

SNARKs with Preprocessing

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G: Someone generates and publishes a common reference string



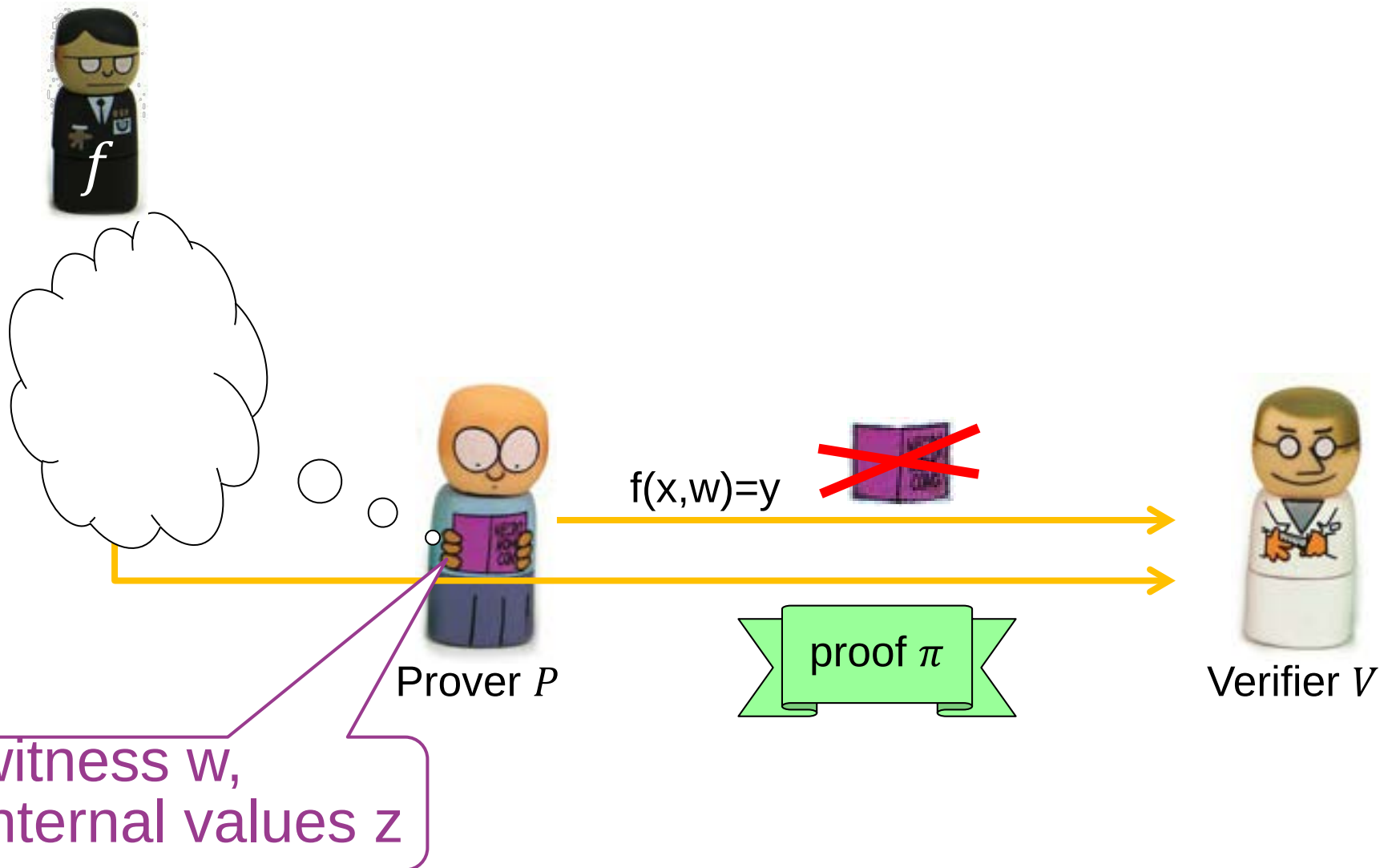
P: Prover picks NP statement “exists w such that $f(x,w)=y$ ” and sends x,y , and a succinct proof



V:
Verifier efficiently checks proof and is convinced that prover knows a witness w .



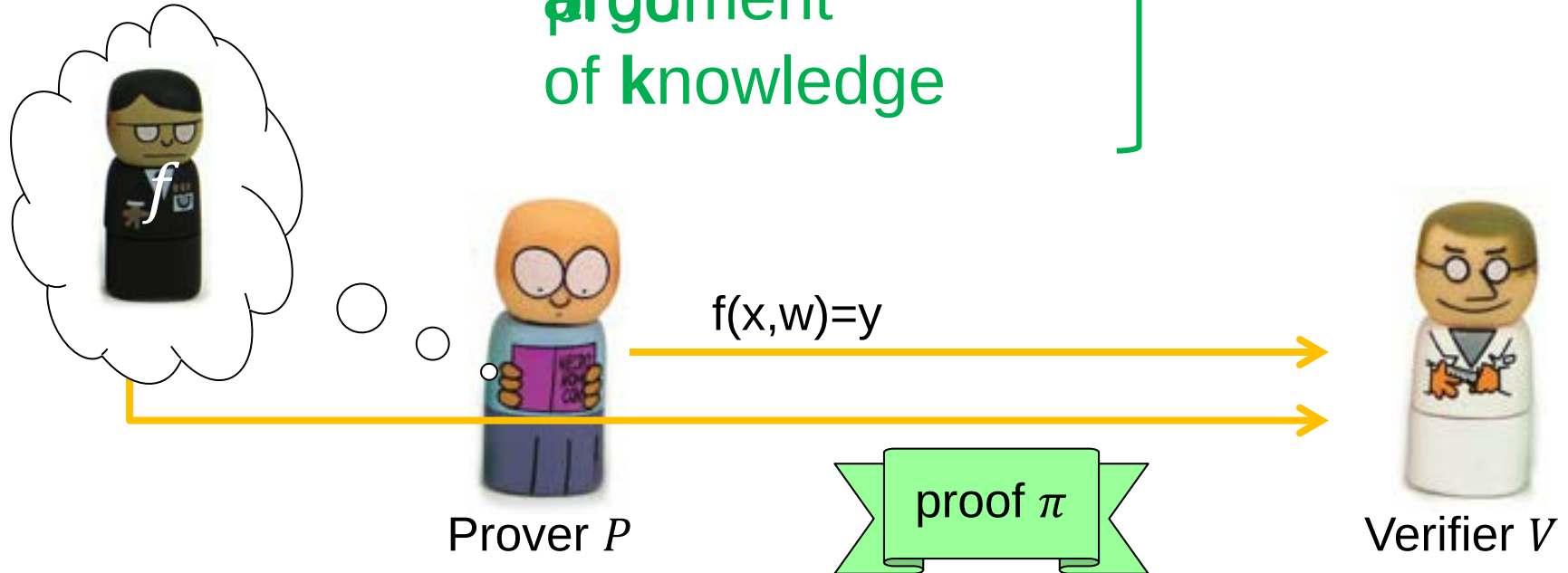
zkSNARK for NP: setting



zkSNARK for NP: setting

zero knowledge
succinct (V and π)
noninteractive
argument
of knowledge

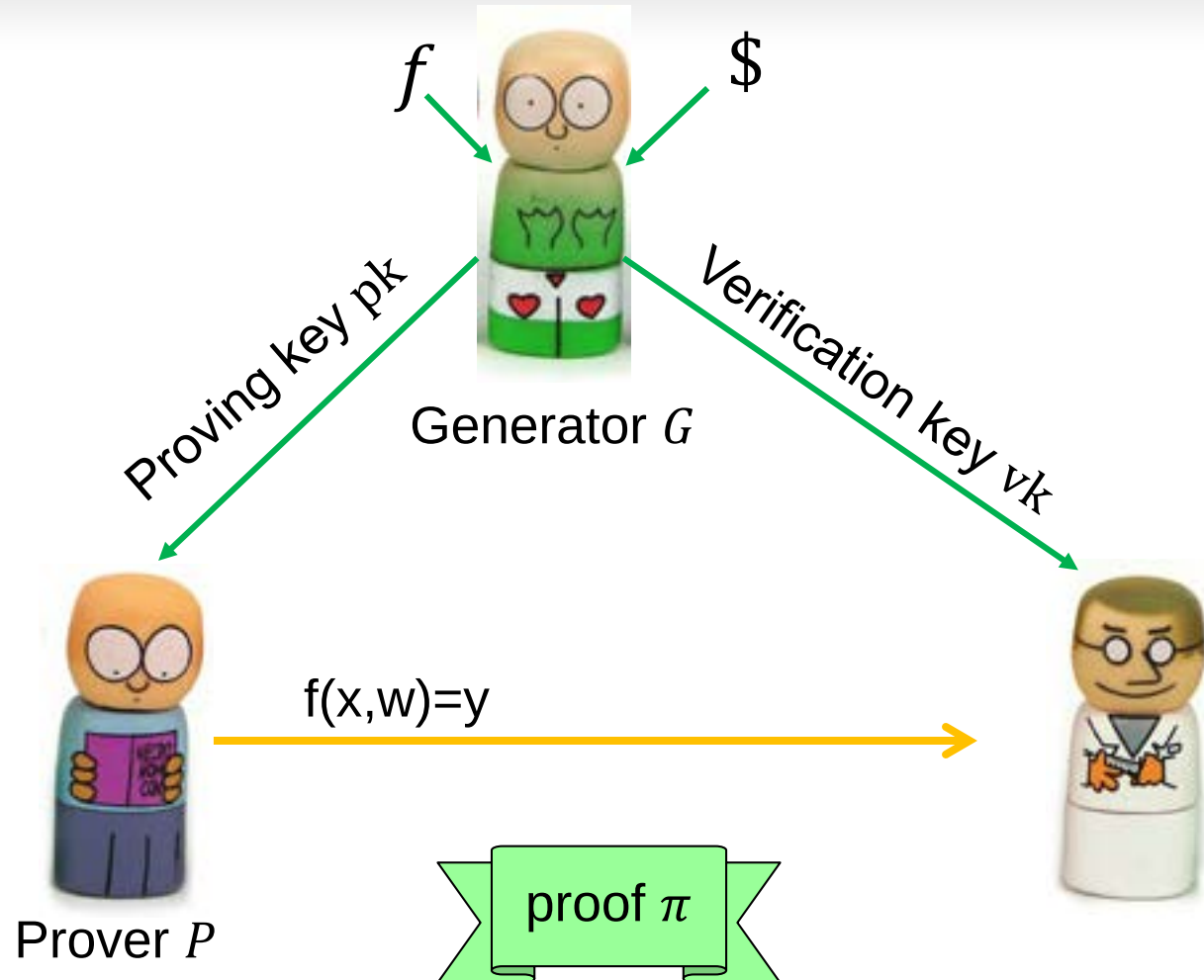
zkSNARK



Preprocessing zkSNARK for NP: setting

Variants:

- Dependence on f
- Cheap / expensive
- Secret / public randomness
- Publicly-verifiable / designated-verifier

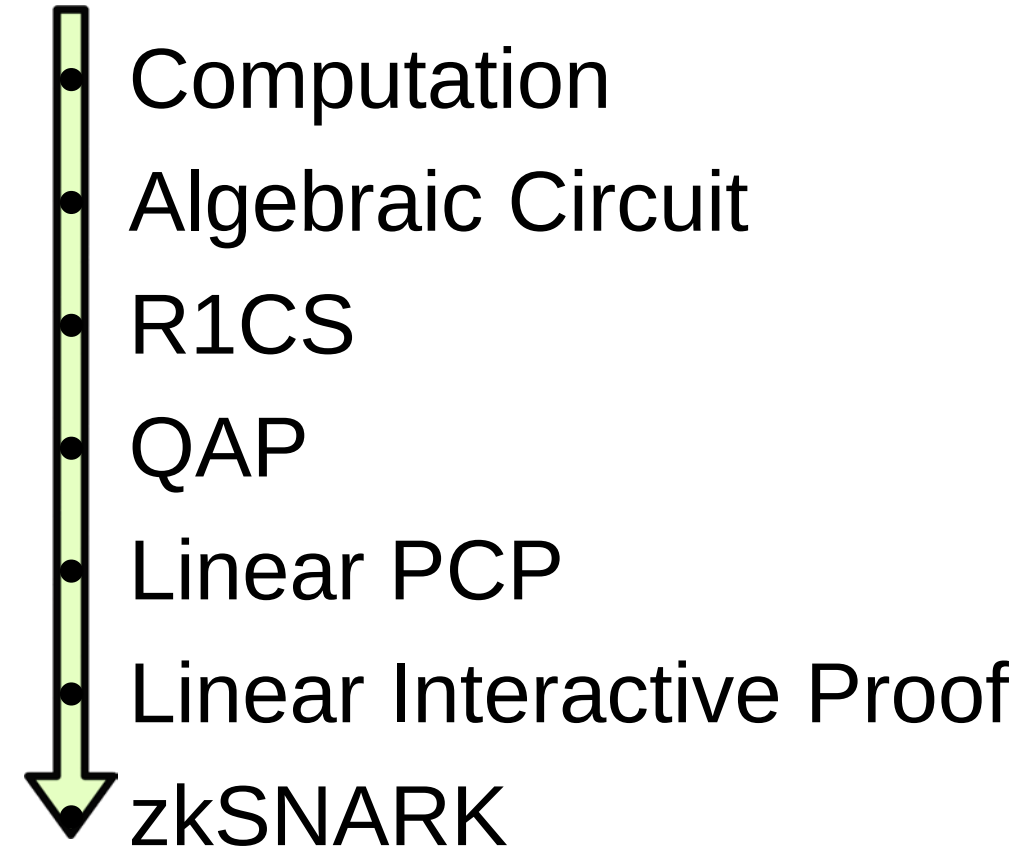


Preprocessing SNARKs for NP

	Proof size (field elements)	CRS size	Prover runtime
[Groth10]	42	$O(s^2)$	$O(s^2)$
[Lipmaa12]	39	$\tilde{O}(s)$	$O(s^2)$
QAP-based [GGPR13] Reinterpreted as linear PCPs: [BCIOP13] [SBVBBW13] Improvements: [PGHR13] [BCTV14usenix] [BBFR15] [CFHKKNPX15] ...	7—8	$O(s)$	$\tilde{O}(s)$
[DFGK14]	4	$O(s)$	$\tilde{O}(s)$

Preprocessing is private-coin and costs $\tilde{O}(s)$

zkSNARK construction via QAP and Linear PCPs



Computation $\Rightarrow$ Arithmetic Circuit

Efficient computation $f(\cdot)$.

- Deterministic $f(x) \rightarrow y$
- Nondeterministic: $\exists w: f(x, w) \rightarrow y$

completeness

soundness,
PoK

Arithmetic circuit $C(\cdot, \cdot)$ over $\mathbb{F}$.

$\exists z: C(x, y)$ accepts with internal values $z \in \mathbb{F}^n$

Arithmetic Circuit $\Rightarrow$ R1CS (Rank-1 Quadratic System)

[GGPR13]

Arithmetic circuit $C(\cdot, \cdot)$ over $\mathbb{F}$.

$\exists z: C(x, y)$ accepts with internal values $z \in \mathbb{F}^n$

completeness

soundness,
PoK

R1CS $(a_j, b_j, c_j)_{j=1}^m$ vectors in $\mathbb{F}^k$.

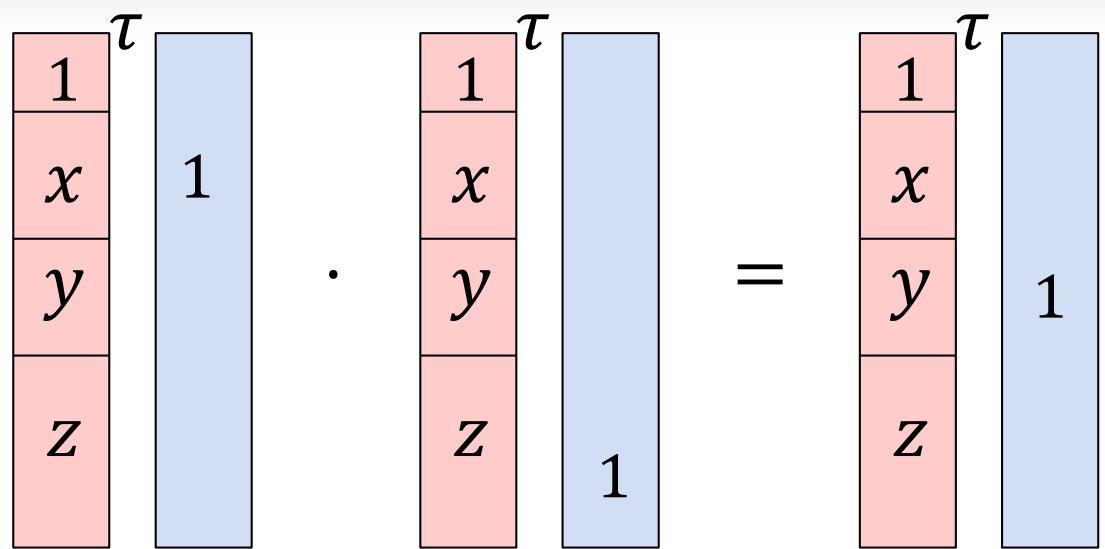
$\exists z \in \mathbb{F}^n:$

$\forall j \in \{1, \dots, m\}:$

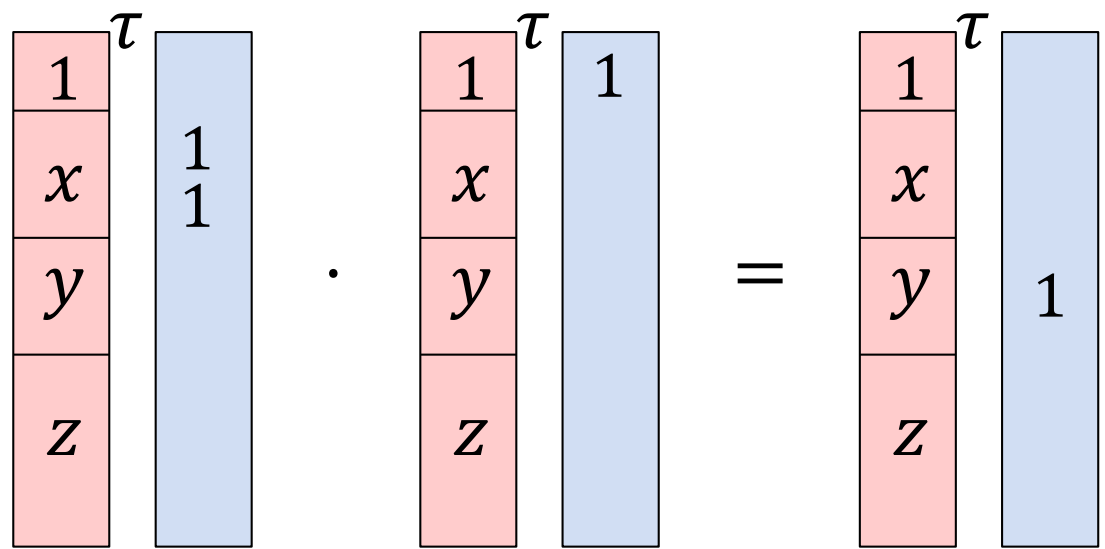
$$\begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array} \begin{array}{|c|} \hline \tau \\ \hline a_j \\ \hline \end{array} \cdot \begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array} \begin{array}{|c|} \hline \tau \\ \hline b_j \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array} \begin{array}{|c|} \hline \tau \\ \hline c_j \\ \hline \end{array}$$

Expressing gates as constraints:

Multiplication gate in $\mathcal{C}$
converted into a
constraint:



Addition gate in $\mathcal{C}$
converted into a
constraint:



Generally, any
bilinear gate.

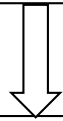
R1CS (Rank-1 Quadratic Constraint System) $\Rightarrow$ QAP (Quadratic Arithmetic Program) [GGPR13]

R1CS $(a_j, b_j, c_j)_{j=1}^m$ vectors in $\mathbb{F}^k$.

$\exists z \in \mathbb{F}^n$:

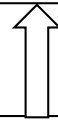
$\forall j \in \{1, \dots, m\}$:

$$\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} a_j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} b_j \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} c_j \end{bmatrix}$$



Matrices (A, B, C) in $\mathbb{F}^{k \times m}$. $\exists z \in \mathbb{F}^n$:

$$\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} A \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} B \end{bmatrix} - \begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}^T \begin{bmatrix} C \end{bmatrix} = \boxed{0^m}$$

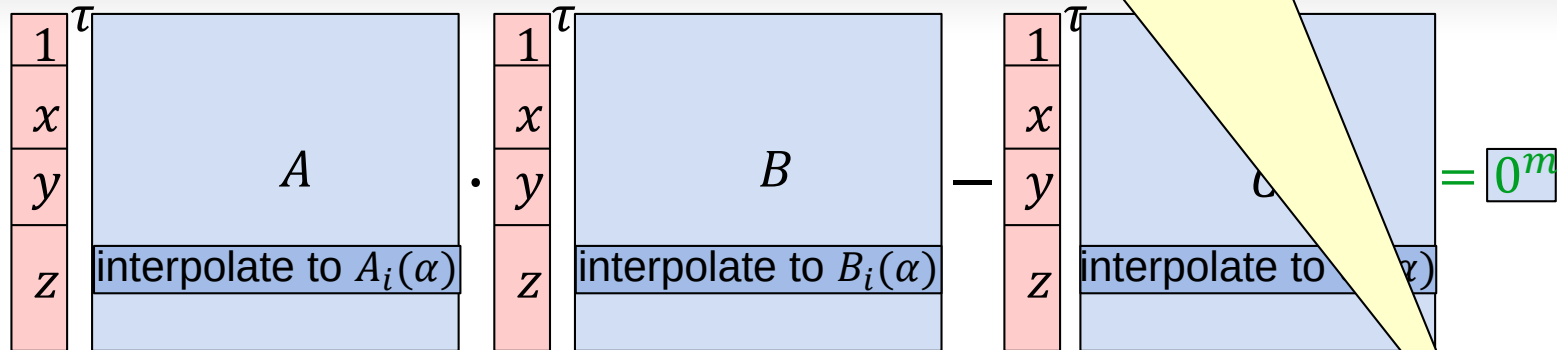


R1CS $\Rightarrow$ QAP (cont.)

Intuition: $V(\alpha)$ generates the ideal of all polynomials that vanish on S .

Matrices
(A, B, C)
in $\mathbb{F}^{k \times m}$.

$\exists z \in \mathbb{F}^m$:



Fix $S = \{\alpha_1, \dots, \alpha_m\} \subset \mathbb{F}$. For $i = 1, \dots, k$ and $j = 1, \dots, m$:

Let $A_i(\alpha)$ be the degree- $(m-1)$ polynomial such that $A_i(\alpha_j) = A_{i,j}$.

Likewise $B_i(\alpha), C_i(\alpha)$. Let $V(\alpha) = \prod_{j=1}^m (\alpha - \alpha_j)$, vanishing on S .

QAP: $(A_i(\alpha), B_i(\alpha), C_i(\alpha))_{i=1}^k$ and V polynomials in $\mathbb{F}[\alpha]$.

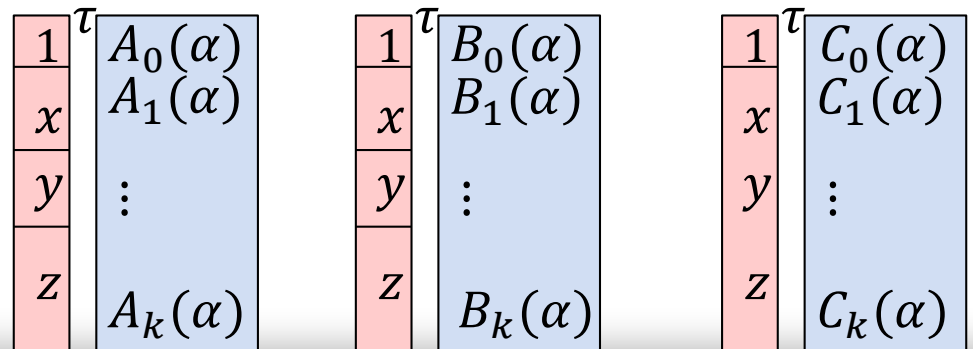
Let $P_{x,y,z}(\alpha) = A_{x,y,z}(\alpha) \cdot B_{x,y,z}(\alpha) - C_{x,y,z}(\alpha)$

$\exists z \in \mathbb{F}^n$:

$V(\alpha)$ divides $P_{x,y,z}(\alpha)$

i.e.,

$\exists H(\alpha): P_{x,y,z}(\alpha) = H(\alpha)V(\alpha)$



QAP $\Rightarrow$ Linear PCP

[GGPR13] [BCIOP13] [BCGTV13]

QAP: $(A_i(\alpha), B_i(\alpha), C_i(\alpha))_{i=1}^k$ and V polynomials in $\mathbb{F}[\alpha]$.

$$\text{Let } P_{x,y,z}(\alpha) = A_{x,y,z}(\alpha) \cdot B_{x,y,z}(\alpha) - C_{x,y,z}(\alpha)$$

$\exists z \in \mathbb{F}^n$:

$\exists H(\alpha)$:

$$P_{x,y,z}(\alpha) = H(\alpha)V(\alpha)$$

$$\begin{array}{c|c} \begin{array}{c} 1 \\ x \\ y \\ z \end{array} & \begin{array}{c} A_0(\alpha) \\ A_1(\alpha) \\ \vdots \\ A_k(\alpha) \end{array} \end{array}^{\tau}$$

$$\begin{array}{c|c} \begin{array}{c} 1 \\ x \\ y \\ z \end{array} & \begin{array}{c} B_0(\alpha) \\ B_1(\alpha) \\ \vdots \\ B_k(\alpha) \end{array} \end{array}^{\tau}$$

$$\begin{array}{c|c} \begin{array}{c} 1 \\ x \\ y \\ z \end{array} & \begin{array}{c} C_0(\alpha) \\ C_1(\alpha) \\ \vdots \\ C_k(\alpha) \end{array} \end{array}^{\tau}$$

Probabilistic check:

Draw $\tau \leftarrow_R \mathbb{F}$ and check $P_{x,y,z}(\tau) \stackrel{?}{=} H(\tau) \cdot V(\tau)$.

- Soundness: polynomial identity testing with degree $< 2m \ll |\mathbb{F}|$

Let $\pi = (1, x, y, z, h)$ where h is the coefficient vector of H .

- Then this check can be done by 4 linear queries to π , retrieving the monomials.

(Plus a 5th for checking x, y via random linear combination.)

- Any $\tilde{\pi}$ still commits to some low-degree $\tilde{H}(\tau) \widetilde{P_{x,y,z}}(\tau)$.

QAP $\Rightarrow$ Linear PCP: the algorithms

$$\text{Let } P_{x,y,z}(\alpha) = A_{x,y,z}(\alpha) \cdot B_{x,y,z}(\alpha) - C_{x,y,z}(\alpha)$$

$\exists H(\alpha):$

$$P_{x,y,z}(\alpha) = H(\alpha)V(\alpha)$$

$$\begin{array}{c} 1 \\ x \\ y \\ z \end{array}^{\tau} \begin{array}{c} A_0(\alpha) \\ A_1(\alpha) \\ \vdots \\ A_k(\alpha) \end{array} \cdot \begin{array}{c} 1 \\ x \\ y \\ z \end{array}^{\tau} \begin{array}{c} B_0(\alpha) \\ B_1(\alpha) \\ \vdots \\ B_k(\alpha) \end{array} - \begin{array}{c} 1 \\ x \\ y \\ z \end{array}^{\tau} \begin{array}{c} C_0(\alpha) \\ C_1(\alpha) \\ \vdots \\ C_k(\alpha) \end{array}$$

- Prover: compute H and its coefficient vector h ;
Output $\pi = (1, x, y, z, h)$ where h is the coefficient vector of H .
Complexity: Dominated by computing the m coefficients of H . With suitable FFT: $\sim m \log m + (\#\text{"nonzero entries in " } A, B, C) \text{ field operations.}$
- Query: Verify: draw $\tau \leftarrow_R \mathbb{F}$, make linear queries to π according to τ .
Complexity: $\sim 4m + 2(\#\text{"nonzero entries in " } A, B, C) \text{ field operations.}$
- Decision: check a simple quadratic equation in the answers.

Later: important for public verifiability (will use of pairings).

QAP $\Rightarrow$ Linear PCP: adding ZK

Let $P_{x,y,z}(\alpha) = A_{x,y,z}(\alpha) \cdot B_{x,y,z}(\alpha) - C_{x,y,z}(\alpha)$

$$\begin{array}{c} \begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array}^T \begin{array}{|c|} \hline A_0(\alpha) \\ \hline A_1(\alpha) \\ \hline \vdots \\ \hline A_k(\alpha) \\ \hline \end{array} \\ +\delta_1 V(\alpha) \end{array} \cdot \begin{array}{c} \begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array}^T \begin{array}{|c|} \hline B_0(\alpha) \\ \hline B_1(\alpha) \\ \hline \vdots \\ \hline B_k(\alpha) \\ \hline \end{array} \\ +\delta_2 V(\alpha) \end{array} - \begin{array}{c} \begin{array}{|c|} \hline 1 \\ \hline x \\ \hline y \\ \hline z \\ \hline \end{array}^T \begin{array}{|c|} \hline C_0(\alpha) \\ \hline C_1(\alpha) \\ \hline \vdots \\ \hline C_k(\alpha) \\ \hline \end{array} \\ +\delta_3 V(\alpha) \end{array}$$

$\delta_1, \delta_2, \delta_3 \leftarrow_R \mathbb{F}$

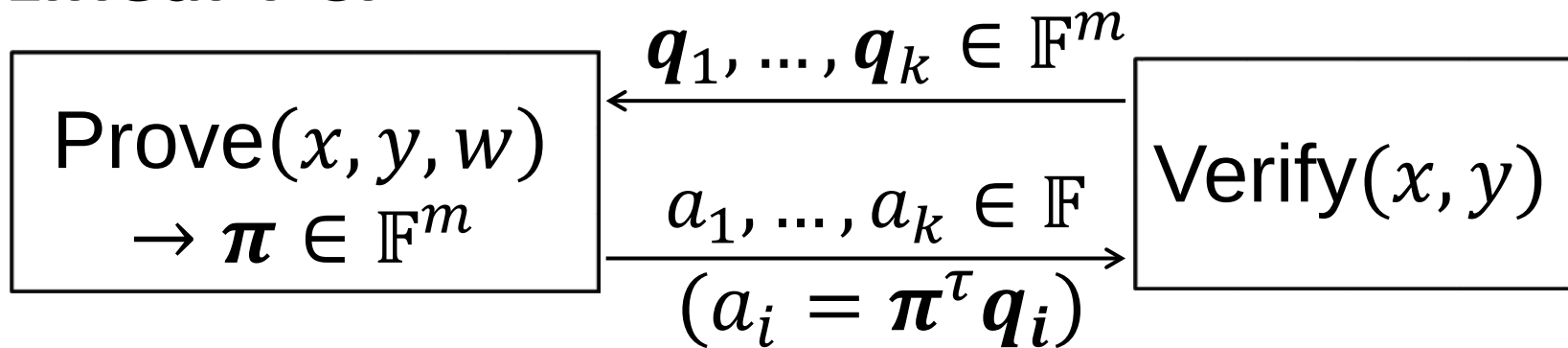
Honest-Verifier Zero Knowledge:

Prover adds random multiple of $V(\alpha)$ to $A, B, C(\alpha)$.

- ZK: The queries to $A_{x,y,z}$, $B_{x,y,z}$, $C_{x,y,z}$ return random independent $\mathbb{F}$ elements. The query to H follows from them. The x, y -consistency query is predictable.

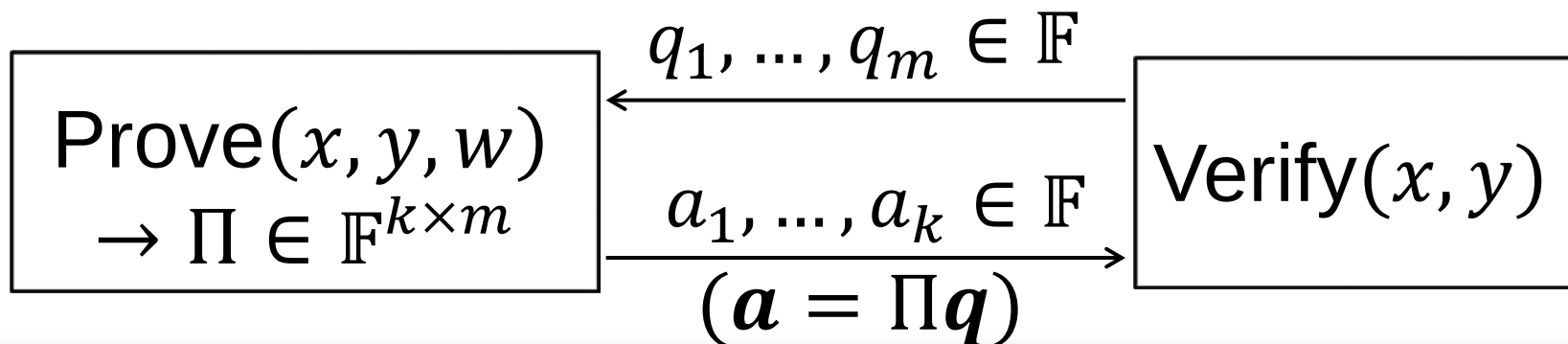
Linear PCP $\Rightarrow$ Linear Interactive Proof [BCIOP13]

Linear PCP



An LPCP is not automatically a LIP, because of consistency (different q_i may “see” different $\boldsymbol{\pi}$)

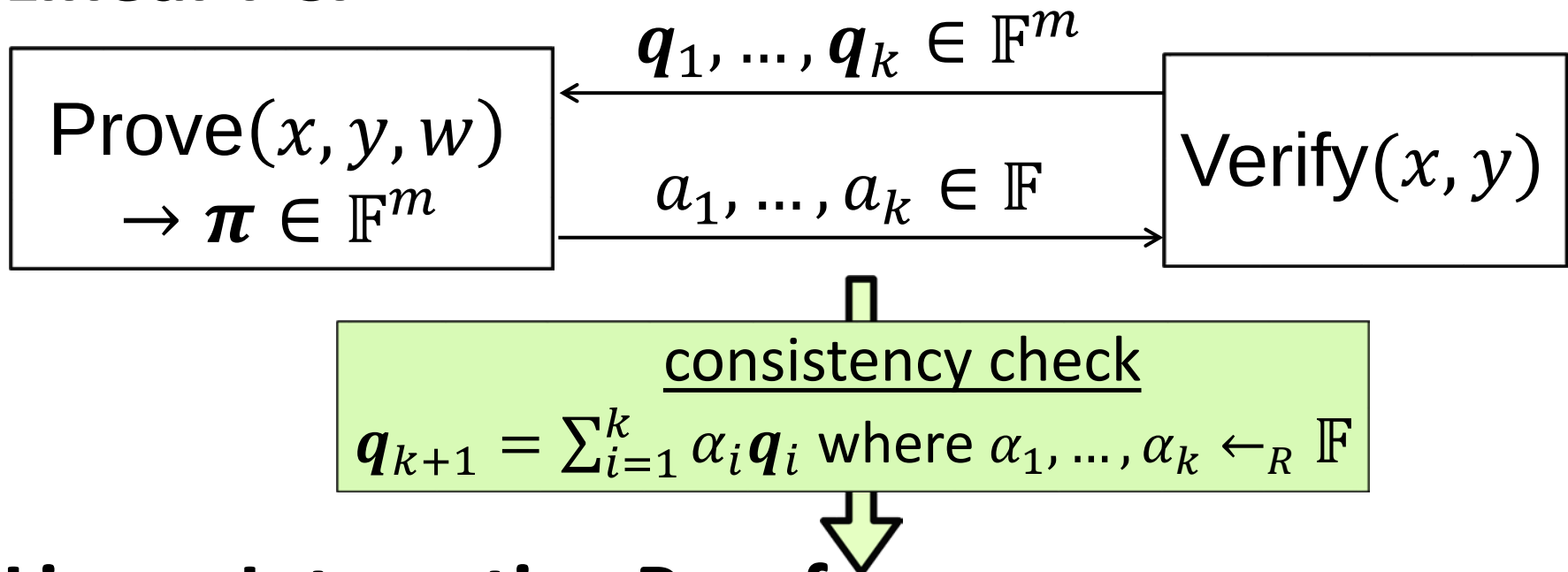
Linear Interactive Proof



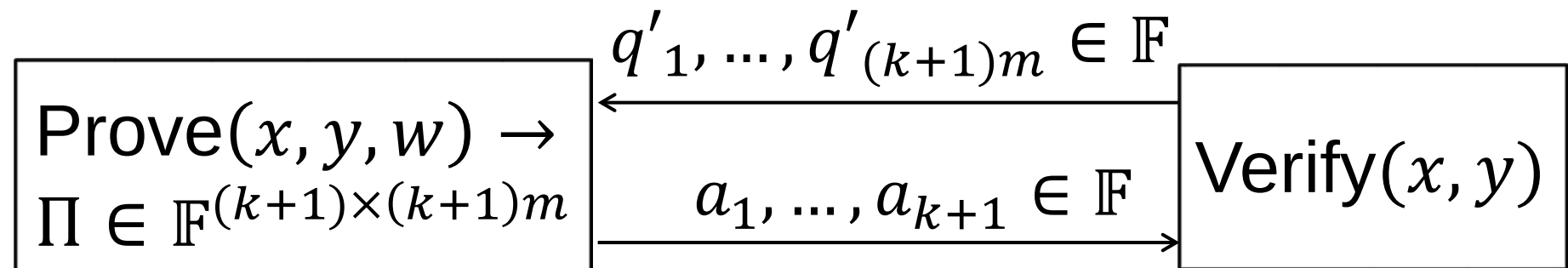
Linear PCP $\Rightarrow$ Linear Interactive Proof: adding a consistency check

[BCIOP13]

Linear PCP



Linear Interactive Proof

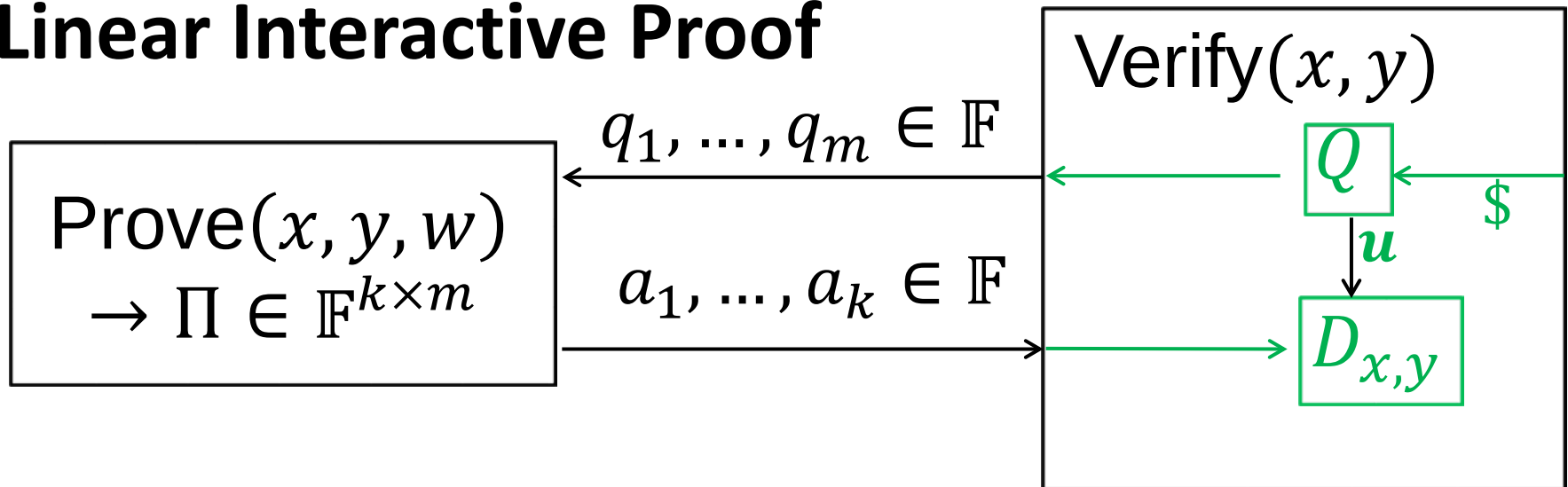


Linear Interactive Proof $\Rightarrow$ designated-verifier SNARK

[BCIOP13]

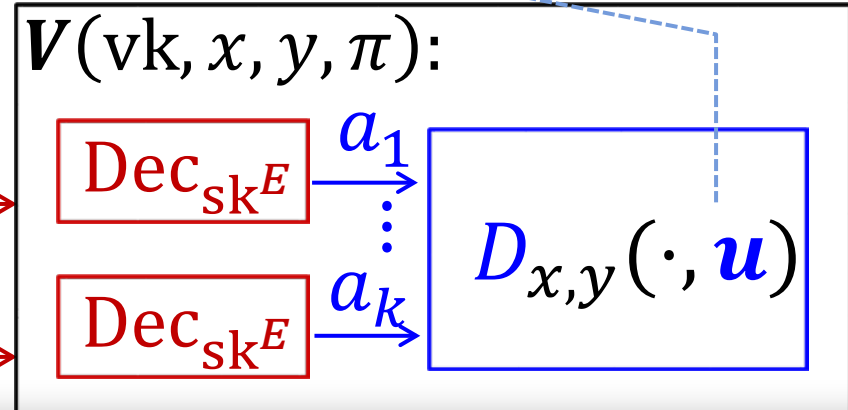
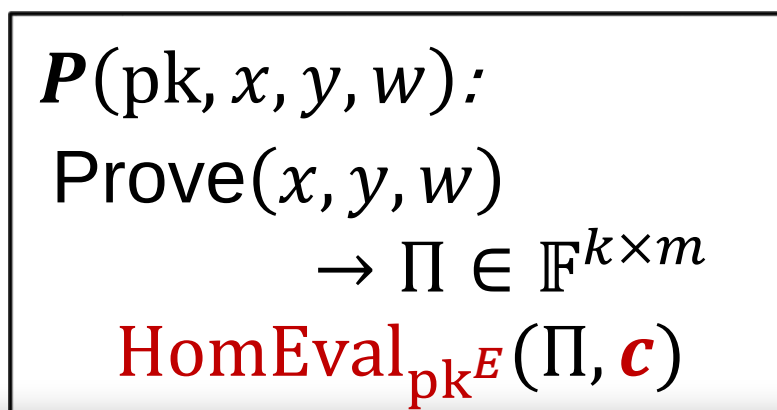
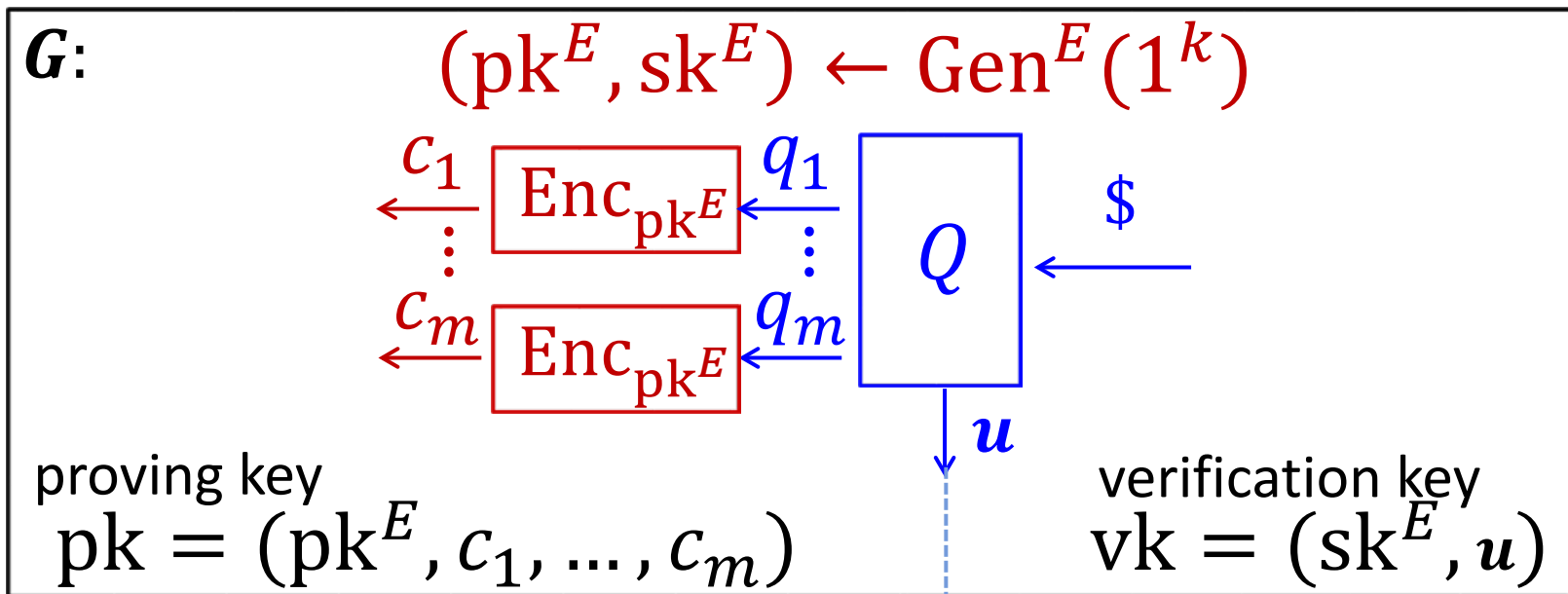
Input-Oblivious

Linear Interactive Proof



**Designated-verifier
SNARK (G, P, V)**

Linear Interactive Proof $\Rightarrow$ designated-verifier SNARK: construction



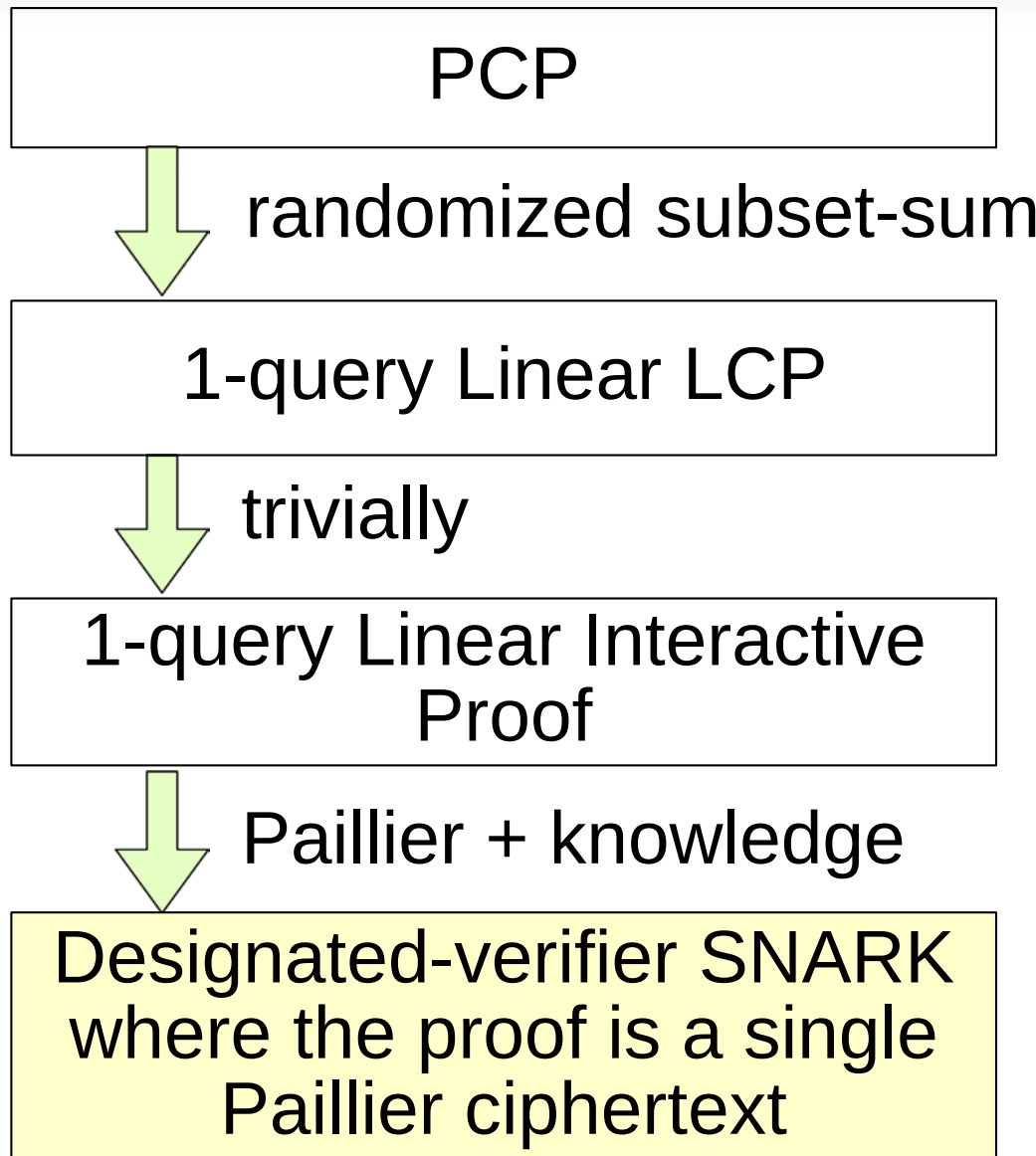
Linear Interactive Proof $\Rightarrow$ designated-verifier SNARK: assumption

Linear-Only Homomorphism ($\sim$ [BSW])
encryption scheme that ONLY allows
 $\mathbb{F}$ -linear homomorphic operations
(e.g., knowledge variant of Paillier)

} non-falsifiable
assumption

Can add ZK and PoK
with a bit more work.

Bonus: ultrasuccinct designated-verifier SNARK [BCIOP13]



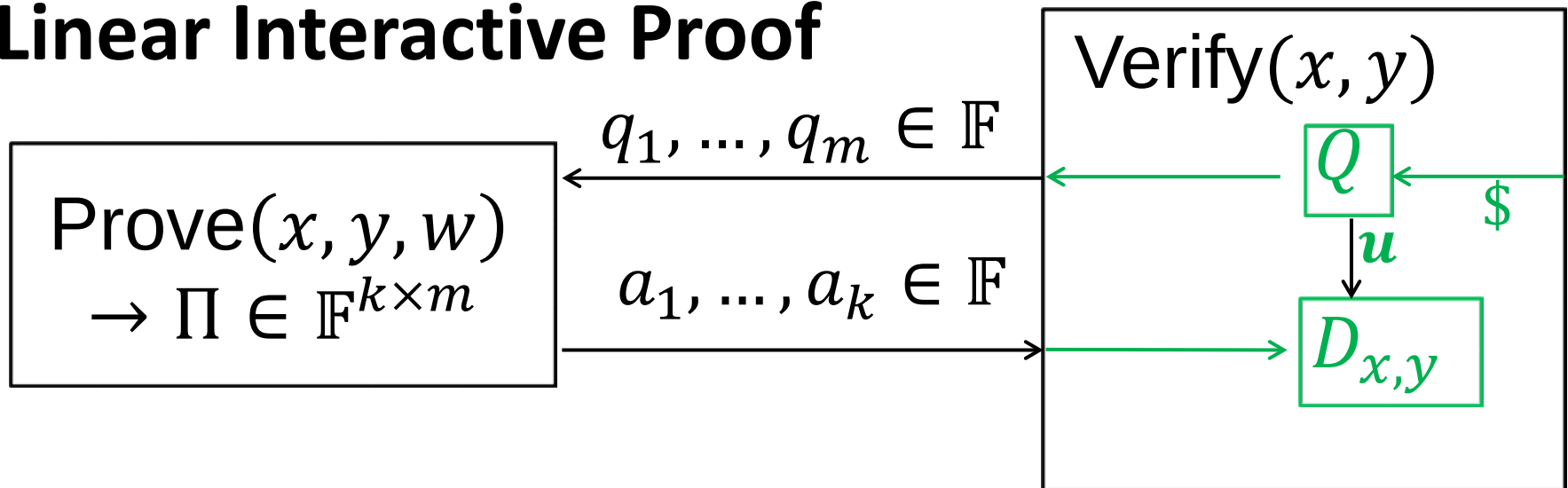
Efficiency caveat:
prover computes
a full-blown
“query-efficient”
PCP.

Linear Interactive Proof $\Rightarrow$ (publicly-verifiable) SNARK

[BCIOP13]

Input-Oblivious

Linear Interactive Proof



**(Publicly-verifiable)
SNARK (G, P, V)**

Linear Interactive Proof $\Rightarrow$ (publicly-verifiable) SNARK: approach [BCIOP13]

$\text{Enc}_{\text{pk}^E}(\cdot)$ satisfies - semantic security
- linear-only homomorphism

Being able to test properties of the prover's answers
implies that we must **give up** semantic security. Need:

$\text{Enc}_{\text{pk}}(\cdot)$ with

- quadratic predicate test
- linear-only homomorphism
- Δ -power one-wayness

← **Bilinear maps**
Dominates V runtime

← **Knowledge of Exponent**
Unfalsifiable.

Also need the LIP, hence, underlying Linear PCP,
to have low-degree Q and D algorithms. QAP works!

zkSNARK construction via QAP and Linear PCPs



**Frank Gehry No Longer
Allowed To Make
Sandwiches For
Grandkids**



- Computation
- Algebraic Circuit
- R1CS
- QAP
- Linear PCP
- Linear Interactive Proof
- zkSNARK

Full [PGHR13] protocol ([BCTV14]_{USENIX} variant)

Public parameters. A prime r , two cyclic groups $\mathbb{G}_1$ and $\mathbb{G}_2$ of order r with generators $\mathcal{P}_1$ and $\mathcal{P}_2$ respectively, and a pairing $e: \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ (where $\mathbb{G}_T$ is also cyclic of order r).

(a) Key generator G

- INPUTS: circuit $C: \mathbb{F}_r^n \times \mathbb{F}_r^h \rightarrow \mathbb{F}_r^l$
 - OUTPUTS: proving key pk and verification key vk
1. Compute $(\vec{A}, \vec{B}, \vec{C}, Z) := \text{QAPinst}(C)$; extend $\vec{A}, \vec{B}, \vec{C}$ via

$$A_{m+1} = B_{m+2} = C_{m+3} = Z,$$

$$A_{m+2} = A_{m+3} = B_{m+1} = B_{m+3} = C_{m+1} = C_{m+2} = 0.$$
 2. Randomly sample $\tau, \rho_A, \rho_B, \alpha_A, \alpha_B, \alpha_C, \beta, \gamma \in \mathbb{F}_r$.
 3. Set $\text{pk} := (C, \text{pk}_A, \text{pk}'_A, \text{pk}_B, \text{pk}'_B, \text{pk}_C, \text{pk}'_C, \text{pk}_K, \text{pk}_H)$ where for $i = 0, 1, \dots, m+3$:

$$\text{pk}_{A,i} := A_i(\tau)\rho_A\mathcal{P}_1, \quad \text{pk}'_{A,i} := A_i(\tau)\alpha_A\rho_A\mathcal{P}_1,$$

$$\text{pk}_{B,i} := B_i(\tau)\rho_B\mathcal{P}_2, \quad \text{pk}'_{B,i} := B_i(\tau)\alpha_B\rho_B\mathcal{P}_1,$$

$$\text{pk}_{C,i} := C_i(\tau)\rho_A\rho_B\mathcal{P}_1, \quad \text{pk}'_{C,i} := C_i(\tau)\alpha_C\rho_A\rho_B\mathcal{P}_1,$$

$$\text{pk}_{K,i} := \beta(A_i(\tau)\rho_A + B_i(\tau)\rho_B + C_i(\tau)\rho_A\rho_B)\mathcal{P}_1,$$
 and for $i = 0, 1, \dots, d$, $\text{pk}_{H,i} := \tau^i\mathcal{P}_1$.
 4. Set $\text{vk} := (\text{vk}_A, \text{vk}_B, \text{vk}_C, \text{vk}_\gamma, \text{vk}_{\beta\gamma}^1, \text{vk}_{\beta\gamma}^2, \text{vk}_Z, \text{vk}_{IC})$ where

$$\text{vk}_A := \alpha_A\mathcal{P}_2, \text{vk}_B := \alpha_B\mathcal{P}_1, \text{vk}_C := \alpha_C\mathcal{P}_2$$

$$\text{vk}_\gamma := \gamma\mathcal{P}_2, \quad \text{vk}_{\beta\gamma}^1 := \gamma\beta\mathcal{P}_1, \quad \text{vk}_{\beta\gamma}^2 := \gamma\beta\mathcal{P}_2,$$

$$\text{vk}_Z := Z(\tau)\rho_A\rho_B\mathcal{P}_2, \quad (\text{vk}_{IC,i})_{i=0}^n := (A_i(\tau)\rho_A\mathcal{P}_1)_{i=0}^n.$$
 5. Output (pk, vk) .

Key sizes. When invoked on a circuit $C: \mathbb{F}_r^n \times \mathbb{F}_r^h \rightarrow \mathbb{F}_r^l$ with a wires and b (bilinear) gates, the key generator outputs:

- pk with $(6a + b + n + l + 26)$ $\mathbb{G}_1$ -elements and $(a + 4)$ $\mathbb{G}_2$ -elements;
- vk with $(n + 3)$ $\mathbb{G}_1$ -elements and 5 $\mathbb{G}_2$ -elements.

Proof size. The proof always has 7 $\mathbb{G}_1$ -elements and 1 $\mathbb{G}_2$ -element.

(b) Prover P

- INPUTS: proving key pk , input $\vec{x} \in \mathbb{F}_r^n$, and witness $\vec{a} \in \mathbb{F}_r^h$
 - OUTPUTS: proof π
1. Compute $(\vec{A}, \vec{B}, \vec{C}, Z) := \text{QAPinst}(C)$.
 2. Compute $\vec{s} := \text{QAPwit}(C, \vec{x}, \vec{a}) \in \mathbb{F}_r^m$.
 3. Randomly sample $\delta_1, \delta_2, \delta_3 \in \mathbb{F}_r$.
 4. Compute $\vec{h} = (h_0, h_1, \dots, h_d) \in \mathbb{F}_r^{d+1}$, which are the coefficients of $H(z) := \frac{A(z)B(z) - C(z)}{Z(z)}$ where $A, B, C \in \mathbb{F}_r[z]$ are as follows:

$$A(z) := A_0(z) + \sum_{i=1}^m s_i A_i(z) + \delta_1 Z(z),$$

$$B(z) := B_0(z) + \sum_{i=1}^m s_i B_i(z) + \delta_2 Z(z),$$

$$C(z) := C_0(z) + \sum_{i=1}^m s_i C_i(z) + \delta_3 Z(z).$$
 5. Set $\tilde{\text{pk}}_A :=$ “same as pk_A , but with $\text{pk}_{A,i} = 0$ for $i = 0, 1, \dots, n$ ”.
Set $\tilde{\text{pk}}'_A :=$ “same as pk'_A , but with $\text{pk}'_{A,i} = 0$ for $i = 0, 1, \dots, n$ ”.
 6. Letting $\vec{c} := (1 \circ \vec{s} \circ \delta_1 \circ \delta_2 \circ \delta_3) \in \mathbb{F}_r^{4+m}$, compute

$$\pi_A := \langle \vec{c}, \tilde{\text{pk}}_A \rangle, \pi'_A := \langle \vec{c}, \tilde{\text{pk}}'_A \rangle, \pi_B := \langle \vec{c}, \text{pk}_B \rangle, \pi'_B := \langle \vec{c}, \text{pk}'_B \rangle,$$

$$\pi_C := \langle \vec{c}, \text{pk}_C \rangle, \pi'_C := \langle \vec{c}, \text{pk}'_C \rangle, \pi_K := \langle \vec{c}, \text{pk}_K \rangle, \pi_H := \langle \vec{h}, \text{pk}_H \rangle.$$
 7. Output $\pi := (\pi_A, \pi'_A, \pi_B, \pi'_B, \pi_C, \pi'_C, \pi_K, \pi_H)$.

(c) Verifier V

- INPUTS: verification key vk , input $\vec{x} \in \mathbb{F}_r^n$, and proof π
 - OUTPUTS: decision bit
1. Compute $\text{vk}_{\vec{x}} := \text{vk}_{IC,0} + \sum_{i=1}^n x_i \text{vk}_{IC,i} \in \mathbb{G}_1$.
 2. Check validity of knowledge commitments for A, B, C :

$$e(\pi_A, \text{vk}_A) = e(\pi'_A, \mathcal{P}_2), e(\text{vk}_B, \pi_B) = e(\pi'_B, \mathcal{P}_2),$$

$$e(\pi_C, \text{vk}_C) = e(\pi'_C, \mathcal{P}_2).$$
 3. Check same coefficients were used:

$$e(\pi_K, \text{vk}_\gamma) = e(\text{vk}_{\vec{x}} + \pi_A + \pi_C, \text{vk}_{\beta\gamma}^2) \cdot e(\text{vk}_{\beta\gamma}^1, \pi_B).$$
 4. Check QAP divisibility:

$$e(\text{vk}_{\vec{x}} + \pi_A, \pi_B) = e(\pi_H, \text{vk}_Z) \cdot e(\pi_C, \mathcal{P}_2).$$
 5. Accept if and only if all the above checks succeeded.

[PGHR13] assumptions

- q -power Diffie-Hellman
- q -strong Diffie-Hellman
- q -power Knowledge of Exponent

$q = \text{poly}(\text{circuit size})$

Assumption 2 (q -PKE [21]) *The q -power knowledge of exponent assumption holds for $\mathcal{G}$ if for all $\mathcal{A}$ there exists a non-uniform probabilistic polynomial time extractor $\chi_{\mathcal{A}}$ such that*

$$\begin{aligned} & \Pr[(p, \mathbb{G}, \mathbb{G}_T, e) \leftarrow \mathcal{G}(1^\kappa) ; g \leftarrow \mathbb{G} \setminus \{1\} ; \alpha, s \leftarrow \mathbb{Z}_p^* ; \\ & \quad \sigma \leftarrow (p, \mathbb{G}, \mathbb{G}_T, e, g, g^s, \dots, g^{s^q}, g^\alpha, g^{\alpha s}, \dots, g^{\alpha s^q}) ; \\ & \quad (c, \hat{c} ; a_0, \dots, a_q) \leftarrow (\mathcal{A} \parallel \chi_{\mathcal{A}})(\sigma, z) : \\ & \quad \hat{c} = c^\alpha \wedge c \neq \prod_{i=0}^q g^{a_i s^i}] = \text{negl}(\kappa) \end{aligned}$$

for any auxiliary information $z \in \{0, 1\}^{\text{poly}(\kappa)}$ that is generated independently of α . Note that $(y; z) \leftarrow (\mathcal{A} \parallel \chi_{\mathcal{A}})(x)$ signifies that on input x , $\mathcal{A}$ outputs y , and that $\chi_{\mathcal{A}}$, given the same input x and $\mathcal{A}$'s random tape, produces z .

zkSNARK backend implementations

- Pinocchio/Geppetto
<https://vc.codeplex.com>
[PGHR13] [CFHKKNPZ15]
- libsnark
github.com/scipr-lab/libsnark
[BCGTV13a] [BCTV14_{crypto}] [BCTV14_{usenix}] ...
- snarklib
github.com/jancarlsso/snarklib
(clone of libsnark with different C++ style by “Jan Carlsson”)

Numerous frontends (some included in the above), to be discussed tomorrow.

Example: libsnark backends

- [PGHR13] backend with [BCTV14_{USENIX}] improvements
 - speed of verifier by merging parts of the pairing computation
 - reduced verification key size to $\sim 1/3$ (when $\#inputs \ll \#gates$)
- Square Span Programs [DFGK14] backend
- ADSNARK backend, [BBFR15] backend
- Tailored libraries for finite fields, ECC, pairings

1M arithmetic gates, 1000-bit input, desktop PC	80-bit security	128-bit security
Generator	97 s	117 s
Prover	115 s	147 s
Verifier	4.9 ms ($4.7 + 0.0004 x $ ms)	5.1 ms ($4.8 + 0.0005 x $ ms)
Proof size	230 B	288 B

- Full code, MIT license github.com/scipr-lab/libsnark

Extensions/variants

- ADSNARK [BBFR15]
 - a preprocessing SNARKs for proving statements on authenticated data
- [DFGK14]
 - A preprocessing SNARK for the language Square Span Programs, more efficient for Boolean circuits