Session 1: Definitions and Oblivious Transfer

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Secure Multiparty Computation

- A set of parties with private inputs
- Parties wish to jointly compute a function of their inputs so that certain security properties are preserved
- Properties must be ensured even if some of the parties maliciously attack the protocol
- Can model any cryptographic task

Applications

- Elections
- Auctions
- Private database search
- Privacy-preserving data mining
- Secure set intersection
- Much much more…

Security Requirements

- **Consider a secure auction (with secret bids):**
  - An adversary may wish to learn the bids of all parties – to prevent this, require **PRIVACY**
  - An adversary may wish to win with a lower bid than the highest – to prevent this, require **CORRECTNESS**
  - But, the adversary may also wish to ensure that it always gives the highest bid – to prevent this, require **INDEPENDENCE OF INPUTS**
  - An adversary may try to abort the execution if its bid is not the highest – require **FAIRNESS**

General Security Properties

- **Privacy:** only the output is revealed
- **Correctness:** the function is computed correctly
- **Independence of inputs**: parties cannot choose inputs based on others' inputs
- **Fairness:** if one party receives output, all receive output
- **Guaranteed output delivery**

Defining Security

- **Option 1: analyze security concerns for each specific problem**
  - Auctions: as in previous slide
  - Elections: privacy, correctness and fairness only (?)
- **Problems:**
  - How do we know that all concerns are covered?
  - Definitions are application dependent and need to be redefined from scratch for each task

Defining Security

- **Option 2: general definition that captures all (most) secure computation tasks**
- **Properties of any such definition**
  - Well-defined adversary model
  - Well-defined execution setting
  - Security guarantees are clear and simple to understand

Modeling Adversaries

- **Adversarial behavior**
  - **Semi-honest:** follows the protocol specification
    - Tries to learn more than allowed by inspecting transcript
  - **Malicious:** follows any arbitrary strategy
  - **Covert:** follows any arbitrary strategy, but is averse to being caught…
- **Adversarial power**
  - **Polynomial-time:** computational security
  - **Computationally unbounded:** information-theoretic security

Modeling Adversaries

- **Corruption strategy**
  - **Static:** the set of corrupted parties is fixed before the execution begins
  - **Adaptive:** the adversary can corrupt parties during the execution, based on what has happened
    - Models modern “hacking”
    - Cannot use strategies that choose a small set of representatives to compute for all
    - In general, **much harder!**

Execution Setting

- **Stand-alone**
  - Consider a single protocol execution only (or that only a single execution is under attack)
- **Concurrent general composition**
  - Arbitrary protocols executed concurrently
  - Realistic setting, very important model
- **Stand-alone vs composition**
  - Stand-alone: a good place to start studying secure computation, techniques and tools are helpful
  - Composition: true goal for constructions

Feasibility of Secure Computation

- **Assuming an honest majority, any functionality can be securely computed**
  - Even information theoretically, and with adaptive security
- **Without an honest majority, it is impossible to achieve fairness in general**
- **Without an honest majority, any functionality can be securely computed without fairness**

Preliminaries

- **Notations:**
  - Security parameter $\boldsymbol{n}$
  - We wish security to hold for **all inputs** of all lengths, as long as $\boldsymbol{n}$ is large enough

- **Function $\boldsymbol{\mu}$ is negligible:** if for every polynomial $p(n)$ there exists an N such that for all $n > N$ we have $\mu(n) < \frac{1}{p(n)}$

Preliminaries

- **Probability ensemble $\boldsymbol{X = \{X(a,n)\}}$**
  - Infinite series, indexed by a string $\boldsymbol{a}$ and natural $\boldsymbol{n}$
  - Each $\boldsymbol{X(a,n)}$ is a random variable
    - In our context: output of protocol execution with input $\boldsymbol{a}$ and security parameter $\boldsymbol{n}$
    - Probability space: randomness of parties

Preliminaries

- **Computational indistinguishability $\boldsymbol{X \approx Y}$**
  - For every (non-uniform) polynomial-time distinguisher D there exists a negligible function μ such that for every $\boldsymbol{a}$ and all large enough $\boldsymbol{n}$'s:
    $|\Pr[D(X(a,n) = 1] - \Pr[D(Y(a,n) = 1]| < \mu(n)$

Notation

- **Functionality**
  - $\boldsymbol{f} = (\boldsymbol{f_1}, \ldots, \boldsymbol{f_m})$: for input vector $\boldsymbol{x}$, each $\boldsymbol{f_i}(\boldsymbol{x})$ is a random variable (for probabilistic functionalities)
  - Party $\boldsymbol{P_i}$ receives $\boldsymbol{f_i}$
  - We denote $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (\boldsymbol{f_1}(\boldsymbol{x}, \boldsymbol{y}), \boldsymbol{f_2}(\boldsymbol{x}, \boldsymbol{y}))$

Semi-Honest Adversaries

- **Simulation:**
  - Given input and output, can generate the adversary's view of a protocol execution
  - Important: since parties follow protocol, the inputs are **well defined**

Semi-Honest Adversaries

- **For every** **semi-honest A**, **there exists a** **simulator S** **such that for every set of corrupted parties I and every vector of inputs x, the following are** ***computationally indistinguishable***
  - The output of $\boldsymbol{A}$, and the outputs of all parties after a protocol execution
  - The output of $\boldsymbol{S}$ given $\boldsymbol{x_i}$ and $\boldsymbol{f_i(x)}$ for all $\boldsymbol{i \in I}$, and all the values $\boldsymbol{f_1(x), \dots, f_m(x)}$

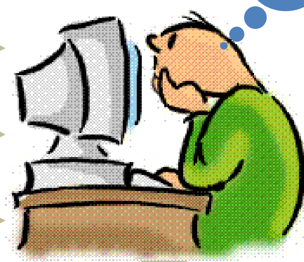
Semi-Honest Adversaries

The REAL execution

x

Protocol

y



Arbitrary Output
(w.l.o.g.: adversary's view in execution)

$f(x, y)$

Simulation

$x, f(x, y)$

Simulator Output

$f(x, y)$

Properties

- **Correctness, independence of inputs, fairness are all non-issues in the semi-honest model**
- **Why is privacy guaranteed by this definition?**
  - The adversary’s view in an execution can be generated from the input and output only
  - If the adversary can compute something after a real protocol execution, it can compute it just from the input/output
  - Very similar to zero-knowledge

Joint Distribution

- **A crucial point: need to consider the joint distribution of adversary's output and honest parties' output**
- **In the definition:**
  - We compare the distribution of all inputs and outputs together with the adversary's output

Joint Distribution

- **Example:**
  - **Functionality:** $\boldsymbol{A}$ outputs random bit, $\boldsymbol{B}$ outputs nothing
    - $\boldsymbol{B}$ should clearly not learn $\boldsymbol{A}$'s output bit
  - **Protocol:** $\boldsymbol{A}$ chooses a random bit, outputs it, and sends the bit to $\boldsymbol{B}$ (who ignores it)
- **This is simulatable when separately looking at distribution of $\boldsymbol{B}$'s view and actual outputs**

Deterministic Functionalities

- **In the case of deterministic functionalities, the outputs are fully determined by the inputs**
- **It suffices to separately prove**
  - Correctness
  - Simulation: can generate view of semi-honest adversary (corrupted parties' view), given inputs and outputs only
    - This is significantly easier!

Malicious Adversaries

- **First attempt: require the existence of a simulator that generates the adversary's view given the inputs/outputs of corrupted**
- **Problem: what are the inputs used by the adversary?**
  – They are not necessarily those written on the input tape
  – They are not explicit: the adversary doesn't run the protocol but arbitrary code

Malicious Adversaries

- **We also need to require independence of inputs, correctness, fairness etc.**
  - These properties are not captured by “view simulation” alone
- **Can we separate correctness and privacy?**
  - Instead of computing f, compute a function that reveals first input bit of other party
  - Correctness or privacy???
- **What about independence of inputs and privacy?**

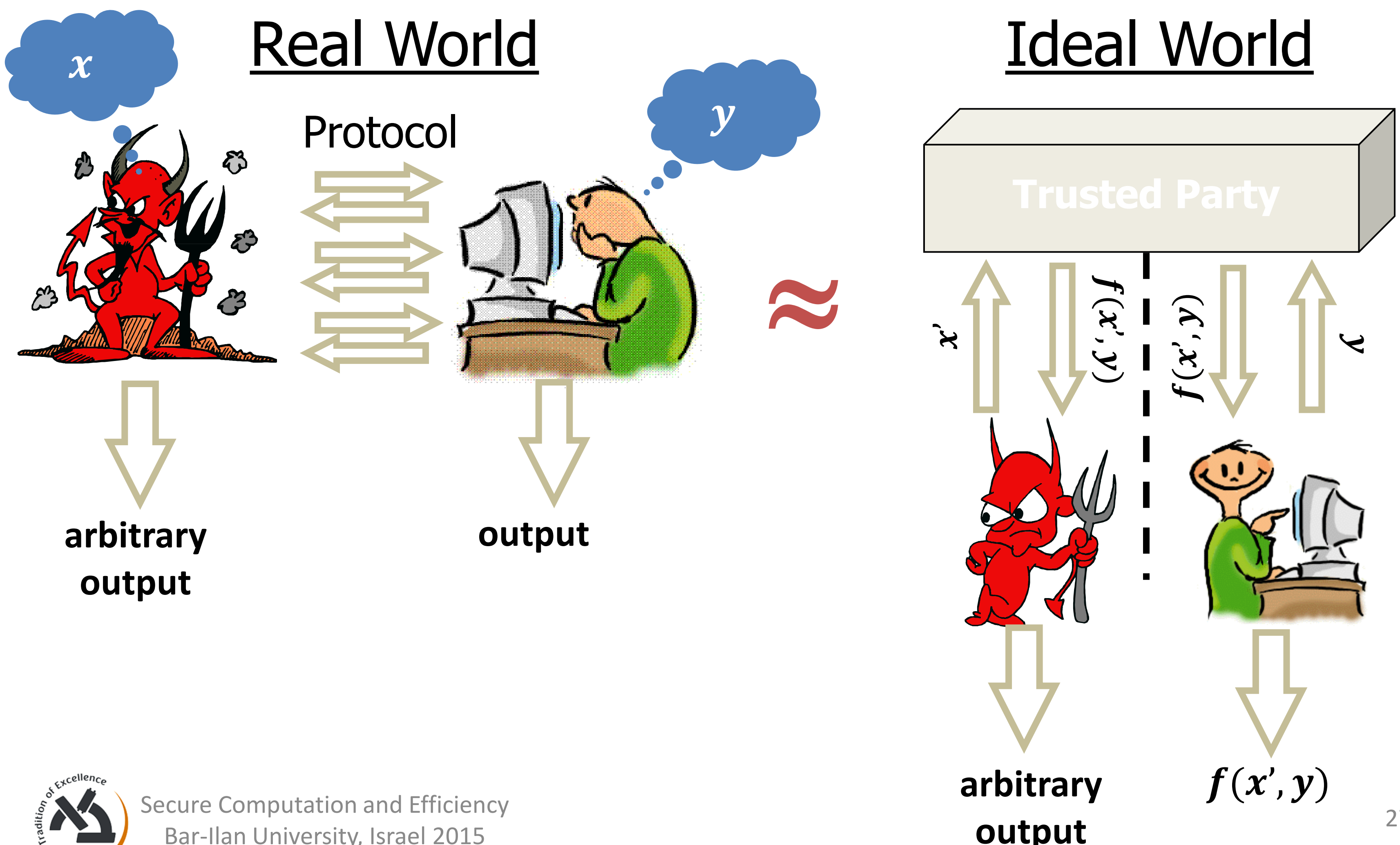
The Ideal/Real Paradigm

- **What is the best we could hope for?**
  - An incorruptible trusted party
  - All parties send inputs to trusted party (over perfectly secure communication lines)
  - Trusted party computes output
  - Trusted party sends each party its output (over perfectly secure communication lines)
  - This is an **ideal world**
- **What can an adversary do?**
  - Just choose its input…

The Ideal/Real Paradigm

- **The real protocol must be like the ideal world**
- **Formalizing this notion:**
  - For *every adversary* ***A*** attacking the real protocol, *there exists an adversary* ***S*** in the ideal model such that the output distributions (of all) are computationally indistinguishable
  - **S** simulates a real protocol execution while interacting in the ideal world
  - Here we always look at the joint output distribution

The Ideal/Real Paradigm



"Formal" Security Definition

- **Protocol π securely computes a function f if:**
  - For every non-uniform polynomial-time real-model adversary A, there exists a non-uniform polynomial-time ideal-model adversary S, such that for all input vectors and auxiliary inputs:
  - the joint outputs of A and the honest parties in a real execution of π is computationally indistinguishable from the joint outputs of S and the honest parties in an ideal execution where the trusted party computes f

Properties

- **The following properties hold**
  - **Privacy**: from adversary's outputs
  - **Correctness**: from honest parties' outputs
  - Independence of inputs: from ideal execution
  - Fairness and guaranteed output delivery: from ideal execution
  - More?

Relaxing the Ideal Model

- In some cases, this ideal model is too strong and cannot be achieved
- Fairness cannot be achieved in general without an honest majority

Relaxing the Ideal Model

- **Change the instructions of the trusted party**
  - Trusted party receives input from all parties
  - Trusted party sends corrupted parties' outputs to adversary
  - Adversary says "continue" or "halt"
  - If "continue", trusted party sends output to honest parties; else, it sends "abort"

Reactive Functionalities

- **Functionalities that obtain inputs and provide outputs in stages**
- **Examples:**
  - Mental poker
  - Commitment schemes
- **This is also useful for relaxing ideal functionalities (give side information to S)**
- **The definition extends naturally to this as well**

Advantages of This Approach

- **General – it captures ALL applications**
- **The specifics of an application are defined by its functionality, security is defined as above**
- **The security guarantees achieved are easily understood (because the ideal model is easily understood)**
  – We can be confident that we did not “miss” any security requirements

Using Secure Computation

- **The ideal-model paradigm**
  - You don't need to understand anything about how a protocol works to use it
  - You just need to imagine an incorruptible trusted party computing the functionality for you

- **Very advantageous for usage**

Sequential Modular Composition

- **Sequential modular composition:**
  - Secure protocols are run sequentially, with arbitrary messages sent in between them
- **Why consider this?**
  - An important security goal within itself
  - Very helpful (if not crucial) tool for analyzing the security of protocols
- **Formalization – Hybrid Model**
  - A trusted party helps to compute a sub-functionality
  - REAL messages & IDEAL messages

Sequential Modular Composition

- Subprotocols ρ_i securely compute functionalities f_i
- Protocol π securely computes g in a hybrid model where a trusted party is used to compute every f_i
  - This is much easier to analyze since each $\boldsymbol{f_i}$ is effectively "perfectly secure"
- **Theorem:** assuming the above, the real protocol π^ρ that uses real calls to each ρ_i instead of a trusted party for f_i, securely computes g.

Concurrent Composition

- **We have considered the stand-alone model**
  - This implies sequential composition
- **What about concurrent composition?**
  - An Internet-like setting where many (arbitrary, secure and insecure) protocols are run concurrently, with the adversary controlling the scheduling
- **This models the real-world setting more accurately**
  - We don’t know what the result is of running stand-alone protocols concurrently with related inputs

Concurrent Composition

- **Concurrent general composition**
  - Strictly harder than the stand-alone model
  - ***Impossible*** without some trusted set-up assumption (like a common reference string)
- **The UC definition (universal composability) guarantees security in this setting**
  - Efficient UC security is a special challenge…
- **Recommended to study UC next, after studying the stand-alone setting**

Relaxed Definitions

- **In order to achieve high efficiency, sometimes can consider weaker definitions**
  - Semi-honest (but this is very weak)
  - Covert adversaries: adversary may be malicious but is guaranteed to be caught cheating with good probability
    - Suitable where adversaries can be penalized for being caught cheating (e.g., business loss)
  - Privacy only (malicious)
    - Problematic…

Summary

- **Semi-honest: simulator given input/output generates the adversary's view**
  - Probabilistic functionalities – must consider joint distribution of view and outputs
  - Deterministic functionalities: easier, suffices to separately consider correctness and view simulation
- **Malicious: ideal-real simulation**
- **Sequential composition**
- **Advanced topics**
  - Concurrent composition
  - Relaxed definition
  - Semi-honest vs malicious

General vs Specific Protocols

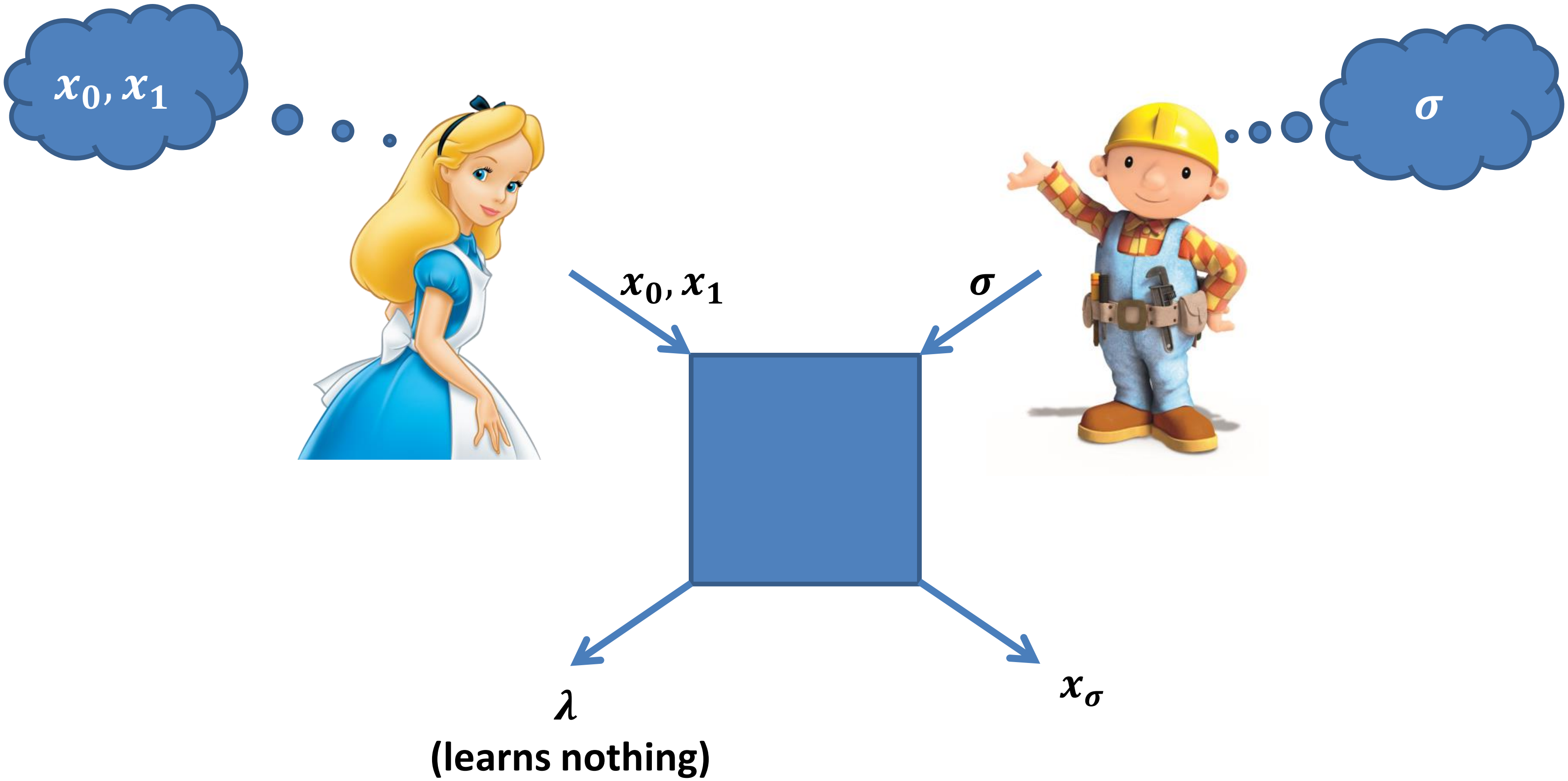
- **Most of the school will focus on general protocols**
  - Convert the function into a Boolean or arithmetic circuit
  - Compute the circuit securely
- **It seems that for specific problems, specific protocols should be more secure**

General vs Specific Protocols

- **General protocols – advantages**
  - Implement once
  - Very flexible: almost no difference between
    - Set intersection
    - Size of set intersection
    - Output 1 if set intersection size is greater than k
  - In many cases is competitive, and in fact the fastest solution known

OBLIVIOUS TRANSFER

Oblivious Transfer (OT)



Called 1-out-of-2 oblivious transfer (OT_1^2)

Fundamental Primitive

- **OT is complete**
  - If can compute OT then can compute any functionality
- **Constructing OT**
  - OT cannot be constructed from PKE in a black box manner
  - Can be constructed from
    - Enhanced trapdoor permutations
    - DDH, RSA, Lattices

Just a Few Important OT Results

- OT is symmetric
- Can construct efficient OT_1^N and OT_k^N from OT_1^2
- Can construct malicious OT from semi-honest OT in a black-box manner (inefficiently)
- Many variants of OT are equivalent
  - Random OT
  - Rabin OT
  - Weak OT

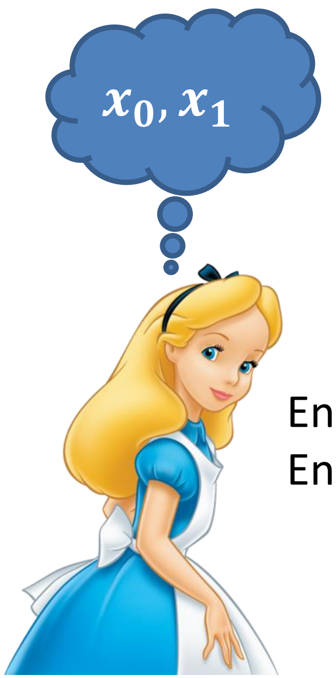
Efficient OT from DDH

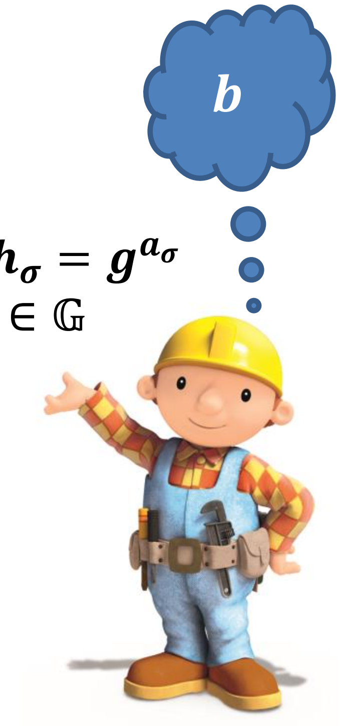
- **Recall the DDH assumption over a group $\mathbb{G}$ of order q with generator g**
  - The DDH assumption says that
    $$\{(g, g^a, g^b, g^{ab})\} \approx \{(g, g^a, g^b, g^c)\}$$
    where $a, b, c \leftarrow \mathbb{Z}_q$ are random

Semi-Honest OT

- **Recall ElGamal encryption**
  - **Secret key**: random $a \leftarrow \mathbb{Z}_q$
  - **Public key**: $h = g^a$
  - **Encrypt** $\boldsymbol{m} \in \mathbb{G}$: $c = (u, v) = (g^r, h^r \cdot m)$, random $r \in \mathbb{Z}_q$
  - **Decrypt** $(\boldsymbol{u}, \boldsymbol{v})$: compute $m = \frac{v}{u^a}$
    - Note: $\frac{v}{u^a} = \frac{h^r \cdot m}{(g^r)^a} = \frac{h^r \cdot m}{(g^a)^r} = \frac{h^r \cdot m}{h^r} = m$

Semi-Honest OT





Choose $\mathbf{a}_{\sigma}$; compute $\boldsymbol{h}_{\sigma} = \boldsymbol{g}^{\boldsymbol{a}_{\sigma}}$
Choose random $\boldsymbol{h}_{1-\sigma} \in \mathbb{G}$

$$\boldsymbol{h}_0, \boldsymbol{h}_1 \longleftarrow$$

Encrypt $\boldsymbol{x}_0$ with $\boldsymbol{h}_0$
Encrypt $\boldsymbol{x}_1$ with $\boldsymbol{h}_1$

$$\xrightarrow{\boldsymbol{c}_0, \boldsymbol{c}_1}$$

Decrypt $\boldsymbol{c}_{\sigma}$ with $\boldsymbol{a}_{\sigma}$

Note:

- Encrypt $\boldsymbol{x}_0$ with $\boldsymbol{h}_0$: $(u_0, v_0) = (g^r, (h_0)^r \cdot x_0)$
- Encrypt $\boldsymbol{x}_1$ with $\boldsymbol{h}_1$: $(u_1, v_1) = (g^s, (h_1)^s \cdot x_1)$

Semi-Honest OT – Security

- **Security:**
  - Alice sees only two public keys, which are two random group elements (and so learns nothing about σ)
    - Formally, simulate by sending two random group elements
  - Bob knows only one private key and so learns only x_σ
    - Formally, simulate by encrypting x_σ with h_σ, and encrypting garbage (e.g., 0) with $h_{1-\sigma}$

Choose $\mathbf{a}_\sigma$; compute $\boldsymbol{h}_\sigma = \boldsymbol{g}^{\boldsymbol{a}_\sigma}$
Choose random $\boldsymbol{h}_{1-\sigma} \in \mathbb{G}$

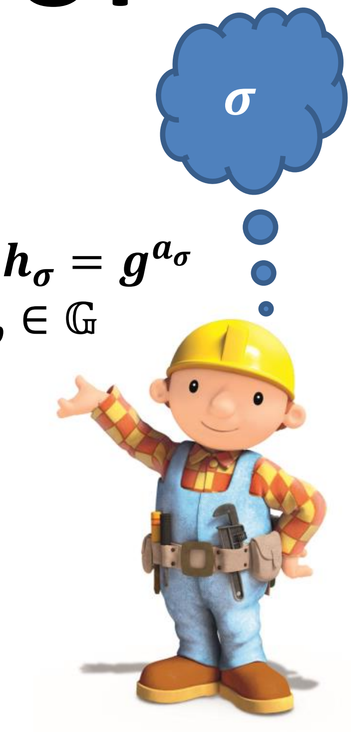
$\xleftarrow{\boldsymbol{h}_0, \boldsymbol{h}_1}$

Encrypt $\boldsymbol{x}_0$ with $\boldsymbol{h}_0$
Encrypt $\boldsymbol{x}_1$ with $\boldsymbol{h}_1$

$\xrightarrow{\boldsymbol{c}_0, \boldsymbol{c}_1}$

Decrypt $\boldsymbol{c}_\sigma$ with $\boldsymbol{a}_\sigma$

More Efficient Semi-Honest OT



Choose $\mathbf{a}_{\boldsymbol{\sigma}}$; compute $\boldsymbol{h_\sigma = g^{a_\sigma}}$
Choose random $\boldsymbol{h_{1-b}} \in \mathbb{G}$

$\boldsymbol{h_0, h_1}$ ←

Choose $r \leftarrow \mathbb{Z}_q$
Compute $u = g^r$
Compute $v_0 = (h_0)^r \cdot x_0$
Compute $v_1 = (h_1)^r \cdot x_1$

$\boldsymbol{u, v_0, v_1}$ →

Output $\boldsymbol{x_\sigma = \frac{v_\sigma}{u^{a_\sigma}}}$

Malicious Adversaries

- **Corrupted sender:**
  - Sender cannot cheat
  - Simulator can "extract" both x_0, x_1 by choosing both h_0 and h_1 so that it knows the secret keys
- **Corrupted receiver:**
  - Receiver can choose both h_0 and h_1 so that it knows the secret keys

Preventing Malicious

- **The idea:**
  - Alice sends a random group element w
  - Bob chooses h_0, h_1 so that $h_0 \cdot h_1 = w$
    - Bob can easily do this by choosing a_σ, computing $h_\sigma = g^{a_\sigma}$ and setting $h_{1-\sigma} = w/h_\sigma$
    - Bob cannot know both DLOGs of h_0, h_1 or it can compute the DLOG of H
- **Encryption uses a random oracle since “not completely knowing” a secret key doesn’t suffice**
  - Encrypt by $(g^r, HASH((h_0)^r) \oplus x_0)$,…

State of the Art – OT

- **Semi-honest adversaries**
  - Receiver: 2 exponentiations + send 2 group elements
  - Sender: 3 exponentiations + send 3 group elements
- **Malicious adversaries (Random Oracle)**
  - Same as semi-honest
- **Malicious adversaries (PVW)**
  - Receiver: 3 exponentiations + send 2 group elements
  - Sender: 8 exponentiations (effectively 6) + send 4 group elements

Proving Malicious Security

- **Proving security in the malicious model is tricky and subtle**
- **The ideal/real model paradigm**
  - Need a simulator who internally runs the real adversary and externally interacts with the trusted party (sending input and getting output)
  - The simulator needs to "**extract**" the real adversary's input, get output, and make the output match
- **We demonstrate the ideal/real proof technique for the problem of coin tossing**

Proving Malicious Security

- **Blum's protocol** **(with ElGamal):**
  - Party P_1:
    - Choose random $b \in \{0,1\}$ and $r, s \leftarrow \mathbb{Z}_q$
    - Compute $h = g^r, u = g^s, v = h^s \cdot g^b$
    - Send (h, u, v) to P_2
  - Party P_2:
    - Choose random $b' \in \{0,1\}$
    - Send b' to P_1
  - Party P_1 sends r, s, b to P_2
  - Party P_2 verifies that $h = g^r, u = g^s, v = h^s \cdot g^b$
  - Both parties output $b \oplus b'$

Intuition

- **Consider a corrupt P_2**
  - By the security of El Gamal encryption, it knows nothing about b when it chooses b'
- **Consider a corrupt P_1**
  - The values (h, u, v) fully define b
    - There exists a single pair (r, s) so that $h = g^r, u = g^s$
    - The value v can either be h^s or $h^s \cdot g$, but **not both**
  - P_2 chooses b' after P_1 sends b; by the above, P_1 cannot change b and so P_1 cannot bias the output

Proving Security - P_1 corrupted

- Let A be an adversary; S works as follows
- S receives a random bit β from the trusted party
- S invokes A and receives (h, u, v)
- S works as follows:
  - S ***internally*** *hands* A the value $b' = 0$
  - S **rewinds** A and internally hands A the value $b' = 1$
  - If A replies correctly both times, S learns the value b, sets $b' = b \oplus \beta$, and outputs this as A's view. In addition, A ***externally*** sends continue to the TTP
  - If A does **not** reply correctly either time, S sends abort to the TTP and outputs a random b' as A's view
  - If A aborts once, then S learns the value b, sets $b' = b \oplus \beta$, and outputs this as A's view. If A aborts on this b' then S sends sends abort to the TTP; else it sends continue to the TTP

Proving Security - P_2 corrupted

- Let A be an adversary; S works as follows
- S receives a random bit β from the trusted party
- S invokes A and works as follows:
  - S chooses a random b and internally hands A the tuple (h, u, v) computed correctly for b
  - S receives b' from A
  - If $b \oplus b' = \beta$ then S outputs (h, u, v) and (r, s, b) as its view, and sends continue to the TTP
  - Else, S **rewinds** A and goes to the beginning again
- Note: there is no abort here since we can just take $b' = 0$ as default if P_2 doesn't respond

Summary

- **Oblivious transfer is a fundamental primitive**
  - It is heavily used in most general secure computation protocols
- **Oblivious transfer is very efficient**
  - But it does cost exponentiations every time!
  - This afternoon we will see how to improve this