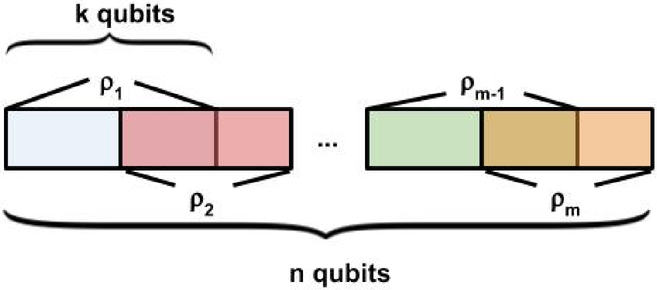
Do “pieces” of quantum state come from the same global state?



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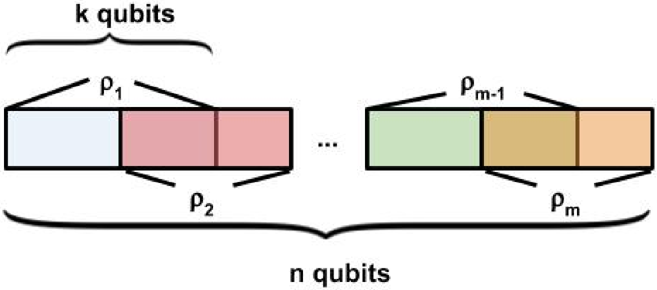

Input: Reduced density matrices $\rho_1, \ldots, \rho_m$ on k-qubits

Output: yes: $\exists \psi$ such that $\forall i : \left\| Tr_{\overline{S_i}}(\psi) - \rho_i \right\| \leq \varepsilon$
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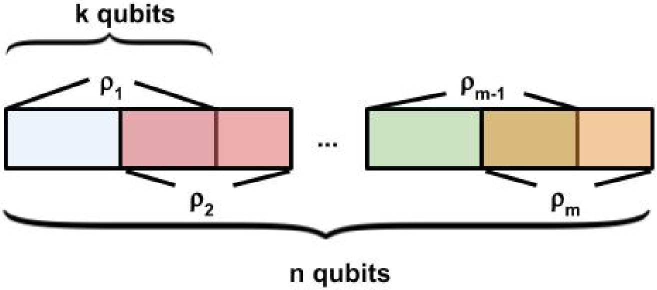

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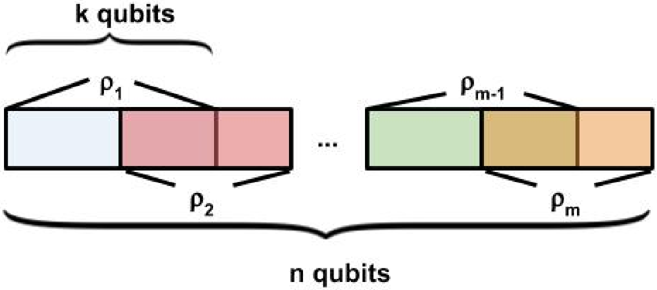

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CLDM is in QMA - overview

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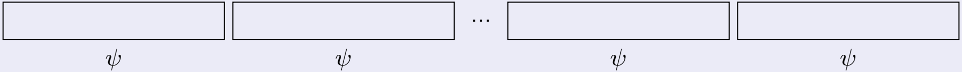

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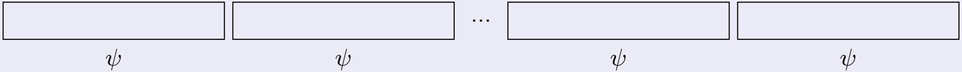

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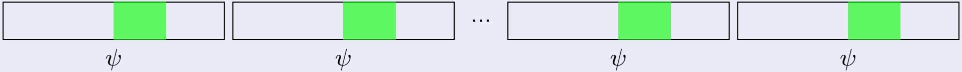

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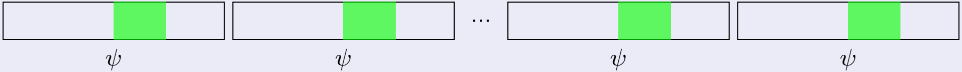

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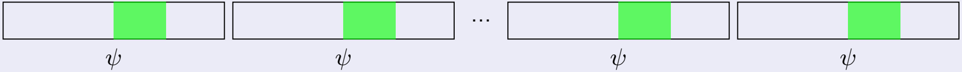

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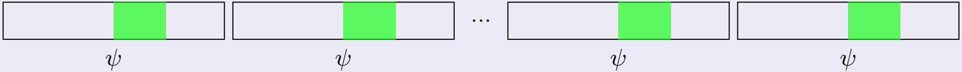

Soundness:

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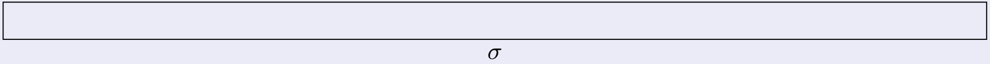

CLDM is in QMA - overview

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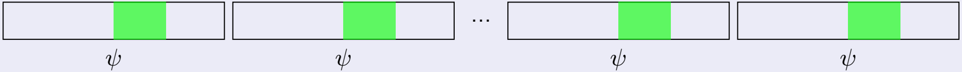

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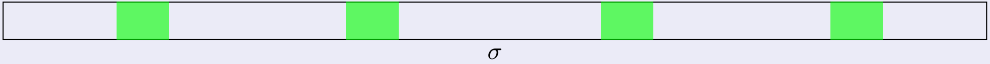

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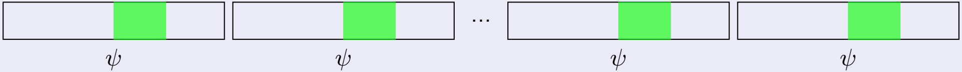

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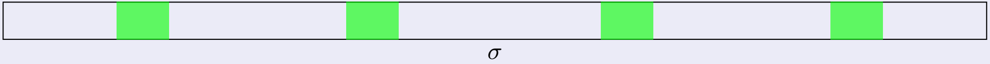

ZK proof for CLDM: BG'20

P V

$\rho_1, \ldots, \rho_m$

ZK proof for CLDM: BG'20

P

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$\psi^{\otimes \ell}$

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ZK proof for CLDM: BG'20

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V

$X^a Z^b \psi^{\otimes \ell} Z^b X^a$

$\rho_1, \ldots, \rho_m$

a_1, b_1

a_2, b_2

$\ldots$

a_{n-1}, b_{n-1}

a_n, b_n

ZK proof for CLDM: BG'20

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ZK proof for CLDM: BG'20

P

$a_1, b_1 \rightarrow 564651$
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$\dots$
$a_n, b_n \rightarrow 894102$

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$\rho_1, \dots, \rho_m$

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…

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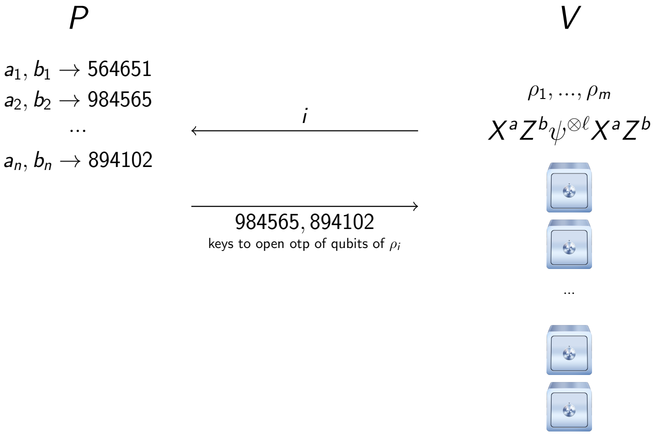

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keys to open otp of qubits of ρ_i

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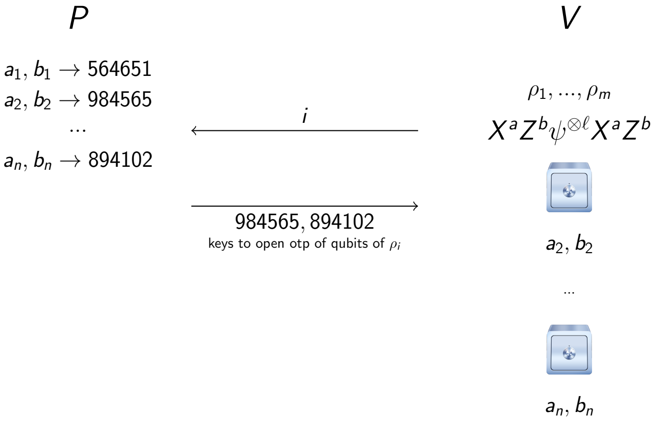

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$\xrightarrow{\quad\quad\quad}$

$984565, 894102$

keys to open otp of qubits of ρ_i

a_2, b_2

...

a_n, b_n

Completeness ✓ Soundness ✓ CZK

Zero-knowledge (sketch of the proof)

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$\mathrm{Sim}_1(\psi)$:

1. Give ψ to $\tilde{V}$.
2. Pick $i \in [m]$ uniformly at random
3. Commit to a state that has ρ_i in the right position
4. Receive a challenge i' from $\tilde{V}$
5. If $i \neq i'$, open the commitment of OTP of the corresponding qubits, and forward output
6. Output $\perp$ from $\tilde{V}$

- If $i = i'$, output of Sim_1 is good
- Sim_1 succeeds with probability $\frac{1}{m}$ $(+\ negl(n))$

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- Runtime of Sim_2 is $poly(|\tilde{V}|, n)$

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Sim$_2(\psi)$:

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- Output of Sim$_2$ is negl(n) close to the output when we have $i = i'$
- Runtime of Sim$_2$ is $poly(|\tilde{V}|, n)$

Corollary

Quantum zero-knowledge proofs for QMA is in MiniQCrypt

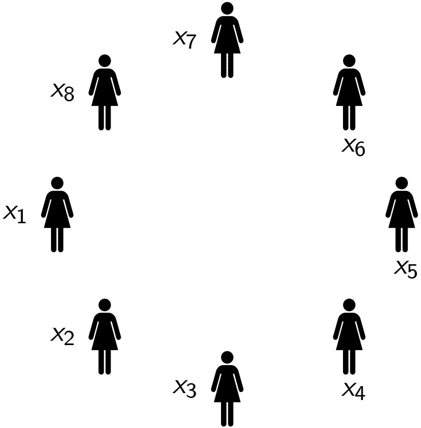
Further development

1. Perfect ZK for multi-prover entangled proof systems (MIP*) [**G**SY'19]
2. Constant round post-quantum ZK for NP/QMA [Bitansky-S'20]
3. Proof of Knowledge
   - Usual soundness: there is no good strategy for no-instance
   - PoK: If Prover passes with high enough probability, then a NP-witness is known

     There is an extractor K, such that if $\tilde{P}$ passes with probability $\geq \kappa$, $K^{\tilde{P}}$ outputs a witness
   - Proof of Knowledge against quantum provers [Unruh'12]
   - Proof of Quantum Knowledge [Broadbent-**G**'20,Coladangelo-VZ'20, Ananth-CLP'20]
4. Classical ZK *arguments* for QMA
   - Computational soundness: no poly-time adversary can make V accept a no-instance
   - Classical argument system for QMA [Mahadev'18,Alagic-C**G**H'20, Chia-CY'20]
   - Classical ZK protocols for QMA [Vidick-Z'20]
5. NIZKs in the quantum setting
   - Post-quantum NIZK for NP [Peikert-S'19]
   - Quantum NIZK for QMA [Broadbent-**G**'20,Coladangelo-VZ'20]
   - Classical NIZK arguments for QMA [Alagic-C**G**H'20]

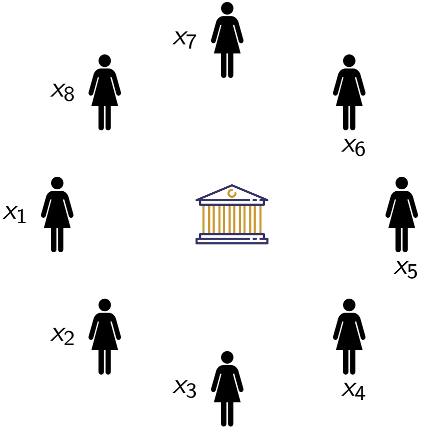
Multi-party (quantum) computation in the quantum world

Multi-party computation



Goal: Compute $f(x_1, \ldots, x_8)$ without revealing their input

Multi-party computation



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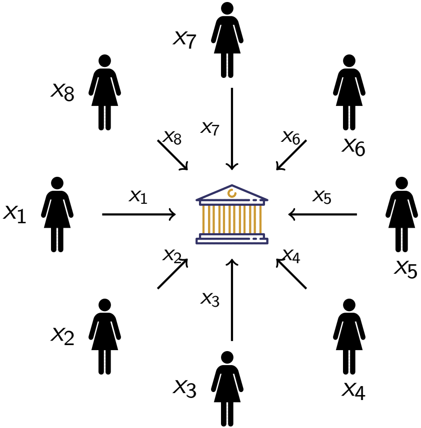
Ideal world

Multi-party computation



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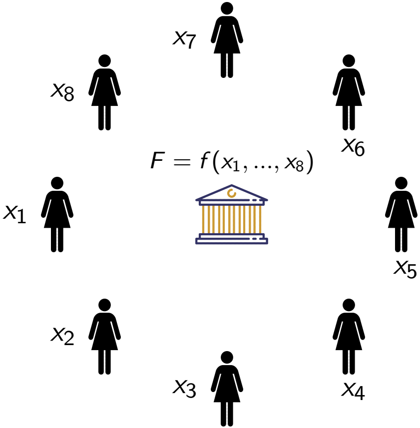

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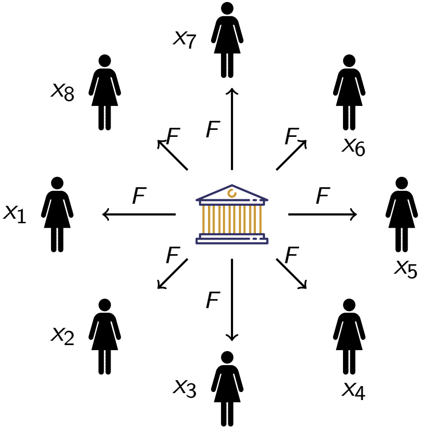

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- Each party learns $F = f(x_1, ..., x_8)$ and nothing else

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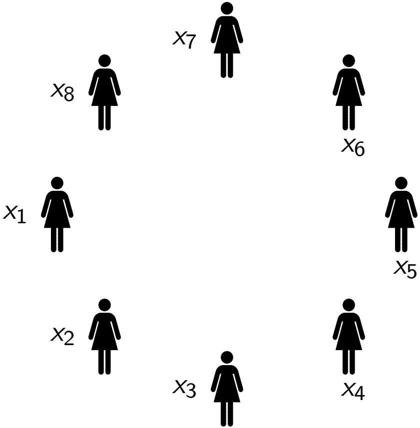

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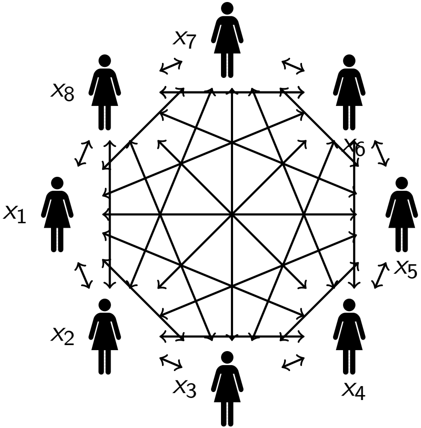

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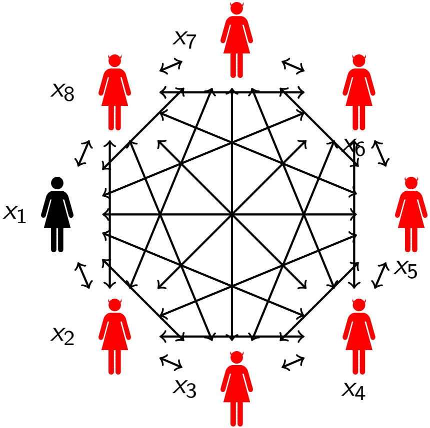
Multi-party computation



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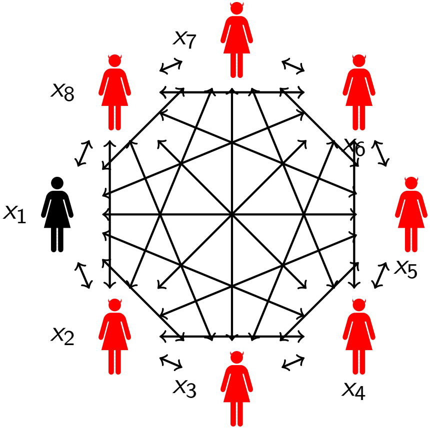

Are classical protocols secure against quantum adversaries?

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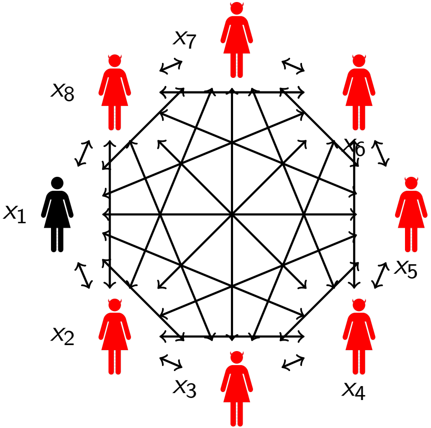

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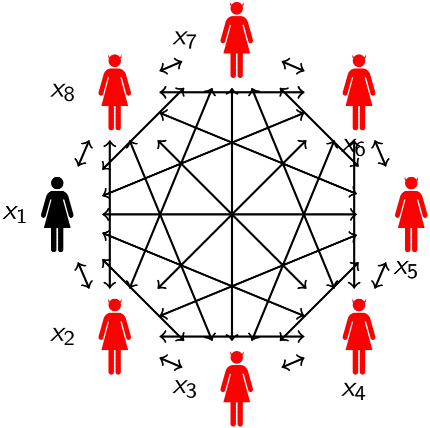
Are there *better* quantum protocols for MPC?

Multi-party computation



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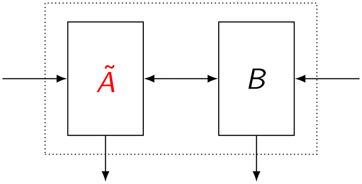
Are there protocols for MP**Q**C?

Security definition (two-party case)



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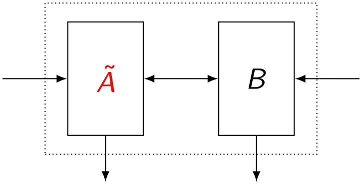
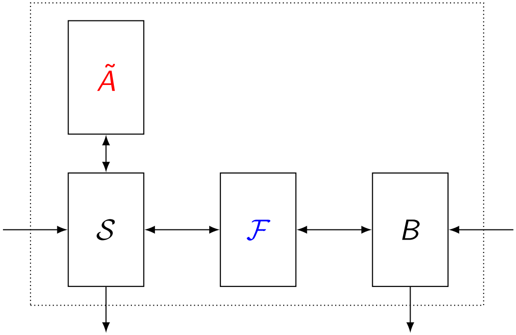


Real world



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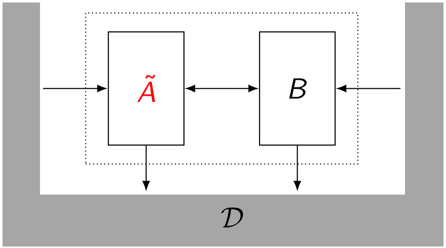
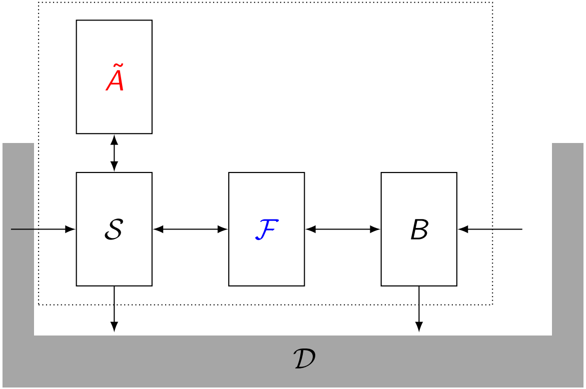
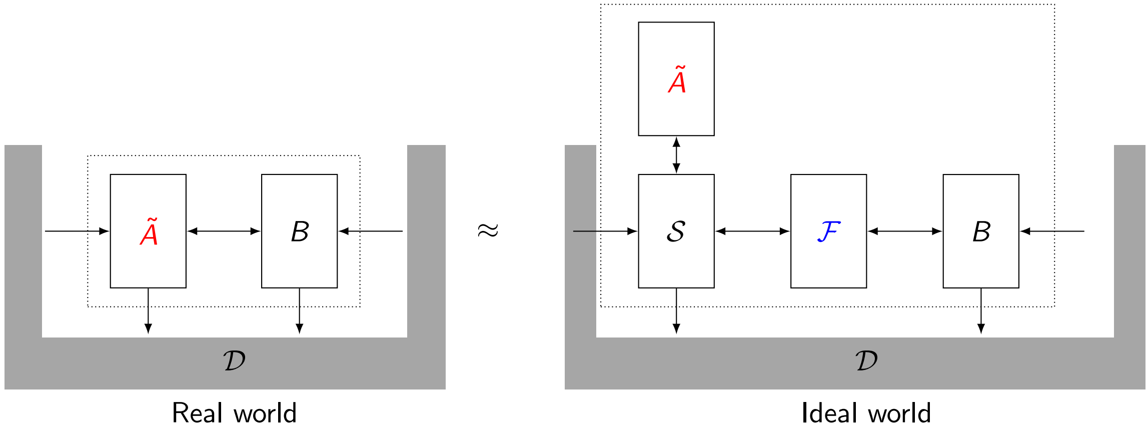


Real world



Ideal world

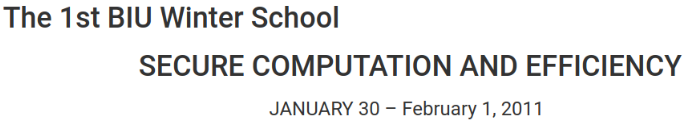
Security definition (two-party case)



For every polynomial-time $\mathcal{D}$, $|Pr[\mathcal{D}(\text{real})] - Pr[\mathcal{D}(\text{ideal})]| \leq negl(\lambda)$

Classical MPC protocols

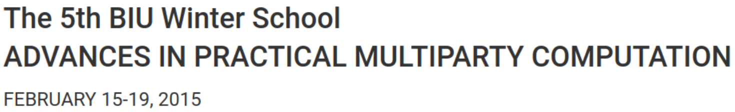
Classical MPC protocols



The 1st BIU Winter School

SECURE COMPUTATION AND EFFICIENCY

JANUARY 30 – February 1, 2011



The 5th BIU Winter School

ADVANCES IN PRACTICAL MULTIPARTY COMPUTATION

FEBRUARY 15-19, 2015

Classical MPC protocols

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GMW family

Honest MPC + $\mathcal{F}_{ZK}$

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Oblivious transfer

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Ideal functionality

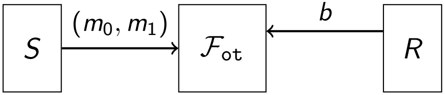
S

$\mathcal{F}_{\mathtt{ot}}$

R

Oblivious transfer

Ideal functionality



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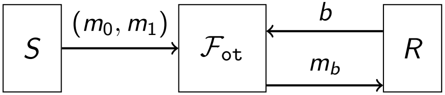

Oblivious transfer

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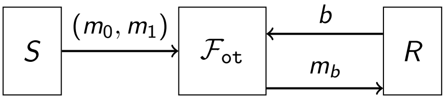
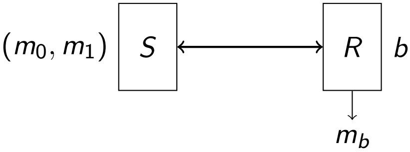


Real world



Oblivious transfer - security definitions

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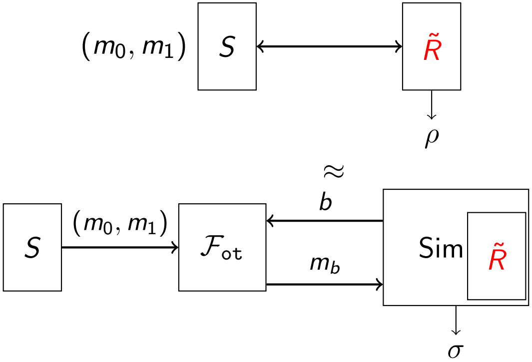
Security against malicious receiver



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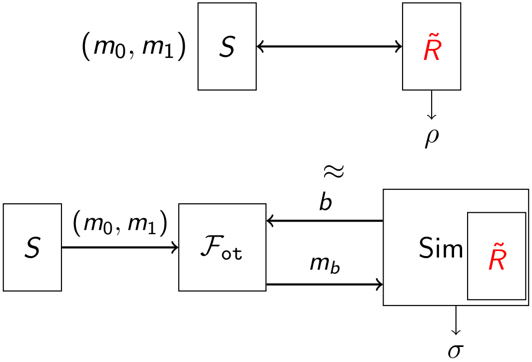
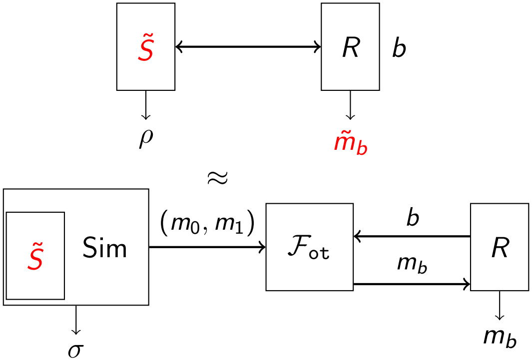


Security against malicious sender



MPC from Quantum+OWF

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Corollary

Quantum protocol for MPC from OWF (i.e. MPC is in Mini**Q**Crypt)

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Quantum protocol for MPC from OWF (i.e. MPC is in Mini**Q**Crypt)
vs.
Impagliazzo-R'91: We don't expect MPC in MiniCrypt!

BBCS protocol (I)

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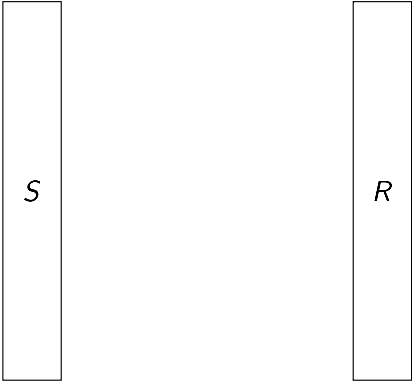
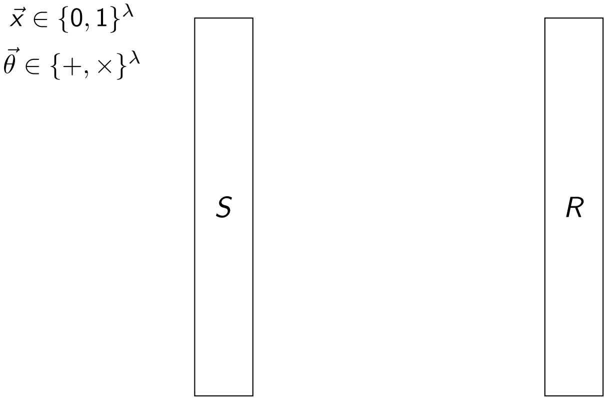


S

R

BBCS protocol (I)



$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$

BBCS protocol (I)

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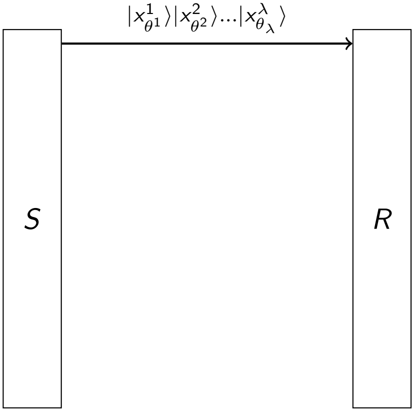

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S

R

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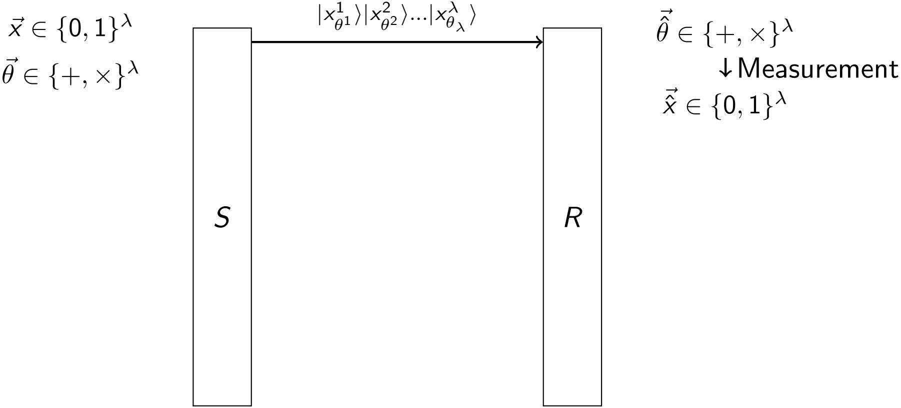

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$\downarrow$Measurement

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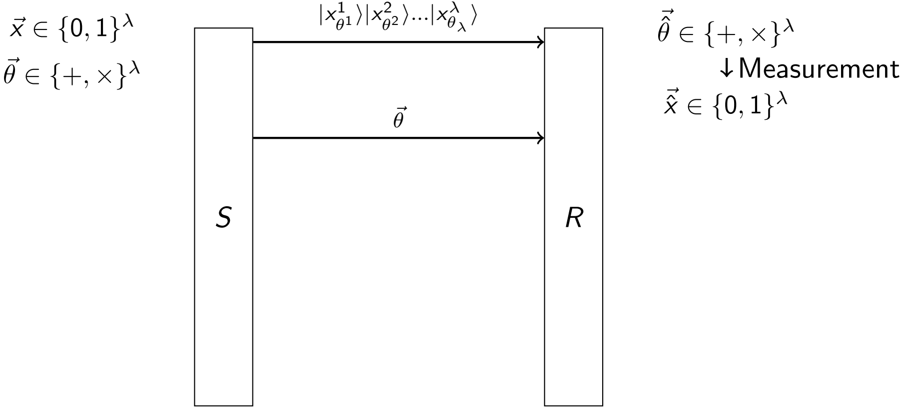

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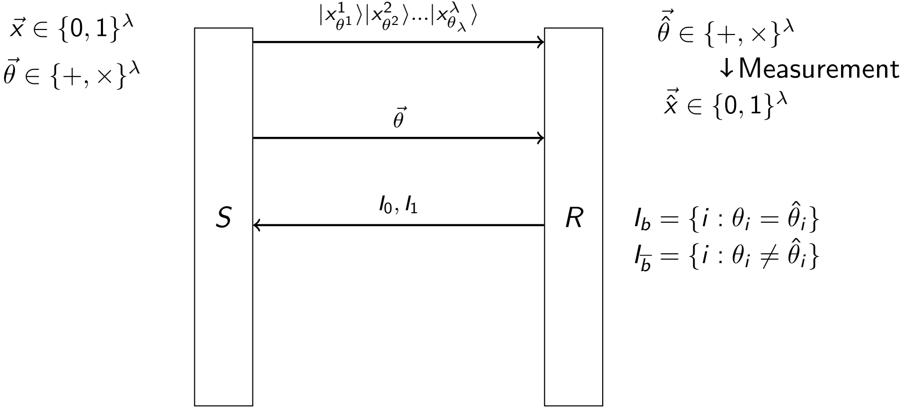

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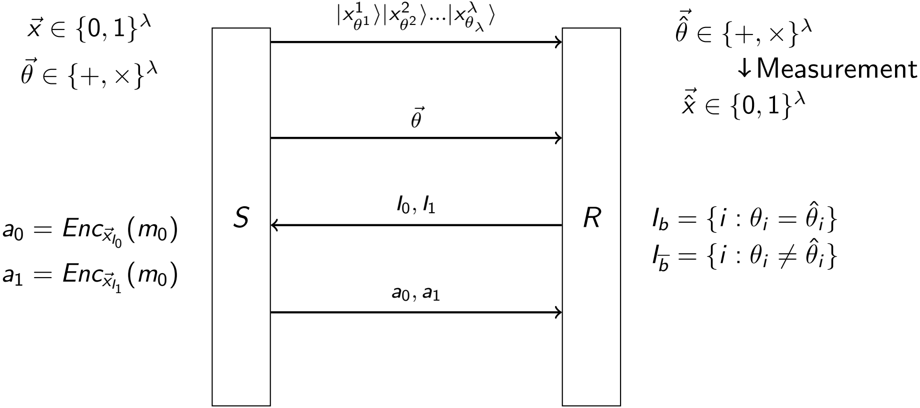

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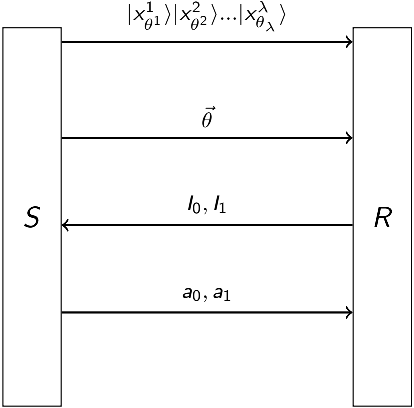

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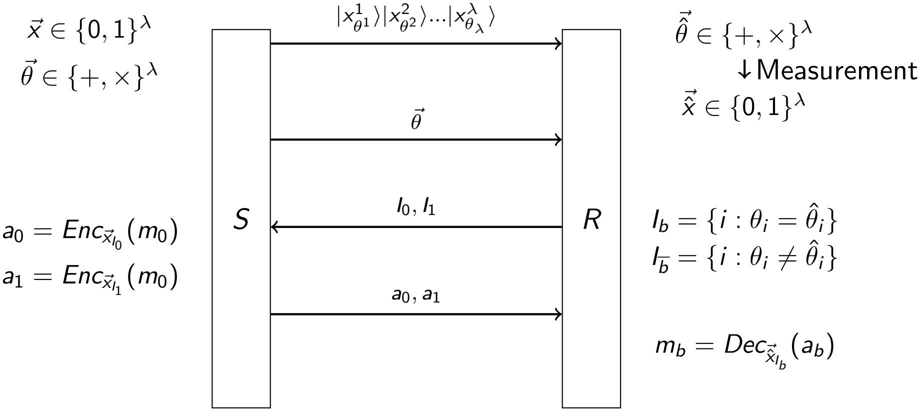

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a_0, a_1

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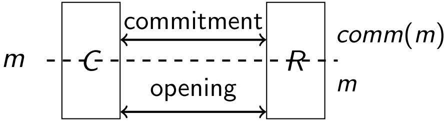
Attack for malicious receiver: $\tilde{R}$ waits $\vec{\theta}$ to measure the qubits using the right basis

Bit-commitment with simulation security



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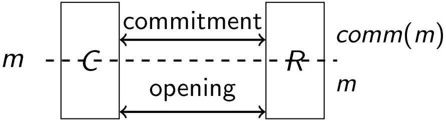
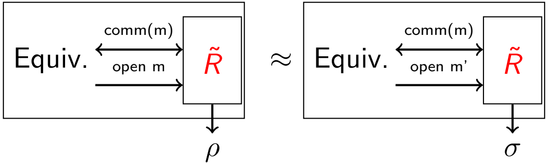


Equivocality: “simulation” hiding



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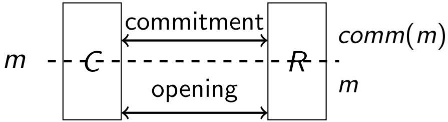

m C commitment opening R $comm(m)$ m

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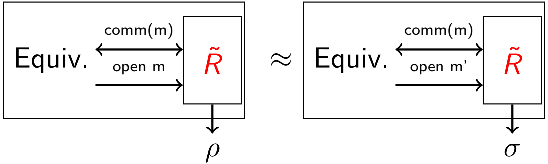
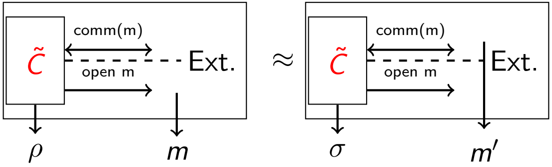


Equiv. comm(m) open m $\tilde{R}$ ρ $\approx$ Equiv. comm(m) open m’ $\tilde{R}$ σ

Extractability: “simulation” binding

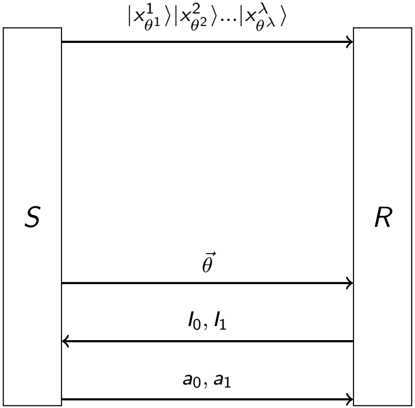


$\tilde{C}$ comm(m) open m Ext. ρ m $\approx$ $\tilde{C}$ comm(m) open m Ext. σ m'

BBCS protocol (II)

$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$



S

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$\vec{\theta}$

I_0, I_1

a_0, a_1

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↓Measurement

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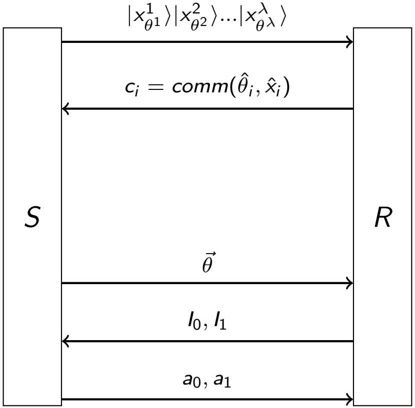

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S

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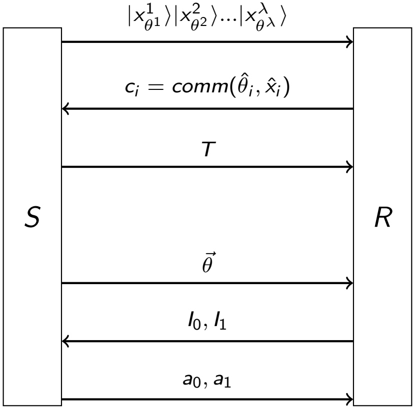

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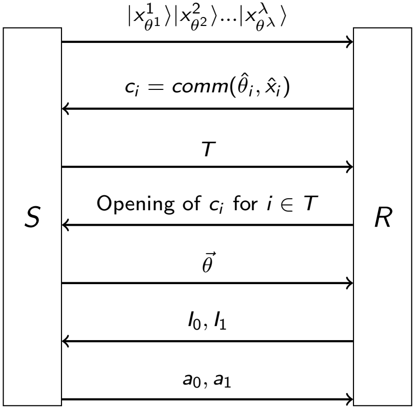

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T

Opening of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

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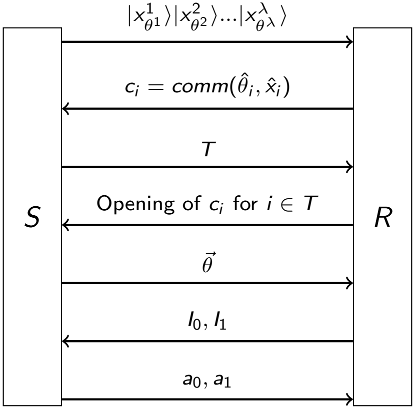

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$\downarrow$Measurement

$\vec{\hat{x}} \in \{0,1\}^\lambda$

$I_b = \{i : \theta_i = \hat{\theta}_i\} \setminus T$

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$a_0 = Enc_{\vec{x}_{I_0}}(m_0)$

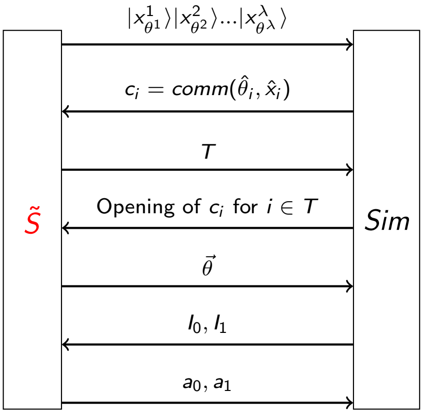
$a_1 = Enc_{\vec{x}_{I_1}}(m_0)$

$m_b = Dec_{\vec{\hat{x}}_{I_b}}(a_b)$

Security of BBCS against malicious sender

$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$



$|x^1_{\theta^1}\rangle|x^2_{\theta^2}\rangle...|x^\lambda_{\theta^\lambda}\rangle$

$c_i = comm(\hat{\theta}_i, \hat{x}_i)$

T

Opening of c_i for $i \in T$

$\tilde{S}$

Sim

$\vec{\theta}$

I_0, I_1

a_0, a_1

$\vec{\hat{\theta}} \in \{+,\times\}^\lambda$

↓Measurement

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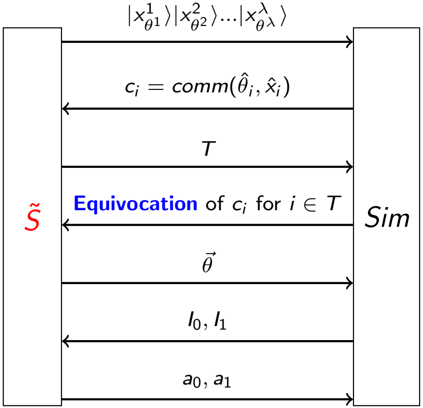

$\tilde{S}$

Sim

$|x^1_{\theta^1}\rangle|x^2_{\theta^2}\rangle...|x^\lambda_{\theta^\lambda}\rangle$

$c_i = comm(\hat{\theta}_i, \hat{x}_i)$

T

Equivocation of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

a_0, a_1

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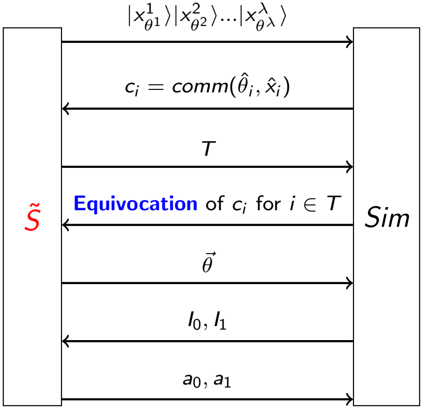

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Equivocation of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

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Sim

Measure qubits in T

$a_0 = Enc_{\vec{x}_{I_0}}(m_0)$

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Security of BBCS against malicious sender

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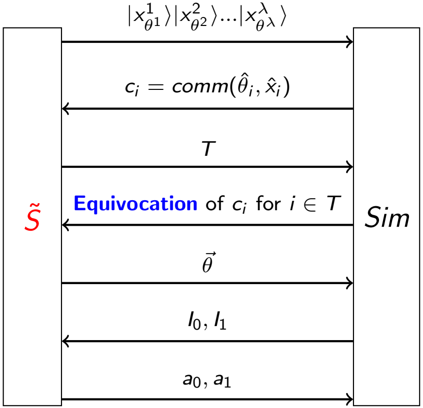

$\tilde{S}$

Sim

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T

Equivocation of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

a_0, a_1

Measure qubits in T

Measure remaining qubits using $\vec{\theta}$ (get $\vec{x}$)

Partition I_0 and I_1 at random

$a_0 = Enc_{\vec{x}_{I_0}}(m_0)$

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Security of BBCS against malicious sender

$\vec{x} \in \{0,1\}^{\lambda}$

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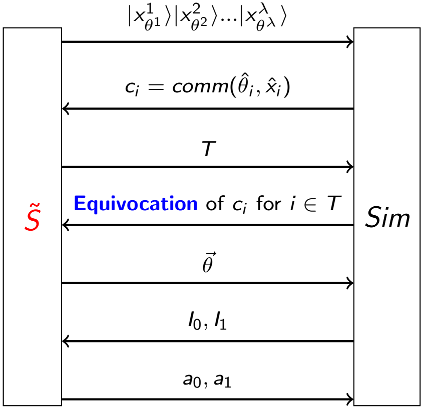

$\tilde{S}$

Sim

$|x^1_{\theta^1}\rangle|x^2_{\theta^2}\rangle...|x^{\lambda}_{\theta^{\lambda}}\rangle$

$c_i = comm(\hat{\theta}_i, \hat{x}_i)$

T

Equivocation of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

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Measure qubits in T

Measure remaining qubits using $\vec{\theta}$ (get $\vec{x}$)

Partition I_0 and I_1 at random

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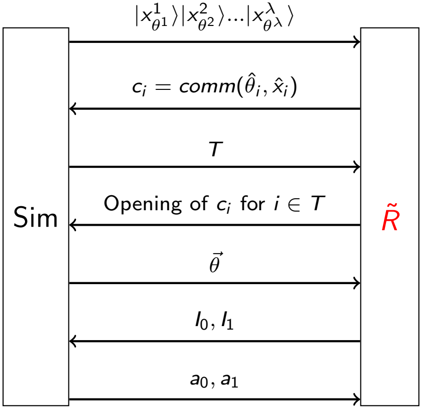
Security of BBCS against malicious receiver

$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$



Sim

$\tilde{R}$

$|x^1_{\theta^1}\rangle|x^2_{\theta^2}\rangle...|x^\lambda_{\theta^\lambda}\rangle$

$c_i = comm(\hat{\theta}_i, \hat{x}_i)$

T

Opening of c_i for $i \in T$

$\vec{\theta}$

I_0, I_1

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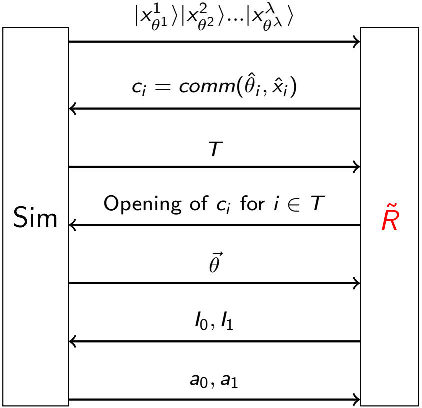
Security of BBCS against malicious receiver

$\vec{x} \in \{0,1\}^\lambda$

$\vec{\theta} \in \{+,\times\}^\lambda$

Extract $\vec{\hat{\theta}}$



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Compute b

$m_b = Dec_{\vec{\hat{x}}_{I_b}}(a_b)$

Implemententing commitment scheme with simulation security from OWF

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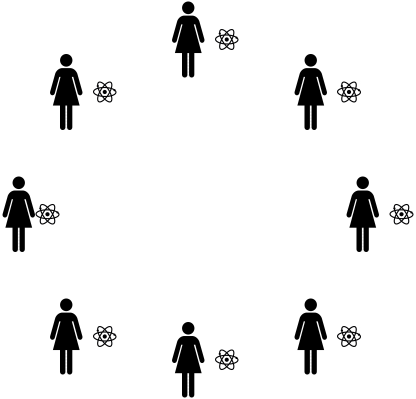
[BCKM21]	[GLSV21]
1. (Black-box) equivocality compiler 2. Extractable commitment from equivocal commitment and quantum communication	1. Equivocal commitment from Naor's commitment and zero-knowledge 2. Unbounded-simulator OT from equivocal commitment 3. Extractable and equivocal commitment from unbounded-simulator OT and quantum communication
- **Black-Box** use of one-way functions - **Statistical** security against malicious receiver	- **Constant-Round** OT in the CRS model - **Statistically binding** extractable commitment

Features:

Multi-party quantum computation

Parties share some input state $\rho_{A_1,\ldots,A_8}$



Goal: Compute U on joint share state ρ without revealing their share

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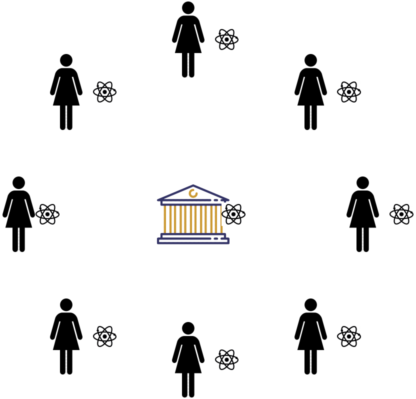

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Ideal world

Multi-party quantum computation

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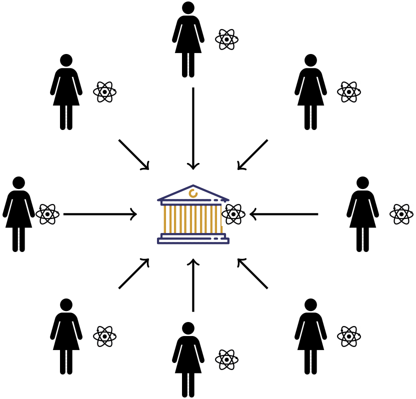

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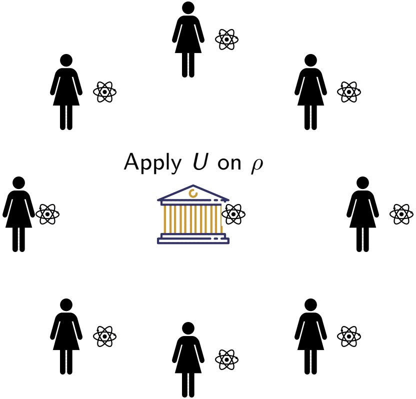

Apply U on ρ

Goal: Compute U on joint share state ρ without revealing their share

Ideal world

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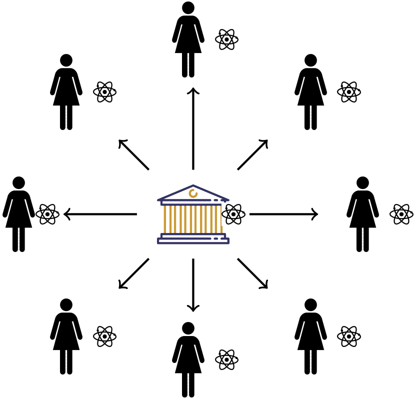

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- Each party gets their share of the output $U\rho U$

Multi-party quantum computation

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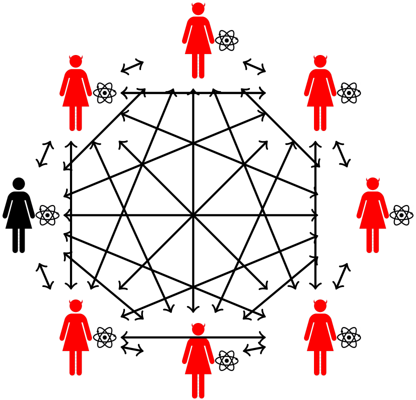

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- Goal: implement the ideal functionality
- Protocols where parties interact, but still they only learn their share even if they behave disonestly

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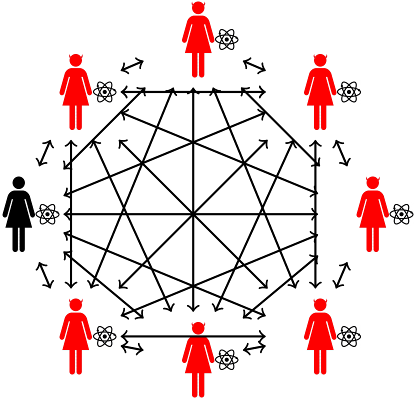

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Real world

- Goal: implement the ideal functionality
- Protocols where parties interact, but still they only learn their share even if they behave disonestly
- Security definition similar to the classical setting

MPQC

- Statistically secure MPQC with honest majority [Crépeau-GS'02,BenOr-CGHS'06]
- Computationally secure 2PQC [Dupuis-NS'10, Dupuis-NS'12, Kashefi-MW'17]
- MPQC with allowed dishonest subsets [Kashefi-P'17]
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- **Computationally secure MPQC with arbitrary dishonest majority** [Dulek-**G**JMS'20]
    - Extends DNS'12 to the multi-party setting
    - Assumes ideal MPC functionality ($\mathcal{F}_{MPC}$)

Clifford encoding

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Clifford operations:

Unitaries generated by $\{H, P, CNOT\}$

$\mathcal{C}_m = \{$Clifford circuits on m qubits $\}$

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Clifford encoding for n-qubit state $|\psi\rangle$ and security parameter λ:

1. Pick a random $(\lambda + n)$-qubit Clifford C
2. $C(|\psi\rangle \otimes |0^\lambda\rangle)$

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Authentication:
For any non-trivial $\mathcal{A}$, trap qubits of $C^\dagger \mathcal{A}(C|\psi\rangle|0^n\rangle)$ will be non-zero w.p. $1 - \text{negl}(\lambda)$

MPQC protocol - General idea

Focus on a single (pure) qubit

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- P_{i^*} holds $C(|\psi\rangle|0^{2\lambda}\rangle)$
- All players (secret) share C
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    - λ trap qubits used in the test
    - remaining λ to keep privacy/authentication even in the test

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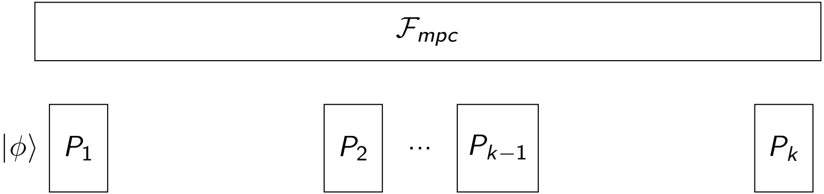
- Computation on encoded data

MPQC protocol - Encoding



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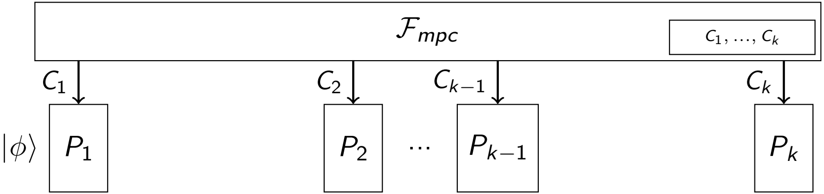

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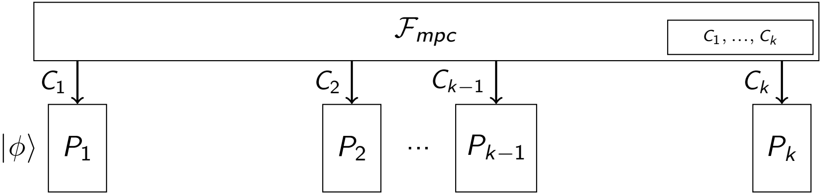
MPQC protocol - Encoding



- $\mathcal{F}_{mpc}$ computes random $\{C_i\}_{2\lambda+1}$, parties apply C_i and send the state around the table

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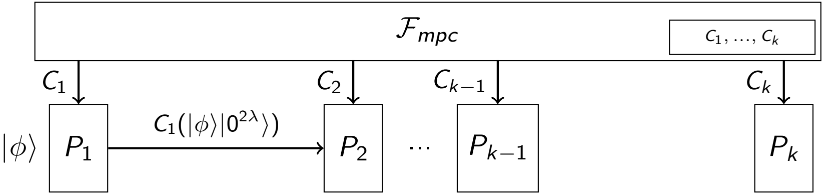

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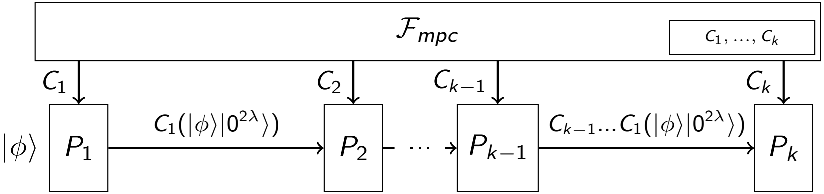

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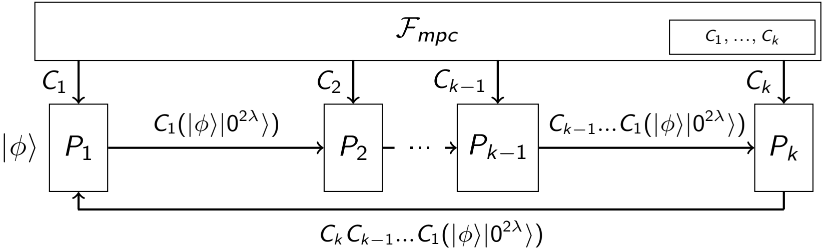

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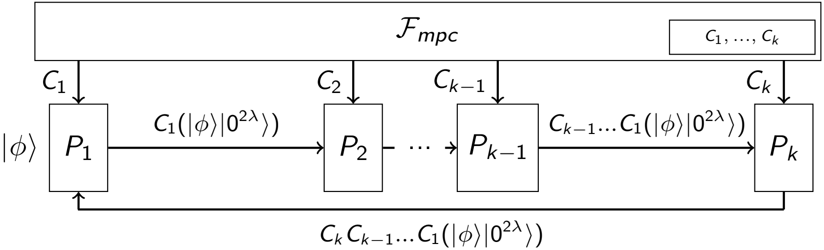

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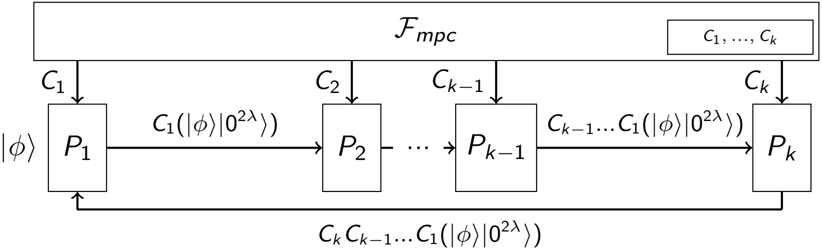
- How to prevent that any of the parties replaces the quantum state (or cheat arbitrarily)?

MPQC protocol - Encoding



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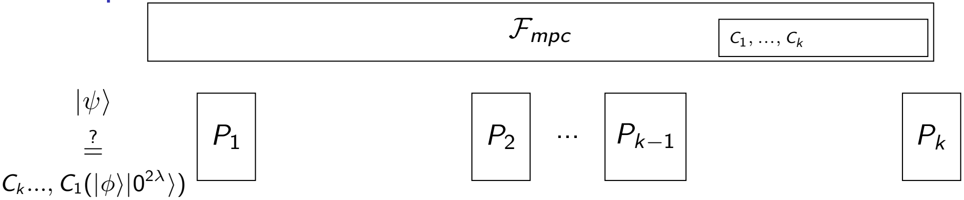
MPQC protocol - Public authentication test



$\mathcal{F}_{mpc}$

$C_1, \ldots, C_k$

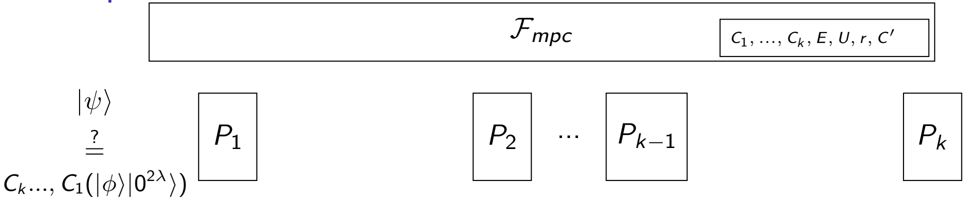
$$|\psi\rangle \overset{?}{=} C_k \ldots, C_1(|\phi\rangle|0^{2\lambda}\rangle)$$

P_1

P_2 $\cdots$ P_{k-1}

P_k

MPQC protocol - Public authentication test



$\mathcal{F}_{mpc}$

$C_1, \ldots, C_k, E, U, r, C'$

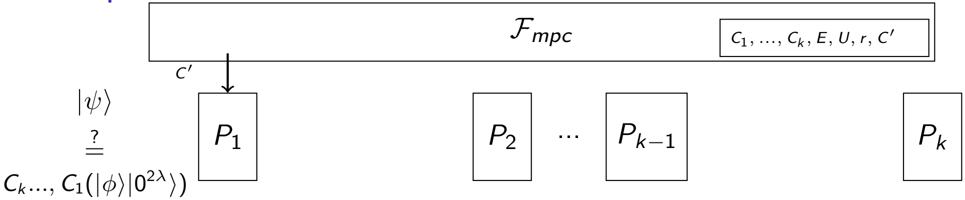
$|\psi\rangle \overset{?}{=} C_k \ldots, C_1(|\phi\rangle|0^{2\lambda}\rangle)$

P_1 P_2 $\cdots$ P_{k-1} P_k

- $\mathcal{F}_{mpc}$ computes random $C' \in \mathcal{C}_{2\lambda+1}$, $E \in \mathcal{C}_{\lambda+1}$, linear function U and $r \in \{0,1\}^{\lambda}$ s.t.

$$C' = (E \otimes X^r)(I_2 \otimes U)C_1^{\dagger} \ldots C_{k-1}^{\dagger} C_k^{\dagger}$$

MPQC protocol - Public authentication test



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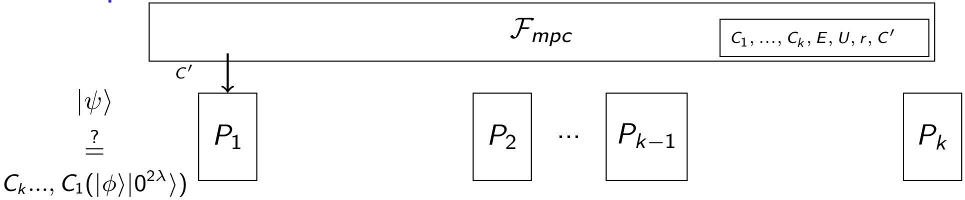
MPQC protocol - Public authentication test



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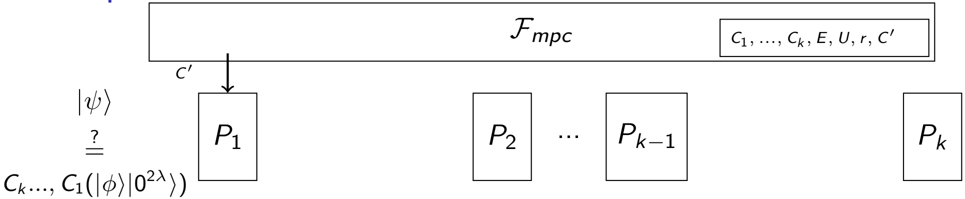
  - Honest case: $E(|\phi\rangle|0^\lambda\rangle)|r\rangle$

MPQC protocol - Public authentication test



$\mathcal{F}_{mpc}$

$C_1, \dots, C_k, E, U, r, C'$

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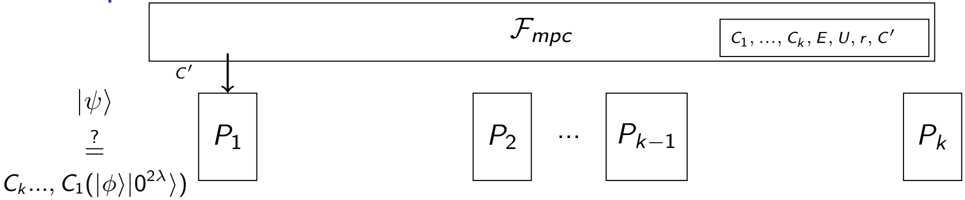
MPQC protocol - Public authentication test



$\mathcal{F}_{mpc}$

$C_1, \dots, C_k, E, U, r, C'$

C'

$|\psi\rangle$

$\overset{?}{=}$

$C_k \dots, C_1(|\phi\rangle|0^{2\lambda}\rangle)$

P_1

P_2 $\cdots$ P_{k-1}

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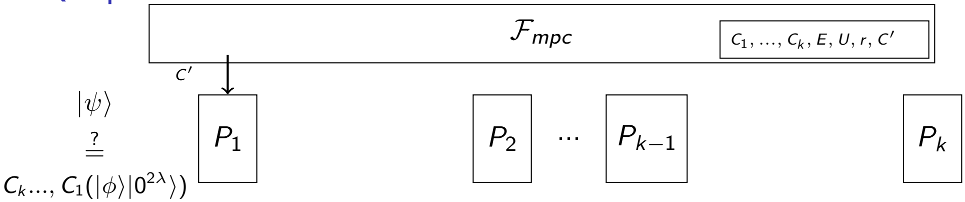

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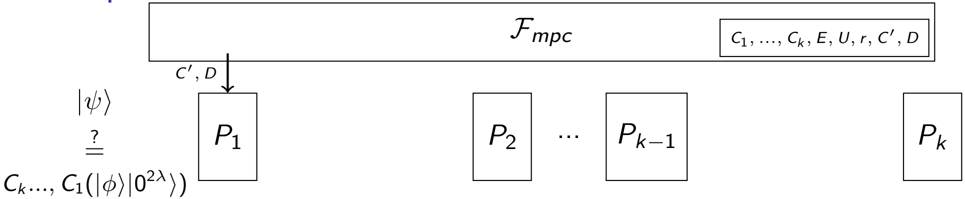

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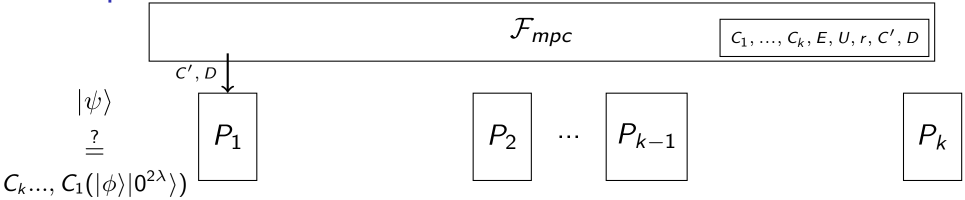
MPQC protocol - Public authentication test



$\mathcal{F}_{mpc}$

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C', D

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- Similar procedure enables (secure) public measurement in the computational basis

MPQC protocol - Applying gates

1. One-qubit Clifford D: can be perfomed by "changing the key"

$$C_k ... C_1(|\phi\rangle|0^{2\lambda}\rangle) = C_k ... C_1'(D|\phi\rangle|0^{2\lambda}\rangle), \text{ for } C_1' = C_1 D^\dagger$$

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    2. Re-encode the two qubits altogether (+ public authentication test)
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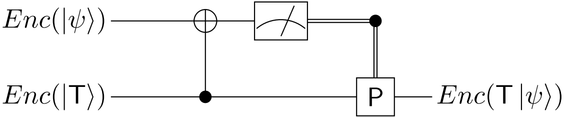
3. T-gate:



MPQC protocol - creating T magic states

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1. P_1 create $poly(\lambda, k)$ T-magic states
2. Parties run sub-protocol to encode the (supposed) magic states
3. Each party tests a random subset
    - Locally decode (with the help of $\mathcal{F}_{mpc}$)
    - Check if the “raw” qubit is indeed $|T\rangle$
4. Use magic state destillation procedure to transform somewhat-good T-magic states into almost-perfect ones
    - Only need Clifford circuit + measurement

MPQC protocol - overall protocol

MPQC protocol - overall protocol

Protocol

1. Parties run sub-protocol to create $Enc(|T\rangle^{\otimes t})$
2. Parties run sub-protocol to encode each qubit
3. For each gate/measurement, parties run the corresponding sub-protocol
4. Each party decodes her own output (with the help of $\mathcal{F}_{MPC}$)

Summary

Zero-knowledge proofs

Central tool in crypto toolbox

1. ZK for NP in MiniCrypt
2. ZK against quantum adversaries
3. ZK for QMA ("quantum NP")

Multi-party computation

Most-general functionality (modulo #rounds)

1. MPC from Oblivious transfer
2. OT is in Mini**Q**Crypt
3. Multi-party quantum computation

Some open questions

1. (Im)possitiblity of constant-round quantum ZK protocol in the plain model
2. Applications of zero-knowledge for quantum proofs
3. (Q)NIZK for QMA with RO/CRS
4. Zero-knowledge with multiple non-signaling provers
5. (Im)possibility of MPQC in constant rounds
6. (Black-box) separations of cryptographic primitives in the quantum setting
7. Further quantum protocols from weaker assumptions
8. Practical quantum cryptographic protocols
9. ...

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Thank you for your attention!