Encryption and Message Authentication

Bar-Ilan Winter School[1]

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[1] These slides are partially based on Benny Chor's slides.

And Finally, Let's Talk Business

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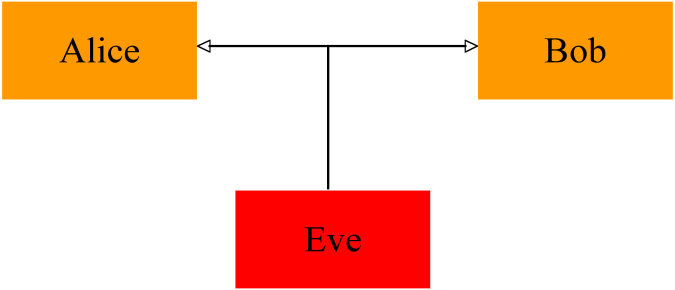
Encryption

Basic Setting



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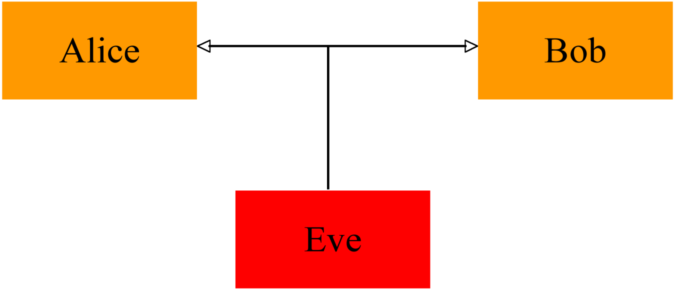

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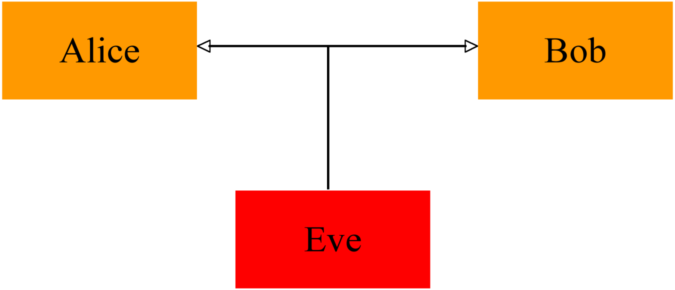

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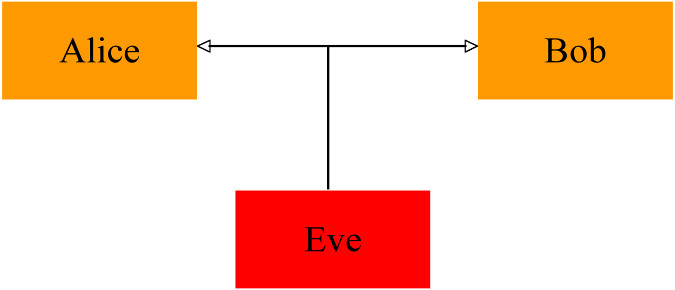

1. Eve listens to the communication.
2. Alice and Bob share a secret random key $k \stackrel{R}{\leftarrow} \{0,1\}^n$.
3. Goal: Alice would like to send Bob a message m confidentially.

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Meta question:

- Can we formalize secrecy mathematically?

Encryption Syntax

Definition

A symmetric encryption scheme consists of:

- Encryption Algorithm: E maps a key $k \in \{0,1\}^*$ and a plaintext $m \in \{0,1\}^*$ into a ciphertext $E_k(m)$.

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Note: Both algorithms are efficient and may be randomized.
So far, no requirement of secrecy.

Security as Indistinguishability

An encryption of m_0 and an encryption of m_1 should "look the same".

Perfect Secrecy (Shannon '49)

For any pair of different messages m_0 and m_1 of equal length: The ciphertexts c_0 and c_1 should be identically distributed.

Experiment 0

Let $k \overset{R}{\leftarrow} \{0,1\}^n$
Output $c_0 = E_k(m_0)$

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- Example: one-time pad ($E_k(m) = k \bigoplus m$) is perfectly secret.
- Unfortunately, perfect secrecy requires long key $|m| = |k|$ (Ex: prove it!)

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The ciphertexts c_0 and c_1 should be indistinguishable for computationally-bounded adversary.

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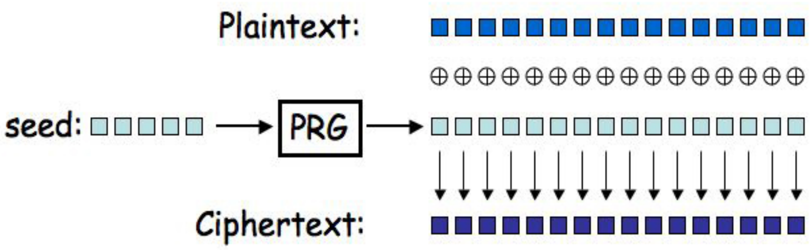
Great ! computational secrecy is a strong notion.
Is it feasible (with a short key)?

Computational analog of "one-time pad"

- Choose a secret random short key k ("seed")
- Expand the seed into a long keying stream $G(k)$
- Encrypt m by $c = G(k) \bigoplus m$
- Decrypt c to $m = c \bigoplus G(k)$.



Pseudorandom Generators (Reminder)

A pseudorandom generator is a polynomial time computable function $G : \{0,1\}^n \mapsto \{0,1\}^\ell$, $\ell \gg n$, which satisfies:

Pseudorandom		Random
Choose $k \xleftarrow{R} \{0,1\}^n$ Output $y_0 = G(k)$	$\stackrel{c}{\equiv}$	Choose $y_1 \xleftarrow{R} \{0,1\}^\ell$ Output y_1

The output of G is computationally indistinguishable from truly random strings of length ℓ.

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Assume that PRG : $\{0,1\}^n \to \{0,1\}^\ell$ *is pseudorandom.*
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- What went wrong?

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- In fact, we would like to grant the adversary the extra power of Chosen Plaintext Attack
- Before that, let us reconsider our original definition

Reminder: Ciphertext Indistinguishability

For any pair of messages m_0 and m_1 of equal length:

Experiment 0

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Ciphertext Indistinguishability: Alternative Formulation

Challenger

$k \xleftarrow{R} \{0,1\}^n$

$b \xleftarrow{R} \{0,1\}$

$\leftarrow (m_0, m_1)$

$E_k(m_b) \rightarrow$

Adversary $\mathcal{A}(1^n)$

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- Exercise: Prove equivalence to the original one.

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- The adversary cannot do much better !

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- Example: Servers may communicate via encryption but (dishonest) users can control the actual requests that are being transferred

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- Example: Servers may communicate via encryption but (dishonest) users can control the actual requests that are being transferred
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Why do we need such a strong definition?

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- Ex: Try to formalize it and prove that it's indeed strictly weaker than CPA and strictly stronger than CI for a single message.

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Sol1: Randomized encryption

Sol2: Stateful encryption

Encrypting via Ideal Cipher

- Suppose that Alice and Bob share a truly random function

$$R : \{0,1\}^n \to \{0,1\}^n.$$

  - For each input $x \in \{0,1\}^n$ choose $R(x) \xleftarrow{R} \{0,1\}^n$.

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- How to achieve CPA security? Randomize the message !

CPA Security from Random Permutation

(Inefficient) Construction

Encrypt m: choose $r \xleftarrow{R} \{0,1\}^n$ and output $(r, F(r \bigoplus m))$
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Overall, the winning probability is upper-bounded by
$\Pr[\text{win}|G]\Pr[G] + \Pr[\bar{G}] \leq \frac{1}{2} + \mathrm{neg}(n)$. □

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Given x output $y = F_k(x)$

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Choose random function

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PPT Adversary can't distinguish with more than negligible probability.

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$$\Pr_k[\mathcal{B}^{F_k} = 1] - \Pr[\mathcal{B}^{Rand} = 1] \geq (\tfrac{1}{2} + \epsilon) - (\tfrac{1}{2} + \mathrm{neg}(n)) \geq \epsilon - \mathrm{neg}(n).$$

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Exercise prove:

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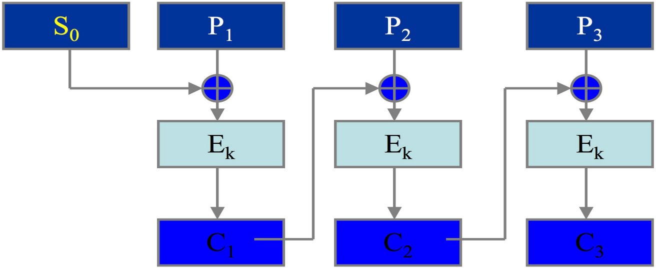
- Is there a better solution?

CBC Mode Encryption



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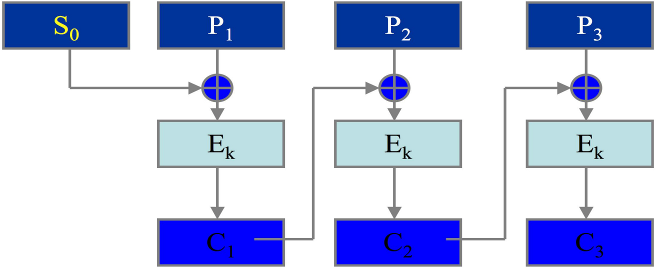

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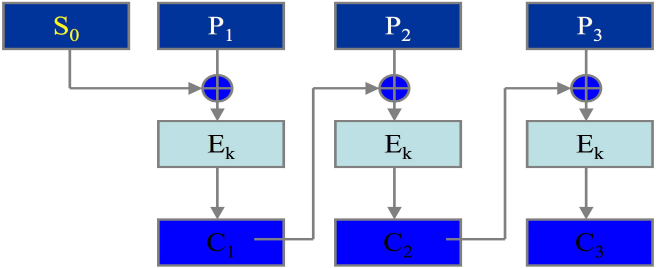

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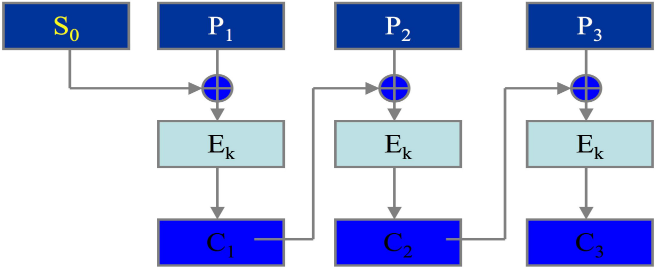

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- New message requires a freshly chosen random seed (why?)

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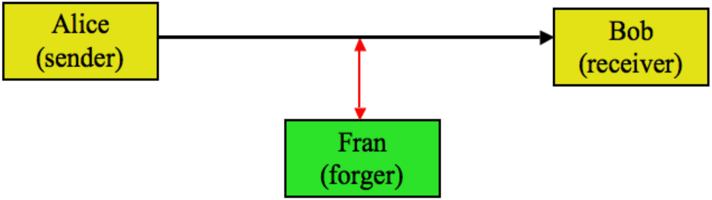
Security: It can be proved that if E is a pseudorandom permutation, then CBC is resistant to chosen plaintext attacks (CPA-secure).

Message Authentication Codes

Authentication – Goal

Ensure integrity of messages against an active adversary

- Adversary hears previous genuine messages



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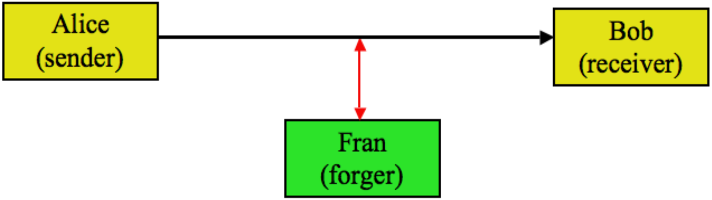

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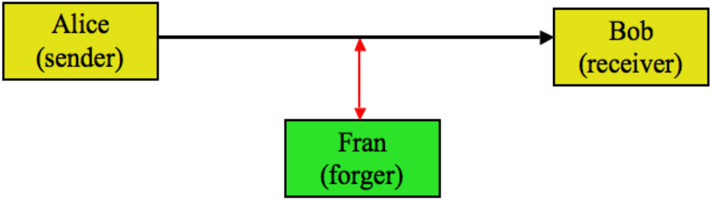

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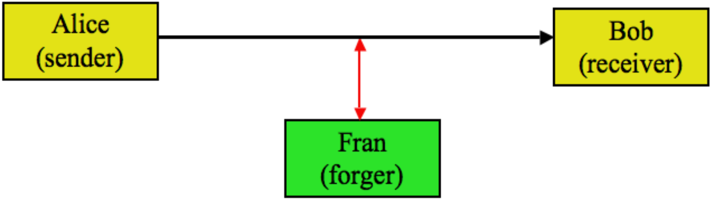

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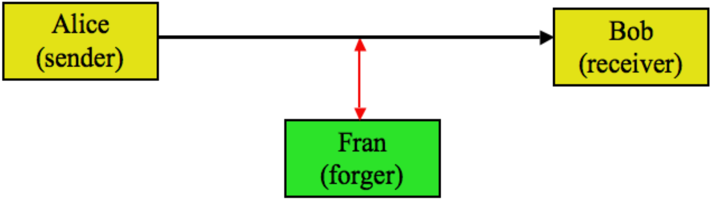

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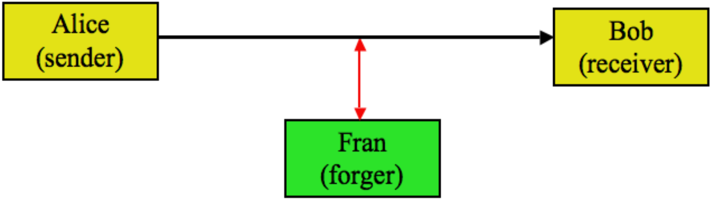

Important Remark: Authentication is orthogonal to secrecy. Secrecy alone usually does not guarantee integrity.

Sol: Message Authentication Code (MAC)

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Security: Intuitively, should be hard to forge a valid tag even after seeing many legal tags

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- The adversary succeeds even if the message being forged is "meaningless". The reason is that it is hard to predict what has and what does not have a meaning in an unknown context, and how will Bob, the receiver, react to such successful forgery.

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- Conclusion: key and tag should not be too short

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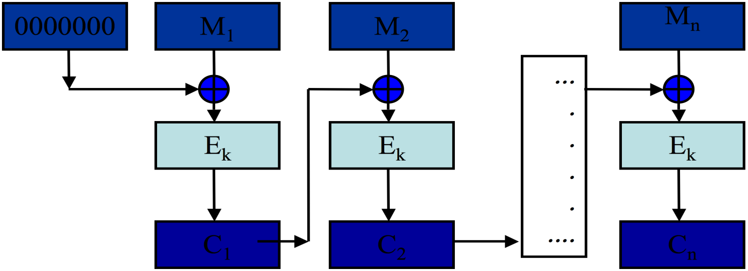
Problem: Impractical due to large communication overhead!

MACs for Long Messages

We will describe an efficient approach based on CBC Mode,
there is an alternative solution (HMAC) based on cryptographic hash functions.

CBC Mode MACs

- Start with the all zero seed.



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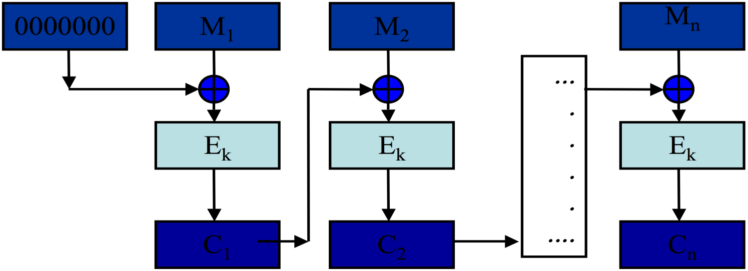

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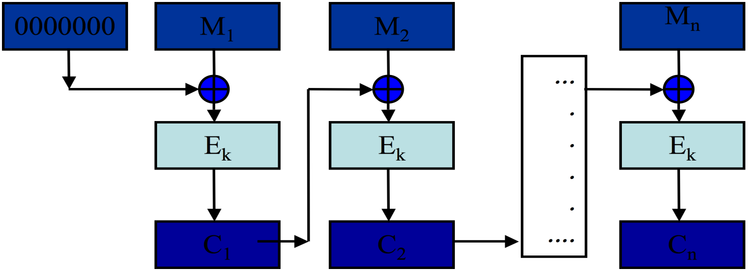

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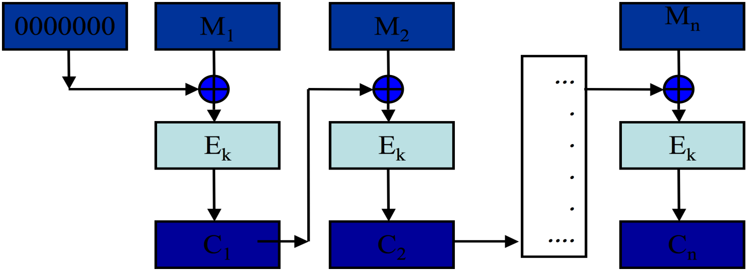

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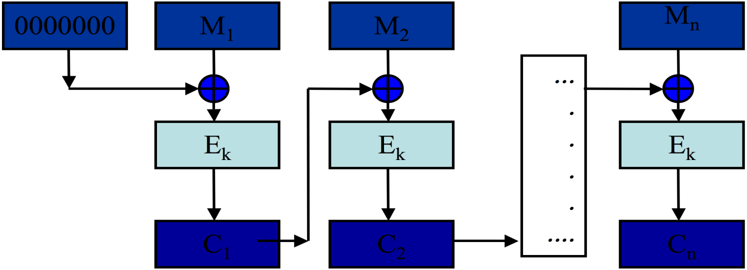

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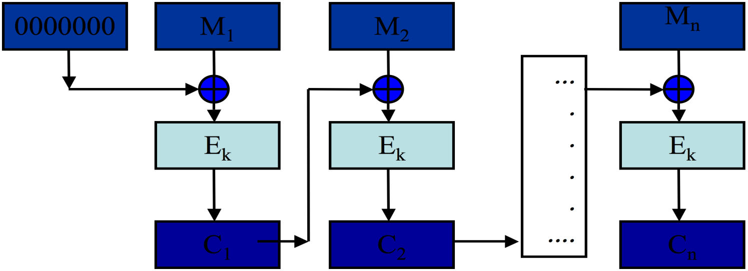

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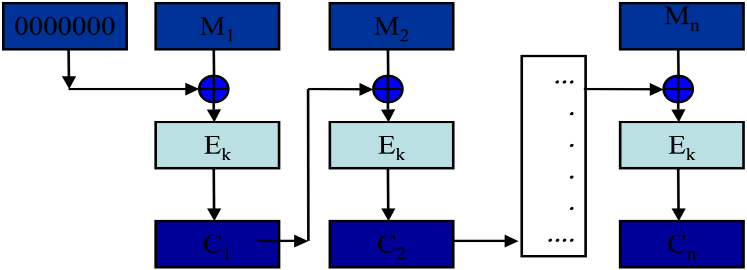

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Q: Can we replace the all-zero seed with a random public string?

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- Theorem: If E_k is a pseudorandom function, then the fixed length CBC MAC is resilient to forgery when authenticating messages of the same length, n.

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- Warning: Construction is not secure if messages are of varying lengths, namely number of blocks varies among messages.

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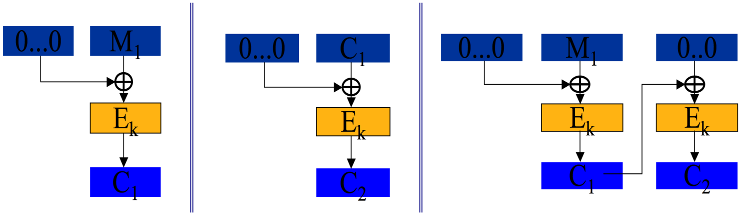

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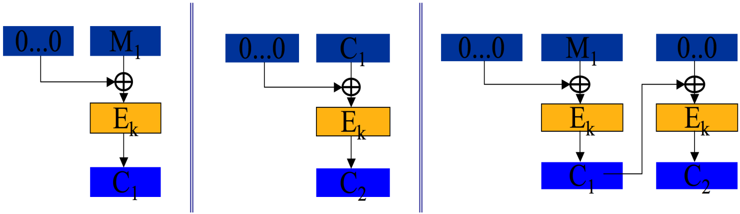

- One can efficiently design, for every n, two messages, one with 1 block, the other with $n+1$ blocks, that have the same $\mathrm{MAC}_k(\cdot)$.

CBC-MAC for Variable Length Messages

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  Apply CBC-MAC to $(n, M_1, \ldots, M_n)$.
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- Solution 3: Encrypted CBC (ECBC MAC):
  Compute $E_{k_2}(CBC - MAC_{k_1}(M_1, \ldots, M_n))$,
  where E is a block-cipher and k_2 is another secret key.
  Essentially the same overhead as CBC-MAC (widely used).

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CPA security is not always enough!
(Some real world attacks follow a similar scenario)

Reminder: Security under Chosen Plaintext Attack (CPA)

Challenger		Adversary $\mathcal{A}$
$k \xleftarrow{R} \{0,1\}^n$	$\leftarrow x_1$	
	$E_k(x_1) \rightarrow$	
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- Decryption queries can be also asked after the challenge as long as $y \neq c^*$.

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- With no useful queries, the decryption oracle isn't really being used
- We can break E via CPA.

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- Authentication is orthogonal to secrecy – combination is tricky.
- MACs and Encryption schemes can be based on PRFs/PRPs via highly efficient (practical) transformations.
- Good design methodology: Reduce a complicated task to a simpler task. Solve the simple task and extend the solution. (E.g., design encryption for a single-block messages and then show how to extend it to longer messages).