

# Hard Problems in Blockchains

Valeria (Lera) Nikolaenko  
for the 13th BIU Winter School on  
cryptography

al6z  
crypto

- 1: Long-Range Attacks on PoS**
- 2: Proposer Election in PoS**
- 3: Post-quantum blockchains**

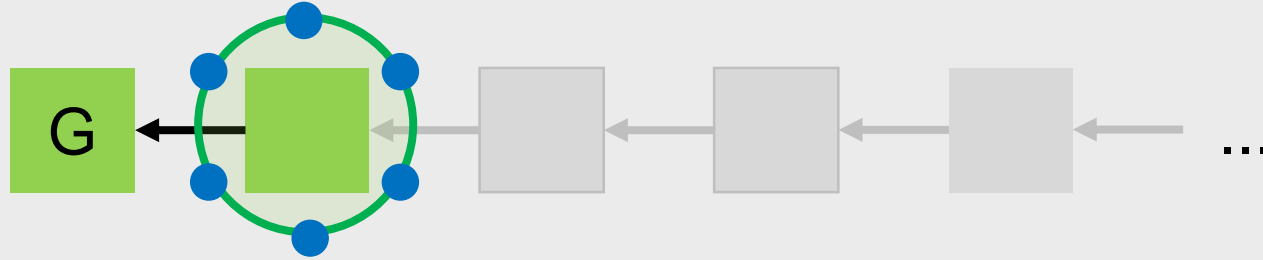


**1: Long-Range Attacks on PoS**

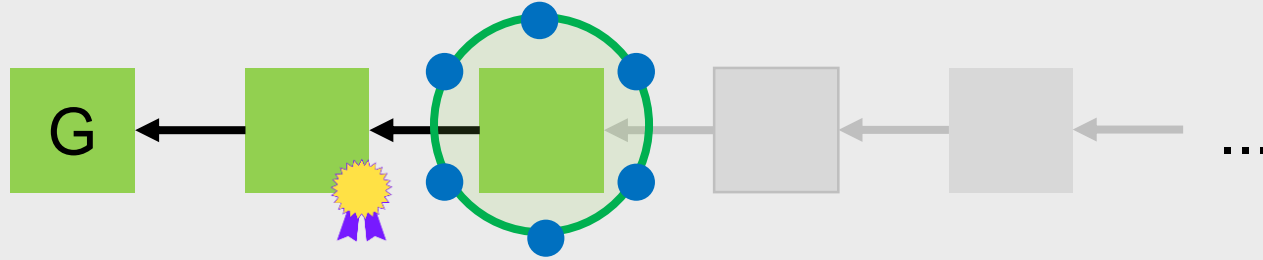
2: Proposer Election in PoS

3: Post-quantum blockchains

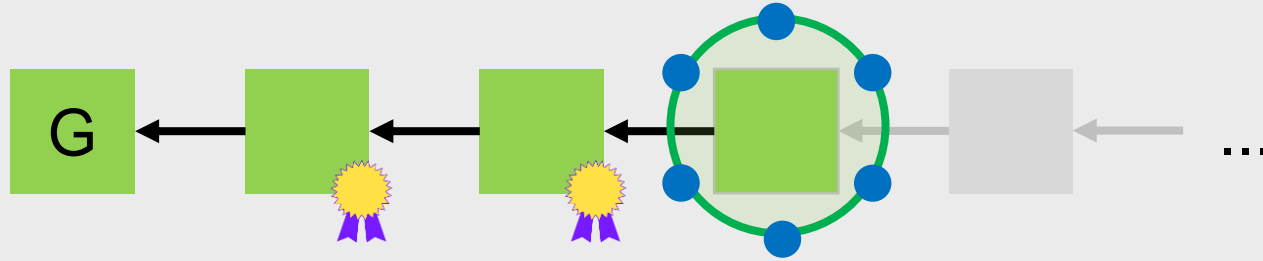
# Proof-of-Stake blockchains



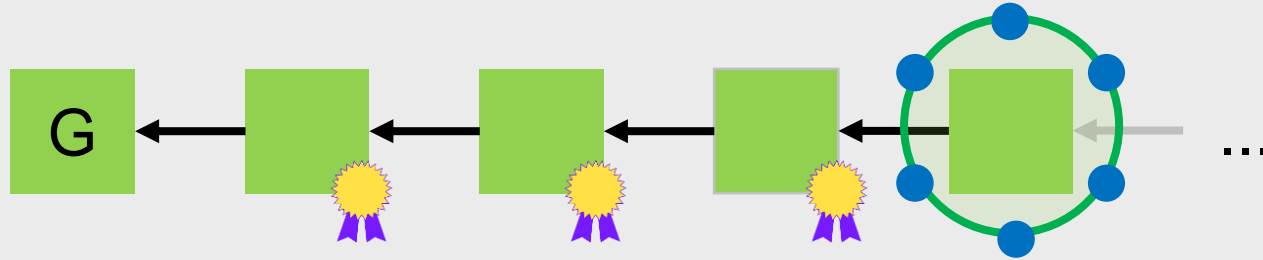
# Proof-of-Stake blockchains



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# Proof-of-Stake blockchains



**Validators “notarize” blocks**

Active validators stay  honest  
due to incentives

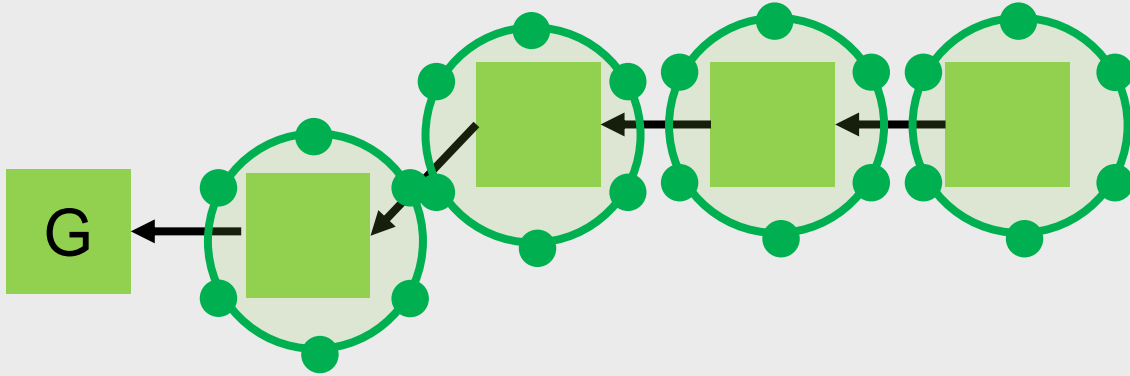


# Validator with no incentives can leak old keys

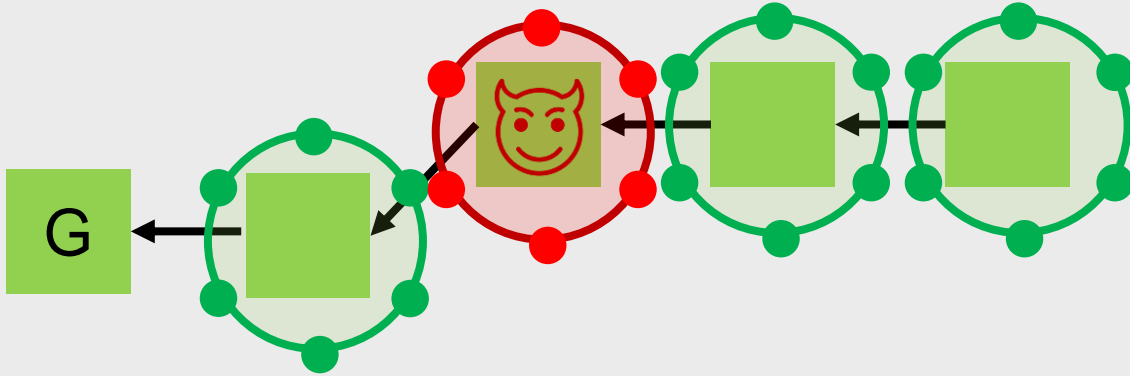


If old validators become  corrupt  
safety is broken

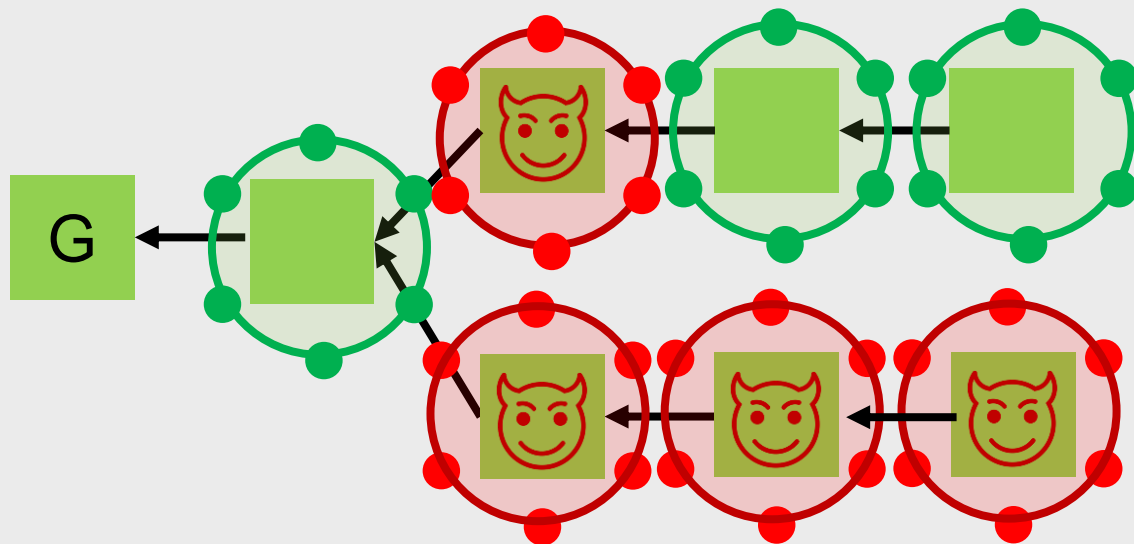
# Corrupt validators may fork history



# Corrupt validators may fork history



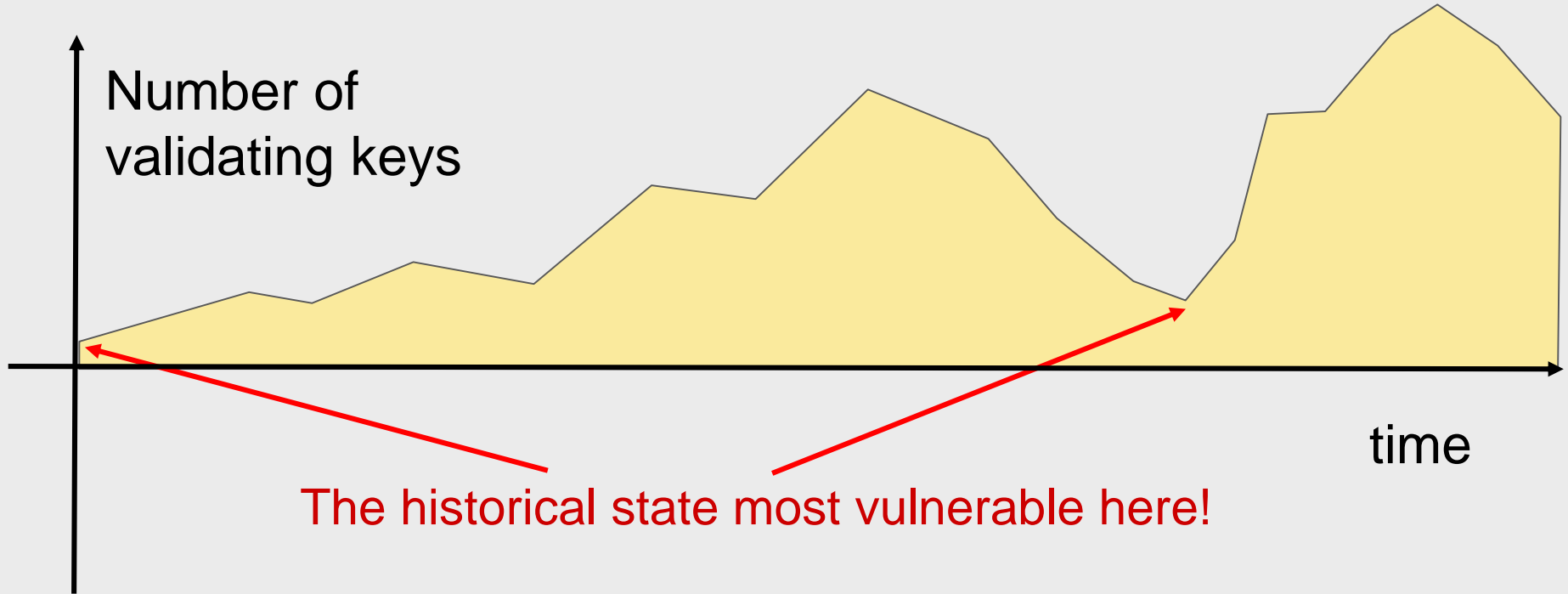
# Corrupt validators may fork history



Users can not differentiate!

unless they've been following all consensus rounds actively

# Low number of keys don't protect history well



# Mitigations for long-range attacks

1. Checkpointing
2. Key-evolving cryptography
3. Keep everybody online
4. Winkle (user-based consensus)

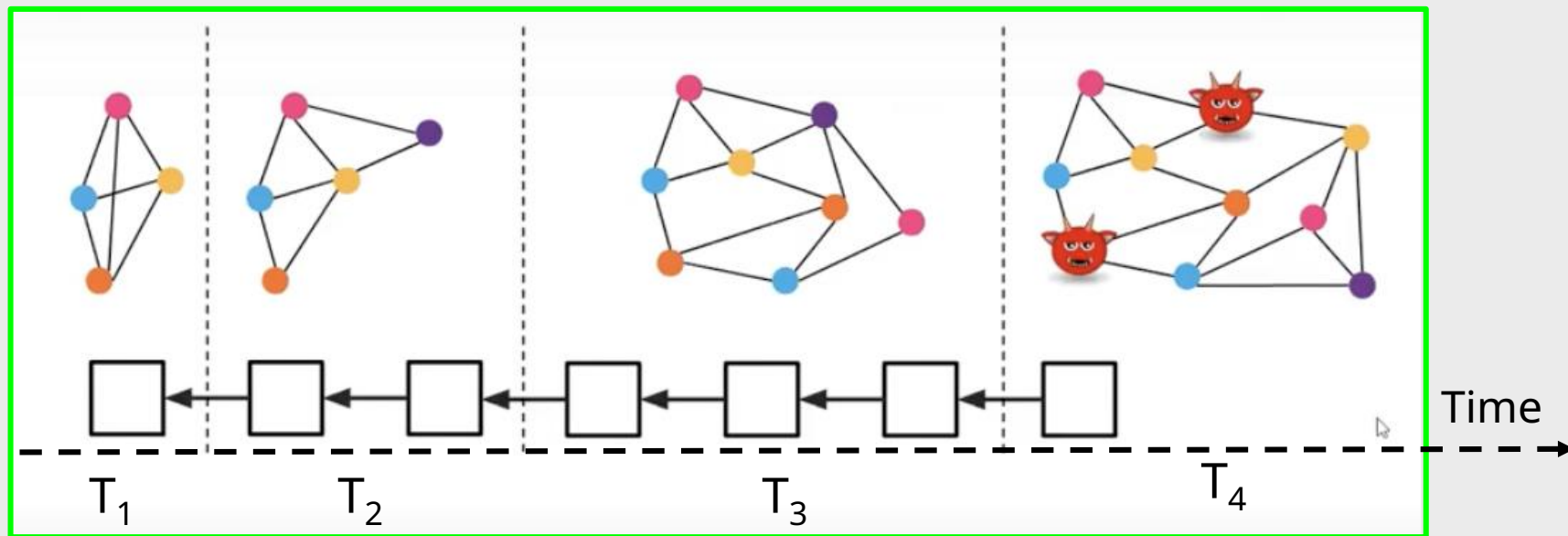
# #1 Checkpointing

- Centralized checkpointing (i.e. hardcode the checkpoints into the github codebase)
  - Checkpoint = hash of a block (very small)
  - When synching check that the checkpoint matches the hardcoded one
- Checkpoint to a PoW chain

**Problem:** centralized!

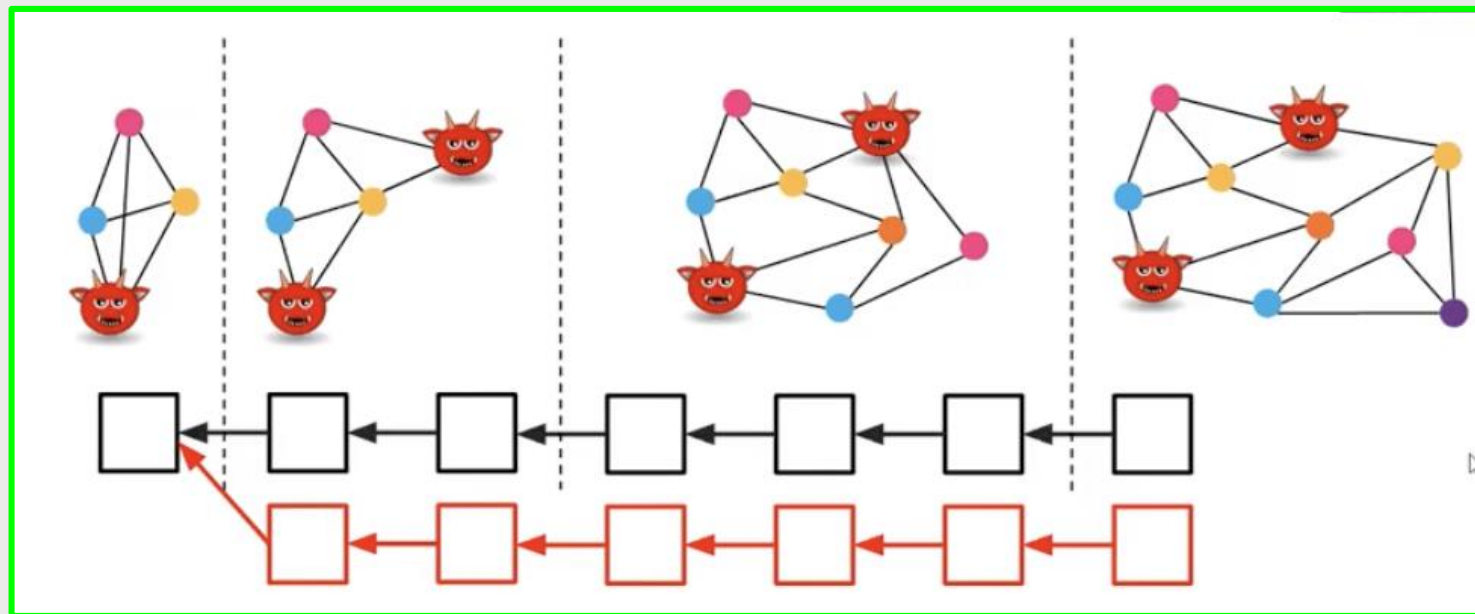


# Easier to attack if validators are not rotating their keys



Attacking 2 at time  $T_4$  equivalent to attacking 2 at time  $T_2$ !

# Easier to attack if validators are not rotating their keys



2-out-of-5 is  
enough to attack!

2-out-of-10 is not  
enough to attack

## #2 Key-Evolving cryptography

- Rotate validator keys frequently:  $(pk, sk) \rightarrow (pk', sk')$
- Time-evolving secret-key (public key stays the same!) [DGNW20]:  
 $(pk, sk_1) \rightarrow (pk, sk_2) \rightarrow (pk, sk_3) \rightarrow (pk, sk_4) \rightarrow \dots$
- Assume honest validators forget old keys.

Does not solve our problem, but good practice anyway!

**Problem:** erasing the old secret keys is incentive incompatible!

## #3: Keep everybody online

When all the nodes are online and monitoring the blockchain closely, it is very hard to make them believe a deep fork.

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### Casper the Friendly Finality Gadget

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Vitalik Buterin and Virgil Griffith  
Ethereum Foundation

■ ■ ■

In simple terms, long-range attacks are prevented by a fork choice rule to never revert a finalized block, as well as an expectation that each client will “log on” and gain a complete up-to-date view of the chain at some regular frequency (e.g., once per 1–2 months). A “long range revision” fork that finalizes blocks older than that will

**Problem** is: clients/validators can be sleepy.

## #4 Winkle :

make users “vote” inside their transactions on the current state of the blockchain

### Winkle: Foiling Long-Range Attacks in Proof-of-Stake Systems

Sarah Azouvi  
University College London,  
Protocol Labs

George Danezis  
University College London,  
Facebook Novi

Valeria Nikolaenko  
Facebook Novi

#### ABSTRACT

Winkle protects any validator-based byzantine fault tolerant consensus mechanisms, such as those used in modern Proof-of-Stake blockchains, against long-range attacks where old validators' signature keys get compromised. Winkle is a decentralized secondary layer of client-based validation, where a client includes a single additional field into a transaction that they sign: a hash of the previously sequenced block. The block that gets a threshold of signatures (confirmations) weighted by clients' coins is called a “confirmed” checkpoint. We show that under plausible and flexible security assumptions about clients the confirmed checkpoints can not be equivocated. We discuss how client key rotation increases security, how to accommodate for coins' minting and how delegation allows for faster checkpoints. We evaluate checkpoint latency experimentally using Bitcoin and Ethereum transaction graphs, with and without delegation of stake.

Validator key rotations help alleviate the problem, assuming secure destruction of older keys. However, validators might have auxiliary incentives to sell their old keys to an adversary, especially when real-world identities of validators are unknown in a permissionless system and reputation is not at risk. When dishonest behaviour of a validator becomes rational, real-world security of the whole system is at great risk. We notice that corrupting a significant number of coin holders, even after they have no more stake in the system, is far more challenging as they are much more numerous than validators (we justify this assumption in Section 4). This observation brings us to introducing Winkle — a novel mechanism that leverages votes from clients creating a decentralized secondary layer of client-based validation to confirm checkpoints (snapshots of the blockchain) and to prevent long-range attacks on proof-of-stake protocols. The voting mechanism is very simple: each client augments their transaction with a single additional field — a hash of a previously sequenced block. Once this transaction gets signed

# Winkle: users vote on blocks when transacting

- New transaction format:  
Tx = [sender, receiver, amount , **LAST\_BLOCK**]<sub>o</sub>
- The block is checkpointed when 50% of all coins vote on it.
- Checkpoint can't be reverted even under Long-Range-Attack.
- To attack the adversary needs to obtain:  
validators' keys AND users' keys



All blockchain users  
1M – 1B

Validators  
100-1,000

To attack the adversary needs to obtain: **validators' keys** AND **users' keys**

# Winkle: second layer of confirmation

## Consensus by users:

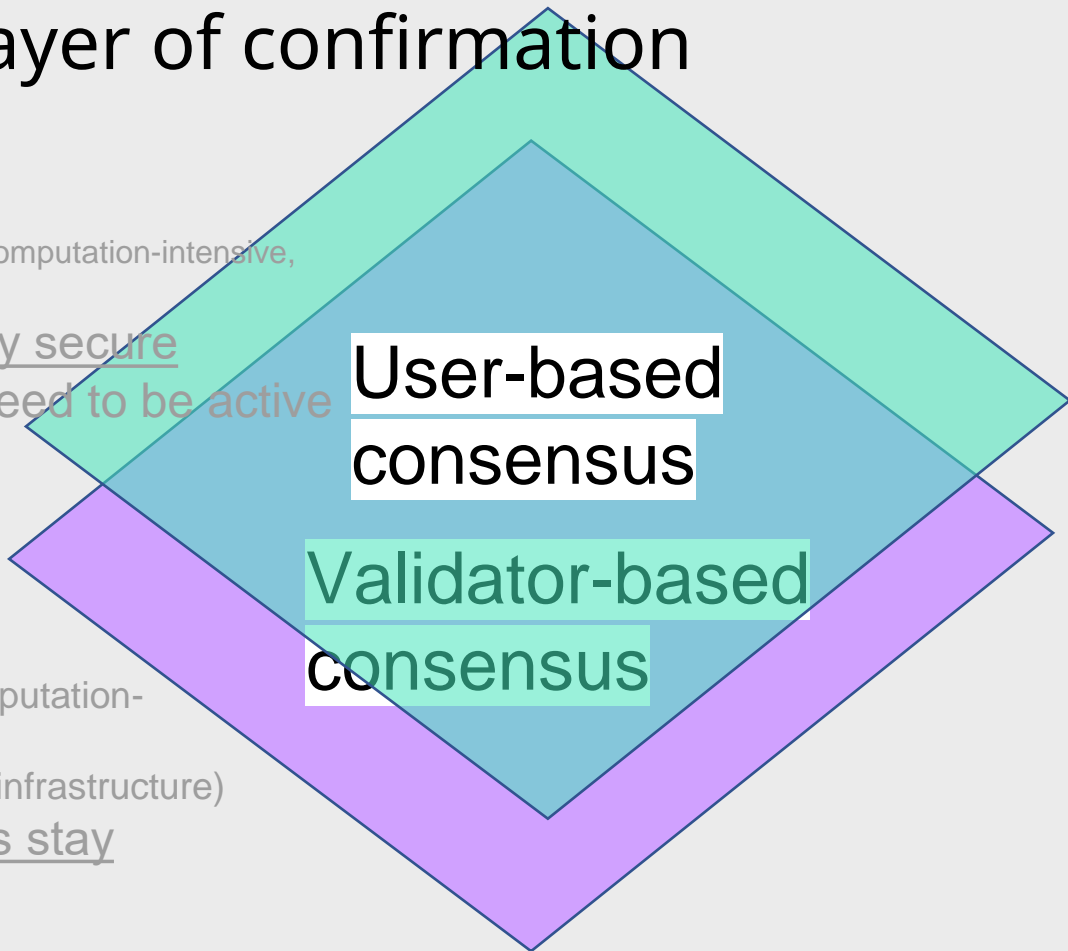
- simple (NON-INTERACTIVE, not computation-intensive, no additional infrastructure)
- safe while users' keys stay secure
- large threshold of users need to be active

User-based  
consensus

## Consensus by validators:

- $< 1/3$  byzantine
- complicated (interactive, computation-intensive, requires dedicated infrastructure)
- safe while validators' keys stay secure

Validator-based  
consensus





# Mitigations for long-range attacks

1. Checkpointing
2. Key-evolving cryptography
3. Keep everybody online
4. Winkle (user-based consensus)

philosophically, a good idea to make all users (not just the validators) work on maintaining the security of blockchain

- 1: Long-Range Attacks on PoS
- 2: Proposer Election in PoS**
- 3: Post-quantum blockchains

# Proposer election goals

- A proposer is elected per time-slot to propose a block.
- A proposer gets rewarded for proposing a good block.
- Election properties
  - **Uniqueness** of elected proposer
  - **Unpredictability<sub>T</sub>**: T is the time between proposer getting publicly known and proposer announcing a block
    - $T > 0$  : “public election”
    - $T = 0$  : “secret election” - the proposer announces itself when published the block
    - $\Rightarrow$  fair (each leader is elected with equal probability)
  - **Unbiasability** (nobody should be able to influence the proposer election in its favor)  
= Unpredictability under active attacks

# #1 Proposer election : Round-Robin

**Round-robin proposer election:** proposers are chosen one after the other in a lexicographical order.

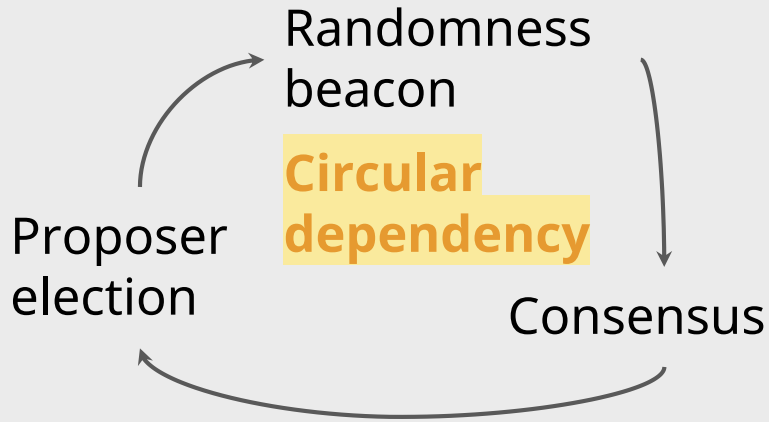
Unique

**Predictable = public:** the proposer for the slot is known well in advance:  
proposers can be DDoSed

**Biasable**

## #2 Proposer election : Randomness Beacon

**Randomness beacon:** a distributed protocol that outputs (pseudo)-random values at regular time intervals



**Circular dependency** is broken by induction:

- fix proposer schedule of the current epoch,
- build randomness beacon to randomize proposer schedule of the next epoch.

**Unique; Unbiasable;**

**Predictable:** schedule known 1 epoch in advance

## #2 Proposer election : Randomness Beacon

Protocol	Network Model	Adaptive Adversary	Liveness	Unpredictability	Bias-Resistance	Fault-tolerance	Communication Complexity	Computation Complexity	Verification Complexity	Cryptographic Primitive	No Trusted Dealer or DKG required
ALBATROSS [18]	syn.	✗	✓	✓	✓	1/2	$\mathcal{O}(n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(n)$	PVSS	✓
Algorand [39]	semi-syn.	✗	✓	✓ <sup>1</sup>	✗	1/3 <sup>0</sup>	$\mathcal{O}(cn)$	$\mathcal{O}(c)$	$\mathcal{O}(1)$	VRF	✓
BRandPiper [8]	syn.	✓	✓	✓	✓	1/2	$\mathcal{O}(n^3)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$	PVSS	✓
Cachin et. al [16]	asyn.	✗	✓	✓	✓	1/3	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	Uniq. thr-sig.	✗
Caucus [2]	syn.	✗	✓	✓ <sup>1</sup>	✓	1/2	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	Hash func.	✓
Continuous VDF [34]	asyn.	✗	✗ <sup>1</sup>	✓	✓	1/2	$\mathcal{O}(1)$	VDF	$\mathcal{O}(1)$	VDF	✓
DFINITY [45]	semi-syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	BLS thr-sig.	✗
Drand [31]	syn.	✗	✓	✓	✓	1/2	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	Uniq. thr-sig.	✗
GLOW [36]	syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(n)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	DVRF	✗
GRandomPiper [8]	syn.	✗	✓	✓ <sup>1</sup>	✓	1/2	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$	PVSS	✓
HERB [20]	syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(n^2)^*$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	PHE	✗
HydRand [66]	syn.	✗	✓	✓ <sup>1</sup>	✓	1/3	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	PVSS	✓
Nguyen-Van et. al [55]	syn.	✗	✗	✓	✓	1/2	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$	PHE, VRF	✓
Ouroboros [47]	syn.	✗	✓	✓	✓	1/2	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	PVSS	✓
Ouroboros Praos [27]	semi-syn.	✓	✓	✓ <sup>1</sup>	✓	1/2	$\mathcal{O}(n)^*$	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$	VRF	✓
Proof-of-Delay [14]	syn.	✗	✓	✓	✓	1/2	$\mathcal{O}(n)$	very high	$\mathcal{O}(\log \Delta)^0$	Hash func.	✓
Proof-of-Work [53]	syn.	✗	✓	✓ <sup>1</sup>	✗	1/2	$\mathcal{O}(n)$	very high	$\mathcal{O}(1)$	Hash func.	✓
RandChain [71]	syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(cn)$	$\mathcal{O}(cn)$	$\mathcal{O}(n)$	VRF	✓
RANDCHAIN [44]	syn.	✓	✓	✓	✓	1/3	$\mathcal{O}(n)$	VDF	$\mathcal{O}(1)$	VDF	✓
RANDAO [59]	asyn.	✗	✓	✗	✗	1/2	$\mathcal{O}(n)$	VDF	$\mathcal{O}(1)$	VDF	✓
RandHerd [69]	syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(c^2 \log n)$	$\mathcal{O}(c^2 \log n)$	$\mathcal{O}(1)$	PVSS, CoSi	✗
RandHound [69]	syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(c^2 n)$	$\mathcal{O}(c^2 n)$	$\mathcal{O}(c^2 n)$	PVSS	✓
RandRunner [65]	syn.	✓	✓	✓ <sup>1</sup>	✓	1/2	$\mathcal{O}(n^2)$	VDF	$\mathcal{O}(1)$	VDF	✓
RandShare [69]	asyn.	✓	✓ <sup>0</sup>	✓	✓	1/3	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	VSS	✓
Rand Extractor [21,13]	asyn. <sup>±</sup>	✓	✓ <sup>II</sup>	✓	✓	1/2	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	Hash func.	✓
SCRAPE [17]	syn.	✗	✓	✓	✓	1/2	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$	PVSS	✓
SPURT [25]	semi-syn.	✗	✓	✓	✓	1/3	$\mathcal{O}(n^2)$	$\mathcal{O}(n)$	$\mathcal{O}(n)$	PVSS, Pairing	✓
Unicorn [49]	asyn.	✗	✓ <sup>1</sup>	✓	✓	1/2	$\mathcal{O}(1)$	high	$\mathcal{O}(1)$	Sloth	✓

### SoK: Decentralized Randomness Beacon Protocols (2022) by M.

Raikwar and D. Gligoroski

### Practical projects:



[drand.love](https://drand.love)  
[github.com/drand/drand](https://github.com/drand/drand)



[blog.chain.link/vrf-v2-mainnet-launch](https://blog.chain.link/vrf-v2-mainnet-launch)

## #2 Simplest Randomness Beacon using VRFs and VDFs

Each node  $i$  pre-registers VRF public key:  $pk_i$

During an epoch:

- node  $i$  submits  $v_i = \text{VRF\_Eval}(sk_i, \text{epoch\_number})$

At the end of the epoch:

- beacon =  $\text{VDF\_Eval}(v_1 \oplus v_2 \oplus \dots \oplus v_n)$

**Contributes deterministic verifiable randomness:**

- can't compute VRF in two possible ways
- output is pseudorandom

**Long computation (longer than one epoch), fast to verify.**

## #2 Simplest Randomness Beacon using VRFs and VDFs

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During an epoch:

- node  $i$  submits  $v_i = \text{VRF\_Eval}(sk_i, \text{epoch\_number})$

At the end of the epoch:

- $\text{beacon} = \text{VDF\_Eval}(v_1 \oplus v_2 \oplus \dots \oplus v_n)$

**Unique; Unbiasable;**

**Predictable:** schedule of leader is known 1 epoch in advance



# #3 Proposer election : SSLE

## **SSLE:** Single Secret Leader Election

1. Each validator publishes a commitment to a secret value.
2. Next, validators take turn shuffling and rerandomizing the list of commitments.
3. The random beacon is used to do the final open shuffle, and the final list determines the sequence of proposers for the next epoch.
4. Only the proposer knows its position in the list.

Unique

Unbiasable

Unpredictable

Expensive!

- “[Single Secret Leader Election](#)” (2020) by D.Boneh,S.Eskandarian,L.Hanzlik,N.Greco
- Ethereum’s SSLE: [Whisk](#)

# Proposer election approaches

		Unique	Unbiasable	Unpredictable
Proof-of-Work		NO	NO	NO
Proof-of-Stake	Round-robin	YES	NO	NO
	Randomness-beacon	YES	YES	NO
	SSLE	YES	YES	YES

- 1: Long-Range Attacks on PoS
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# Progress in quantum computing

- \* 1998 - 3 qubits
- \* 2000 - 7 qubits
- \* 2005 - 8 qubits
- \* 2006 - 12 qubits
- \* 2011 - 14 qubits
- \* 2017 - 50 qubits (IBM)
- \* 2018 - 72 qubits (Google)
- \* 2019 - 27 qubits IBM Falcon
- \* 2020 - 65 qubits IBM Hummingbird
- \* 2021 - 127 qubits IBM Eagle
- \* 2022 - 433 qubits [IBM Osprey](#)

- 6,146 logical qubits to break RSA-3072 ([Håner-Roetteler-Svore 2017](#))
- 2,330 logical qubits to break discrete log over NIST P-256 curve ([Roetteler-Naehrig-Svore-Lauter 2017](#))

23 Dec 2022

**Factoring integers with sublinear resources on a superconducting quantum processor**

Bao Yan,<sup>1,2,\*</sup> Ziqi Tan,<sup>3,\*</sup> Shijie Wei,<sup>4,\*</sup> Haocong Jiang,<sup>5</sup> Weilong Wang,<sup>1</sup> Hong Wang,<sup>1</sup> Lan Luo,<sup>1</sup> Qianheng Duan,<sup>1</sup> Yiting Liu,<sup>1</sup> Wenhao Shi,<sup>1</sup> Yangyang Fei,<sup>1</sup> Xiangdong Meng,<sup>1</sup> Yu Han,<sup>1</sup> Zheng Shan,<sup>1</sup> Jiachen Chen,<sup>1</sup> Xuhao Zhu,<sup>3</sup> Chuanyu Zhang,<sup>3</sup> Feitong Jin,<sup>3</sup> Hekang Li,<sup>3</sup> Chao Song,<sup>3</sup> Zhen Wang,<sup>3,1</sup> Zhi Ma,<sup>1,†</sup> H. Wang,<sup>3</sup> and Gui-Lu Long<sup>2,4,6,7,§</sup>

<sup>1</sup>State Key Laboratory of Mathematical Engineering  
<sup>2</sup>State Key Laboratory of Low-Dimensional Quantum Physics  
<sup>3</sup>School of Physics, ZJU-Hangzhou Global Scientific and Technological Innovation Center, Zhejiang University  
<sup>4</sup>Beijing Academy of Quantum Information Sciences  
<sup>5</sup>Institute of Information Technology, Jiaxing University  
<sup>6</sup>Beijing National Research Center for Information Science and Technology  
<sup>7</sup>Frontier Science Center for Quantum Information

Shor's algorithm has seriously challenged the widely used RSA-2048. However, to break the widely used RSA-2048 far beyond current technical capabilities, factorization by combining the classical lattice reduction (QAOA). The number of qubits required of the integer  $N$ , making it the most qubit algorithm experimentally by factoring integers factored on a quantum device. We estimate that a depth of thousands is necessary to challenge RSA-2048.

**Peter Shor** @PeterShor1  
Replying to @wayintothedeep  
There are apparently possible problems with this paper.

**Craig Gidney** @CraigGidney · Dec 26, 2022  
arxiv.org/abs/2212.12372 sure doesn't mention the expected number of circuit shots it requires. It's critical to the entire premise of the paper to have a small bound on this number, and as far as I can tell they just don't talk about it. Very bad sign.

2:23 PM · Dec 27, 2022 · 2,981 Views  
6 Retweets 1 Quote Tweet 25 Likes

# The power of a quantum adversary on a blockchain

- Quantum adversary can forge currently used digital signatures = steal funds or fork consensus
  - ECDSA, Schnorr/EdDSA, RSA - breakable by a quantum computer
  - There are secure alternatives
- Solve PoW-puzzles faster:  $D^{1/2}$  instead of  $D$  to search  $D$ -size space
  - Classical miner one thread:  $T$  time to search  $T$  space
  - Quantum miner one thread:  $T$  time to search  $T^2$  space (Grover's search-1996)
  - Superlinearity problem: quantum miners have more advantage [[Park-Spooner-2022](#)]
- Hash functions stay secure
  - Collision: classical algorithm  $O(2^{n/2})$  quantum algorithms  $O(2^{n/3})$  [[Brassard-Hoyer-Tapp-1997](#)]
  - Quantum speed-up is not practical [[Bernstein-2009](#)]
  - Wide-believe is that SHA-256 still provides 128-bits collision resistance even post-quantum

# NIST post-quantum standardization

- 2016 - NIST announced a [competition](#)
- 2022 (summer) - NIST [announced finalists](#)
- 2023 - to open drafts for public comments
- 2024 - to have the first PQC standards

Digital Signatures to be standardized: Crystals-Dilithium, Falcon, Sphincs+

Today

- 3 standard signature schemes: ECDSA, RSA and EdDSA
- BLS is widely used but is not standardized

# Post-quantum signature finalists

Compared at level 3 (AES 192)

	pk	sig	sigs (ms)	Verifies (ms)	Assumption
Dilithium 3	<b>1.9 KB</b>	<b>3.3 KB</b>	AVX: 0.06 0.2	AVX: 0.06 0.2	Lattices
Falcon 1024	<b>1.8 KB</b>	<b>1.3 KB</b>	AVX: 0.7 <b>10</b>	AVX: 0.1 0.1	Lattices
Sphincs+ 192s,f+ SHAKE	48 B	<b>16 KB</b> <b>35 KB</b>	<b>5,000</b> <b>250</b>	<b>4</b> <b>10</b>	Hashes

BLS	<b>96 B</b>	<b>48 B</b>	<b>0.2</b>	<b>0.9</b>	<b>BDH</b>
EdDSA	<b>32 B</b>	<b>64 B</b>	<b>0.02</b>	<b>0.07</b>	<b>DH</b>
ECDSA	<b>32 B</b>	<b>72 B</b>	<b>0.04</b>	<b>0.07</b>	<b>Ideal model</b>
RSA	<b>348 B</b>	<b>348 B</b>	<b>2.5</b>	<b>0.05</b>	<b>RSA</b>

# Post-quantum signature finalists

Compared at level 3 (AES 192)

	pk	sig
Dilithium 3	<b>1.9 KB</b>	<b>3.3 KB</b>
Falcon 1024	<b>1.8 KB</b>	<b>1.3 KB</b>
Sphincs+ 192s,f+ SHAKE	48 B	<b>16 KB</b> <b>35 KB</b>
LMS/XMSS	48 B	<b>1-5 KB</b>

Post-quantum signatures  
to be standardized by 2024

Stateful (few-times) post-quantum signatures  
that are standardized (2020)

BLS	<b>96 B</b>	<b>48 B</b>
EdDSA	<b>32 B</b>	<b>64 B</b>
ECDSA	<b>32 B</b>	<b>72 B</b>

Our current signatures, quantum-breakable



# NIST: new call for digital signatures



Ask: non-lattice-based signatures  
and/or short signatures

**Deadline for submission: June 1, 2023**

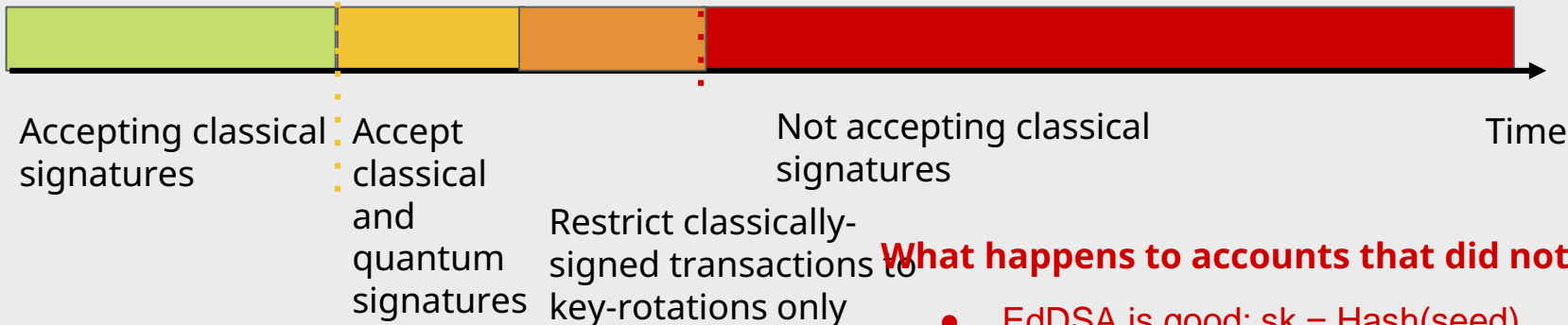
# A happy transition of a blockchain to post-quantum

Could tweak keys:  
 $sk = \text{Hash}(\text{seed})$

Then: prove knowledge of hash-  
preimage with a STARK!

Quantum attacker might exist

Good post-quantum signatures available



**What happens to accounts that did not rotate?**

- EdDSA is good:  $sk = \text{Hash}(\text{seed})$
- Bitcoin is good:  $\text{Address} = \text{Hash}(\text{pk})$

# Schnorr/EdDSA

vs.

# Dilithium

KeyGen():

$sk \leftarrow \text{random integer mod } p$   
 $pk = \mathbf{G} * sk,$   
// G generates a p-order

group

Sign():

$r \leftarrow \text{random mod } p$   
 $R = \mathbf{G} * r$   
 $h = \text{Hash}(R, tx)$   
 $\sigma = (\mathbf{h}, r + \mathbf{h} * sk)$

KeyGen():

$sk \leftarrow \text{random vector}$   
 $e \leftarrow \text{random small vector}$

$pk =$


$$pk = \mathbf{A} \cdot \mathbf{sk} + \mathbf{e}$$

Sign():

$r \leftarrow \text{random vector}$   
 $e \leftarrow \text{random small vector}$

$R =$


$$R = \mathbf{A} \cdot \mathbf{r} + \mathbf{e}'$$

$h = \text{Hash}(R, tx)$   
 $\sigma = (\mathbf{h}, r + \mathbf{h} * sk)$

Dilithium is essentially Schnorr but with matrix A instead of generator G

1: Long-Range Attacks on PoS

2: Proposer Election in PoS

3: Post-quantum blockchains

4: More:

**Threshold Signatures**

# Threshold signatures

- In t-out-of-n threshold signature
  - A single public key, the secret key is split between n nodes.
  - Any t nodes can reconstruct the secret key, or sign through a multi-party protocol.
- Important for wallets: split the key between servers
- Blockchains signatures: ECDSA, Schnorr/EdDSA, BLS
- ECDSA is most widely used on blockchains (NIST standard 1994)
  - Hard to thresholdize
  - [CGGMP-2021]: 4 rounds (= 3 offline + 1 online)
- Schnorr/EdDSA (patent expired 2008)
  - Easy to thresholdize
  - FROST [KG-2020]: 2 rounds (= 1 offline + 1 online)
  - Being [standardized by IETF](#)
- BLS
  - Trivial to thresholdize: 1 round
  - Being [standardized by IETF](#) (expired)

**Need good implementations!**

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Thank you!



## #2 Simplest Randomness Beacon using VRFs and VDFs

Each node  $i$  pre-registers VRF public key:  $pk_i$

In each epoch:

Contributes deterministic verifiable randomness

- node  $i$  submits  $v_i = \text{VRF\_Eval}(sk_i, \text{epoch\_number})$
- $\text{beacon} = \text{VDF\_Eval}(v_1 \oplus v_2 \oplus \dots \oplus v_n)$

VRF = Verifiable Random Function

- Setup  $\rightarrow (sk, pk)$
- $\text{Eval}(sk, x) \rightarrow (y, [\pi])$
- $\text{Verify}(pk, x, y, [\pi]) \rightarrow \{\text{yes/no}\}$
- can't prove different  $y_1 \neq y_2$  same  $x$
- $y$  is indistinguishable from random

Instantiated from unique signatures (BLS or RSA).

VDF = Verifiable Delay Function

- Setup  $\rightarrow (pk)$
- $\text{Eval}(pk, x) \rightarrow (y, [\pi])$
- $\text{Verify}(pk, x, y, [\pi]) \rightarrow \{\text{yes/no}\}$
- long to Eval, fast to Verify

Some VDFs require a trusted setup



# VDFs = Verifiable Delay Functions

VDF = Verifiable Delay Function

- $\text{Setup} \rightarrow (\text{pk})$
- $\text{Eval}(\text{pk}, x) \rightarrow (y, \pi)$
- $\text{Verify}(\text{pk}, x, y, \pi) \rightarrow \{\text{yes/no}\}$

Long to Eval, fast to Verify.

Some VDFs require a trusted setup

- Unbiasable randomness beacons
- Time-release encryption or time-release commitments
- Proof-of-storage

# Dilithium

```
Gen
01  $\mathbf{A} \leftarrow R_q^{k \times \ell}$ 
02  $(\mathbf{s}_1, \mathbf{s}_2) \leftarrow S_\eta^\ell \times S_\eta^k$ 
03  $\mathbf{t} := \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2$ 
04 return  $(pk = (\mathbf{A}, \mathbf{t}), sk = (\mathbf{A}, \mathbf{t}, \mathbf{s}_1, \mathbf{s}_2))$ 

Sign $(sk, M)$ 
05  $\mathbf{z} := \perp$ 
06 while  $\mathbf{z} = \perp$  do
07    $\mathbf{y} \leftarrow S_{\gamma_1 - 1}^\ell$ 
08    $\mathbf{w}_1 := \text{HighBits}(\mathbf{A}\mathbf{y}, 2\gamma_2)$ 
09    $c \in B_\tau := H(M \parallel \mathbf{w}_1)$ 
10    $\mathbf{z} := \mathbf{y} + c\mathbf{s}_1$ 
11   if  $\|\mathbf{z}\|_\infty \geq \gamma_1 - \beta$  or  $\|\text{LowBits}(\mathbf{A}\mathbf{y} - c\mathbf{s}_2, 2\gamma_2)\|_\infty \geq \gamma_2 - \beta$ , then  $\mathbf{z} := \perp$ 
12 return  $\sigma = (\mathbf{z}, c)$ 

Verify $(pk, M, \sigma = (\mathbf{z}, c))$ 
13  $\mathbf{w}'_1 := \text{HighBits}(\mathbf{A}\mathbf{z} - c\mathbf{t}, 2\gamma_2)$ 
14 if return  $\|\mathbf{z}\|_\infty < \gamma_1 - \beta$  and  $[c = H(M \parallel \mathbf{w}'_1)]$ 
```

**Figure 1:** Template for our signature scheme without public key compression.

**Key Generation.** The key generation algorithm generates a  $k \times \ell$  matrix  $\mathbf{A}$  each of whose entries is a polynomial in the ring  $R_q = \mathbb{Z}_q[X]/(X^n + 1)$ . As previously mentioned, we will always have  $q = 2^{23} - 2^{13} + 1$  and  $n = 256$ . Afterwards, the algorithm samples random secret key vectors  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . Each coefficient of these vectors is an element of  $R_q$  with small coefficients of size at most  $\eta$ . Finally, the second part of the public key is computed as  $\mathbf{t} = \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2$ . All algebraic operations in this scheme are assumed to be over the polynomial ring  $R_q$ .

- 1: Long-Range Attacks on PoS
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**Threshold signatures**

VDFs

Merkle/Verkle trees

# Threshold Signatures

## Other options

- **Double-sign:** generate a post-quantum signature alongside the classical signature
- **Onion-key-generation:** generate keys (qpk, qsk) for a stateful hash-based signature (LMS/XMSS), generate classical key:  $sk = \text{Hash}^*(qpk)$ .

# Categories of quantum breakable signatures

1. The quantum-adversary can fully recover the secret key from on-chain information
  - a. ECDSA, BLS, Schnorr
2. The quantum-adversary can forge signatures but can not recover the secret key from on-chain information
  - a. ECDSA, BLS, Schnorr with tweaks or EdDSA
3. The quantum adversary can't forge signatures from on-chain information
  - a. Only new post-quantum secure signatures

# Post-quantum signature finalists

Compared at level 3 (AES 192)

	pk	sig	keygen (ms)	sigs (ms)	Verifies (ms)	assumption
Dilithium 3	<b>1.9 KB</b>	<b>3.3 KB</b>	AVX: 0.07 0.2	AVX: 0.06 0.2	AVX: 0.06 0.2	Lattices
Falcon 1024	<b>1.8 KB</b>	<b>1.3 KB</b>	AVX: <b>25</b> <b>50</b>	AVX: 0.7 <b>10</b>	AVX: 0.1 0.1	Lattices
Sphincs+ 192s,f+ SHAKE	48 B	<b>16 KB</b> <b>35 KB</b>	<b>500</b> <b>8.3</b>	<b>5,000</b> <b>250</b>	<b>4</b> <b>10</b>	Hashes

BLS	<b>96 B</b>	<b>48 B</b>		<b>0.2</b>	<b>0.9</b>
EdDSA	<b>32 B</b>	<b>64 B</b>	<b>0.003</b>	<b>0.02</b>	<b>0.07</b>
ECDSA	<b>32 B</b>	<b>72 B</b>	-	<b>0.04</b>	<b>0.07</b>
RSA	<b>348 B</b>	<b>348 B</b>	-	<b>2.5</b>	<b>0.05</b>

# More post-quantum crypto-primitives

- Threshold signatures
- Zero-knowledge proofs
  - STARKs



# Leader election approaches: RANDAO

## RANDAO (Ethereum 2.0 approach):

Leaders of the previous epoch contribute verifiable deterministic randomness (BLS signing epoch number) to the next epoch.

Leader schedule gets known 1 epoch in advance.

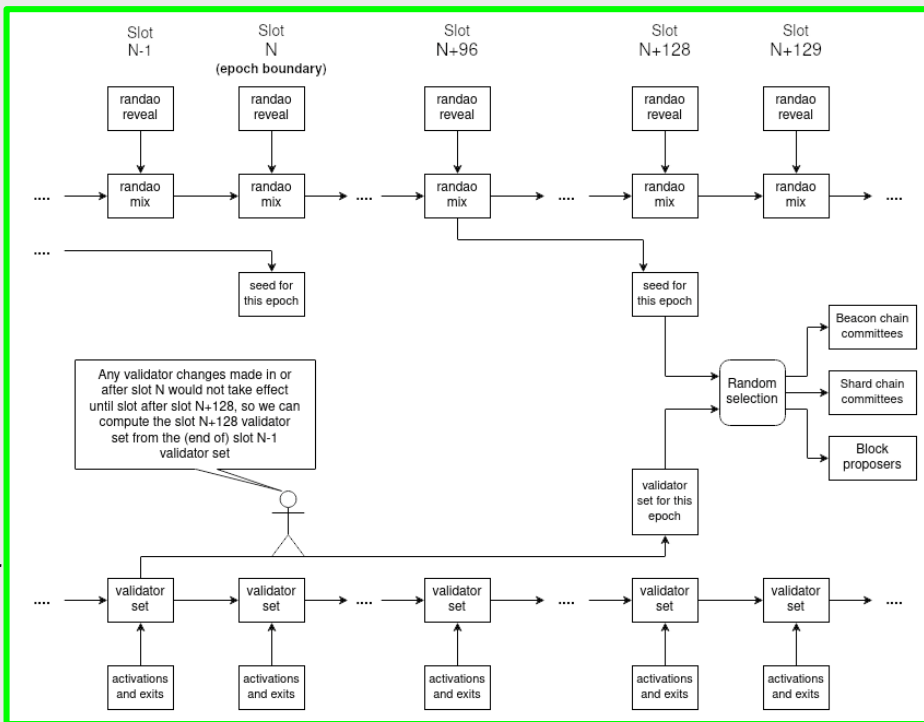
**Public:** a leader learns they are a leader at the same time

the public learns.

**Fair:** all nodes are given equal chances to be leaders (random sampling).

**Unbiasable:** last leader has only 1 bit of bias by either revealing/withholding block

**Unpredictable:** the leader for the slot is known only 1 epoch in advance.



Good for leaders that take longer than an epoch (6.4 min) to DDoS

# Leader election approaches: unbiased and unpredictable

	Assumption	Setup	Single round	Self-cerifying
Albatross, HydRand, RandHerd	PVSS	Free	No	No
RandRunner, RANDAO++, cVDF, Veedo	VDF	Yes/No	Yes	Yes
Dfinity, drand	Threshold-signatures	DKG	Yes	Yes

- **Public:** a leader learns they are a leader at the same time the public learns.
- **Fair:** all nodes are given equal chances to be leaders.
- **Unbiasable:** no node can increase its probability of being selected.
- **Unpredictable**



- Random lotteries
- Randomness API for smart contracts
- Bootstrapping asynchronous consensus\*

Good for leaders that are hard to DDoS within the time it takes them to create and broadcast a proposal (< 10 sec))!

# VDF-based leader election: biasable $\rightarrow$ unbiased

VDF = Verifiable Delay Function

- Setup  $\rightarrow$  (pk)
- Eval(pk, x)  $\rightarrow$  (y,  $\pi$ )
- Verify(pk, x, y,  $\pi$ )  $\rightarrow$  {yes/no}

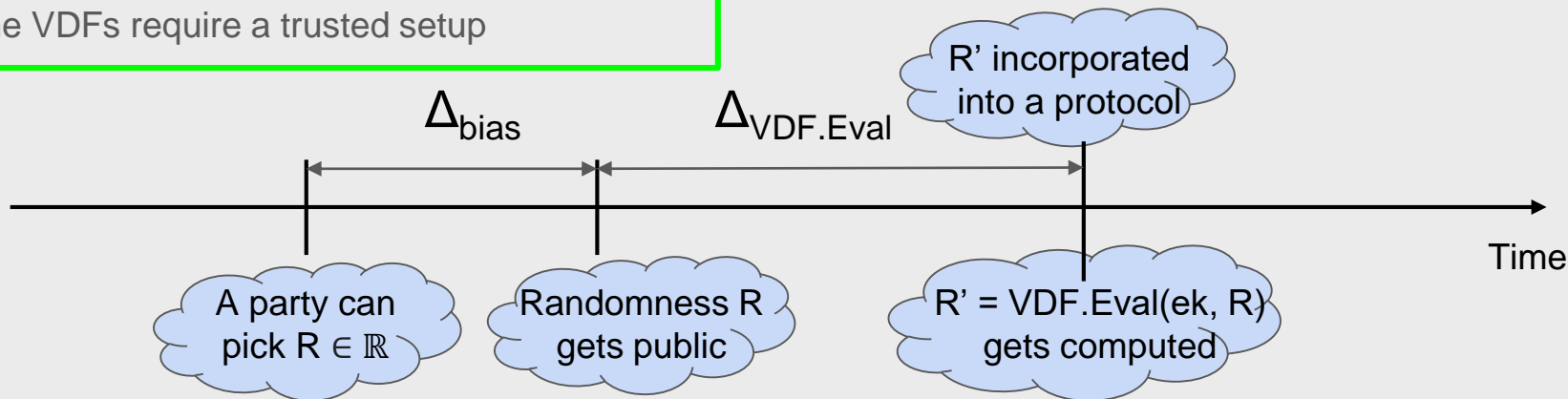
Long to Eval, fast to Verify.

Some VDFs require a trusted setup

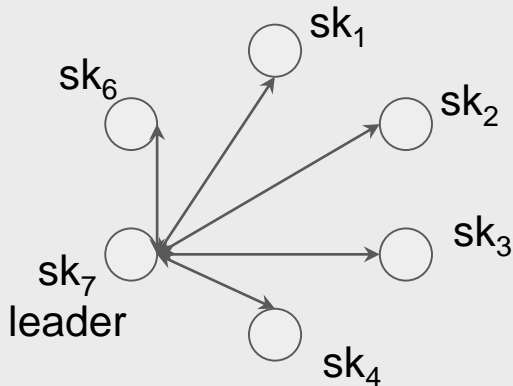
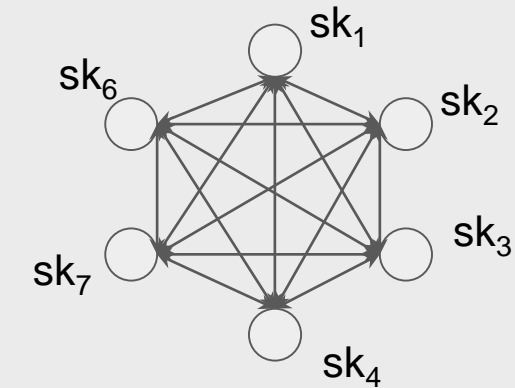
Passing randomness through a VDF makes it unbiased!

$R'$  is unbiased as long as

$$\Delta_{\text{bias}} < \Delta_{\text{VDF.Eval}}$$



# Threshold-Signature-based leader election



- Setup: collectively generate  $sk$  (DKG)
  - $sk$  can be reconstructed from any  $\frac{2}{3}n$  subset of  $\{sk_1, sk_2, \dots, sk_n\}$
- Parties collectively-sign a slot number by submitting signature shares
$$\sigma_i = \text{Sign}(sk_i, \text{msg} = \text{slot-number})$$
- Full signature  $\sigma$  is reconstructed: from  $\frac{2}{3}n$  subset of  $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$
- **Randomness is generated as  $\text{Hash}(\sigma)$**
- Communication can be pipelined through a leader

Only works with unique signatures e.g. BLS

# Private leader election : VRF-based

VRF = Verifiable Random Function

- $\text{Setup} \rightarrow (\text{sk}, \text{pk})$
- $\text{Eval}(\text{sk}, x) \rightarrow (y, \pi)$
- $\text{Verify}(\text{pk}, x, y, \pi) \rightarrow \{\text{yes/no}\}$

$y$  is indistinguishable from random

Instantiated from unique signatures (BLS or RSA).

- Each leader computes  $\text{VRF.Eval}(\text{sk}, \text{\#slot}) \rightarrow (y, \pi)$ , if  $y < \text{threshold}$ , the leader is elected
- The leader creates a block and broadcasts it together with  $\pi$ 
  - No DDoS window!
- **Problem**: multiple leaders can get elected or none!
  - Consensus protocol needs to handle that (e.g. Algorand)